Stochastic Dynamic Programming for Network Resource Allocation

Yahong Rosa Zheng

Department of Electrical & Computer Engineering
Missouri University of Science and Technology, USA

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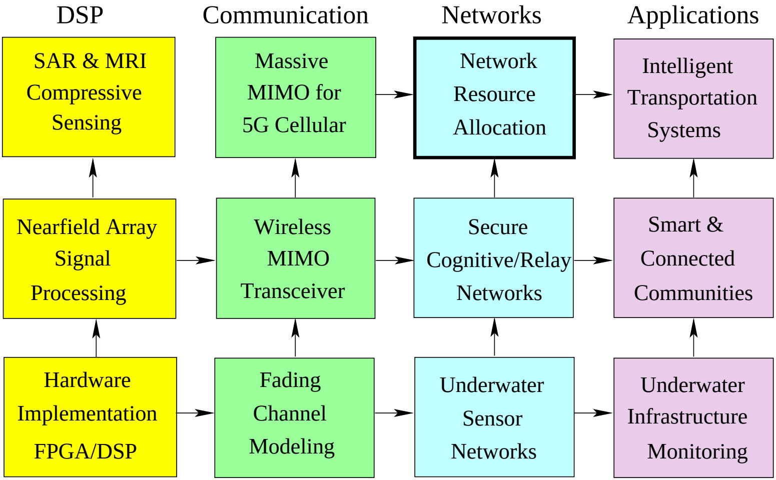
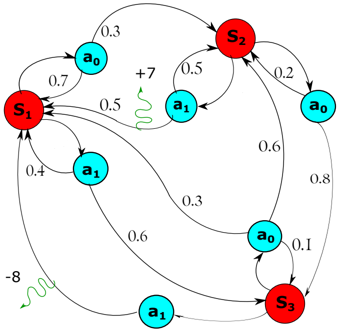
Summary of My Research Projects

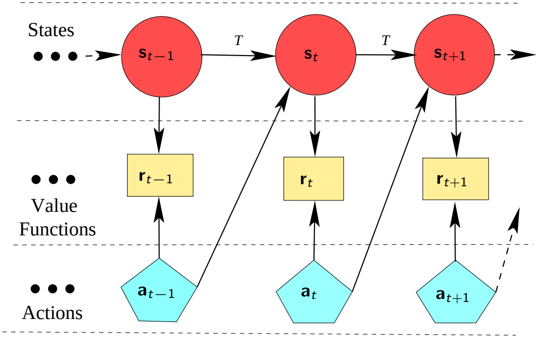


Markov Decision Process

- States: $S_1, S_2, S_3, \cdots$ and Actions: $a_0, a_1, \cdots$;
- Transition Probabilities: $T(S_j|a_i, S_i)$
  – probability of transition to State S_j from State S_i by taking Action a_i;
- Expected Rewards: $R(S_j|a_i, S_i)$ - reward of transition to State S_j by taking Action a_i at State S_i.



State Transition Diagram



MDP model

Solution to an MDP

A policy $\pi : S \rightarrow A$ maps each state to a desirable action.
Optimal solution optimizes the expected discounted rewards over a horizon $[0, T]$:

$$\sum_{t=0}^{T} \gamma^t R\left(s_{t+1} | \pi(s_t), s_t\right)$$

Value function of State s is defined as

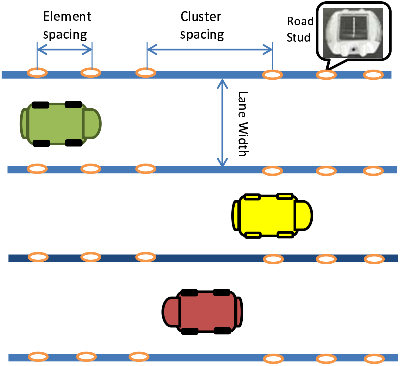
$$V(s) = \sum_{s'} T(s'|a, s)\left[R(s'|a, s) + \gamma V(s')\right]$$

Bellman equation: value iteration

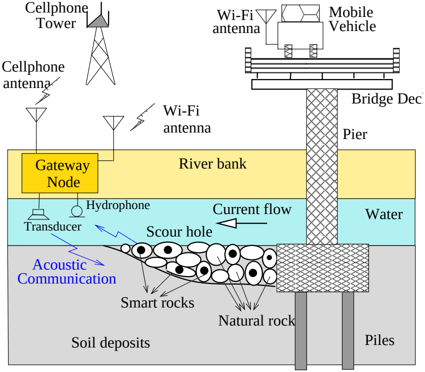
$$V_{i+1}(s) = \max_{a} \left\{ \sum_{s'} T(s'|a, s)\left[R(s'|a, s) + \gamma V_i(s')\right] \right\}$$

Applications: Wireless Network with Energy Harvesting

- Wireless Internet of Things (IoT) for ITS;
- Smart sensors for underwater infrastructure monitoring;
- Cellular base stations powered by renewable energy.



IoT for Intelligent Transportation Systems



Smart Rocks for Bridge Scour Monitoring

Network Resource Allocation

Design Goals:

- Utilize harvested energy as much as possible;
- Ensure network performance.

Research Challenges:

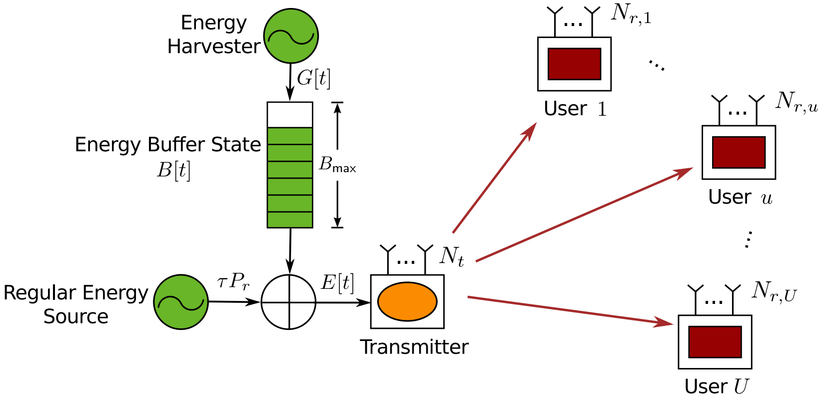
- Harvested energy is non-stable: follows statistical models;
- Mobile communication channels are time-variant and stochastic.

Existing Approaches:

- Online vs. Offline $\Rightarrow$ Dynamic vs. Static;
- Channel State Information: Instantaneous vs. Statistics
- Gaussian Inputs vs. Finite Alphabet Inputs

Wireless Base Station Powered by Renewable Energy Sources

System Model



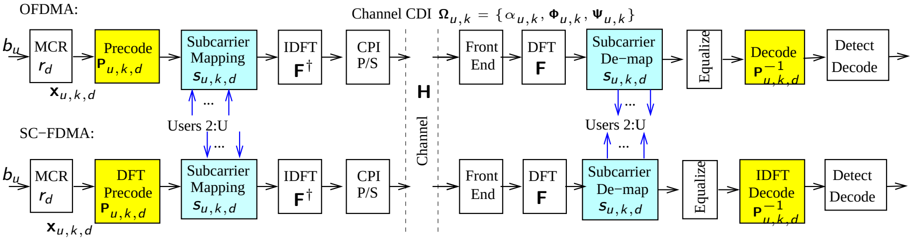
$$\mathbf{y}_{u,k,d}^{(i)} = \mathbf{H}_{u,k}^{(i)} \cdot \mathbf{P}_{u,k,d} \cdot \mathbf{x}_{u,k,d}^{(i)} + \mathbf{n}_{u,k}^{(i)} \quad \text{with} \quad \mathbf{H}_{u,k}^{(i)} = \sqrt{\alpha_{u,k}} \cdot \mathbf{\Psi}_{u,k}^{\frac{1}{2}} \mathbf{\Theta}_{u,k}^{(i)} \mathbf{\Phi}_{u,k}^{\frac{1}{2}}$$

i : index for channel realization; $u = 1 : U$ user index

$k = 1 : K$ subchannel index; $d = 1 : D$ MCR index.

MIMO Communication Networks

OFDMA – Orthogonal Frequency-Division Multiple Access
SC-FDMA – Single-Carrier Frequency-Division Multiple Access



Spectral efficiency $r_d = c_d \cdot N_t \cdot \log_2 M_d$.
where c_d – coding rate; M_d – Constellation Size of Modulation;
Linear precoders $\mathbf{P}_{u,k,d}$: $N_t \times N_t$ complex matrices;
Binary indicators $s_{u,k,d}[t] \in \{0, 1\}$ – resource assignment table

Objectives of Resource Allocation

Network Throughput = Expected Sum of Average Mutual Information (AMI)

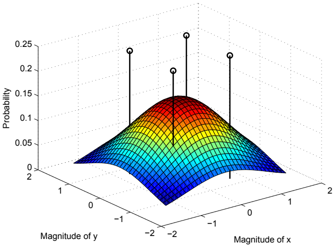
$$\mathcal{I}(\mathbf{P}_{u,k,d}|\mathbf{\Omega}_{u,k}) = N_t \log M_d - \frac{1}{M_d^{N_t}} \sum_{m=1}^{M_d^{N_t}} \mathsf{E}_{\mathbf{H}_{u,k}} \mathsf{E}_{\mathbf{n}_{u,k}} \log \sum_{n=1}^{M_d^{N_t}} e^{v_{mn}/\sigma^2}$$

where $v_{mn} = \|\mathbf{H}_{u,k}\mathbf{P}_{u,k,d}(\mathbf{x}_{m,d} - \mathbf{x}_{n,d}) + \mathbf{n}_{u,k}\|^2 - \|\mathbf{n}_{u,k}\|^2$.

Contrast to Gaussian Inputs with Instantaneous Channel State Information:

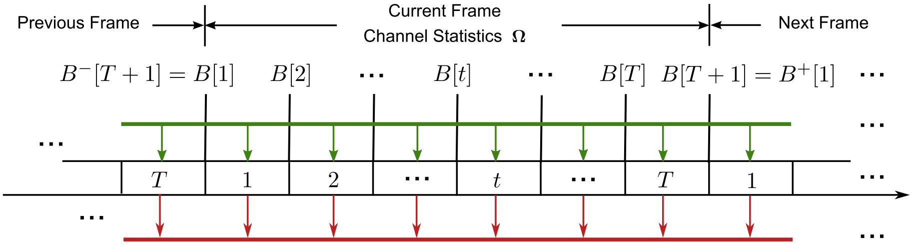
$$\mathcal{I}^{\mathcal{G}}(\mathbf{P}_{u,k}|\mathbf{H}_{u,k}) = \log\det\left(\mathbf{I} + \mathbf{H}_{u,k}\mathbf{P}_{u,k}\mathbf{P}_{u,k}^{\dagger}\mathbf{H}_{u,k}^{\dagger}/\sigma^2\right)$$

Solution is classic waterfilling for Gaussian inputs or mercury waterfilling for finite alphabet.
However, Rx cannot distinguish symbol from noise;
Channel estimated at t_1 is different from that at t_2.



QPSK modulation vs. Gaussian inputs

Online vs. Offline



Energy dynamics: $B[t+1] = \min(B[t] + G[t] - (E[t] - \tau P_r)^+, B_{\max})$;
Battery states: $B[t] = B_1 : B_M$; Harvested energy $G[t] = G_1 : G_J$;
Actions: energy allocation $E[t] = E_1 : E_N$;

where

$$E[t] = \sum_{u=1}^{U} \sum_{k=1}^{K} \sum_{d=1}^{D} s_{u,k,d}[t] \cdot E_{u,k,d}[t]$$

with

$$E_{u,k,d}[t] = \tau \cdot \mathrm{Tr}\left(\mathbf{P}_{u,k,d}[t] \cdot \mathbf{P}^{\mathbf{H}}_{u,k,d}[t]\right)$$

Online Resource Allocation

SDP:

$$\underset{\mathcal{P},\mathcal{S}}{\text{maximize}} \quad R_{\text{SUM}}^{(j)}(\mathcal{P},\mathcal{S}) \quad (j = 1:T; \text{time slot index})$$

$$\text{subject to} \quad B[T+1] \geq Q_B, \quad E[t] \leq \tau P_r + B[t]$$

$$\sum_{u=1}^{U}\sum_{d=1}^{D} s_{u,k,d}[t] \leq 1, \quad \forall k, t.$$

Value function:

$$R_{\text{SUM}}^{(j)}(\mathcal{P},\mathcal{S}) = R_j\left(\mathcal{P}[1],\mathcal{S}[1]\middle|\mathbf{\Omega}, B[1]\right) + \sum_{t=j+1}^{T} \mathrm{E}_{\mathcal{B}}\left\{R_t\left(\mathcal{P}[t],\mathcal{S}[t]\middle|\mathbf{\Omega}, B[t]\right)\right\}$$

where

$$\mathcal{P}[t] = \{\mathbf{P}_{u,k,d}[t] : \forall u,k,d\} \longrightarrow \mathcal{P} = \{\mathcal{P}[t] : \forall t\}$$

$$\mathcal{S}[t] = \{\mathbf{s}_{u,k,d}[t] : \forall u,k,d\} \longrightarrow \mathcal{S} = \{\mathcal{S}[t] : \forall t\}$$

$$R_t\left(\mathcal{P}[t],\mathcal{S}[t]\middle|\mathbf{\Omega}, B[t]\right) = W \cdot \sum_{u=1}^{U} \rho_u \sum_{k=1}^{K}\sum_{d=1}^{D} r_d \cdot s_{u,k,d}[t]$$

with

$$\mathcal{I}\left(\mathbf{P}_{u,k,d}[t]\middle|\mathbf{\Omega}_{u,k}\right) \geq r_d \cdot s_{u,k,d}[t], \quad \forall u,k,d$$

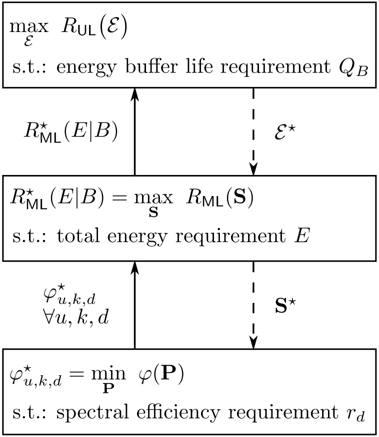
Break the Curse of Dimensionality

Existing energy harvesting designs

- Offline designs with non-causal energy arrival profile: [Ozel2011,JSAC], [Gregori2013,TC]
- Online designs with causal energy arrival profile: SISO – [Ho2012,TSP] Heuristic – [Ng2013,TWC]

Our approach:

- Layered decomposition achieves global optimal solution with low complexity



W. Zeng, **Y.R. Zheng**, and R. Schober, "Online Resource Allocation for Energy Harvesting Downlink Multiuser Systems: Precoding with Modulation, Coding Rate, and Subchannel Selection," *IEEE Trans. Wireless Commun.*, Oct. 2015.

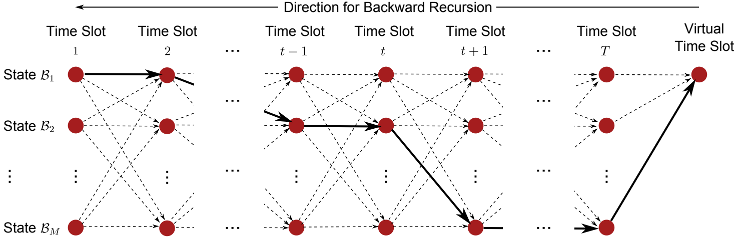
The Upper Layer: a One-Dimensional SDP Problem

$$\underset{\mathcal{E}}{\text{maximize}} \quad R_{\mathrm{UL}}(\mathcal{E}) = R^{\star}_{\mathrm{ML}}(E[1]|B[1]) + \sum_{t=2}^{T} \mathrm{E}_{\mathcal{B}}\left\{R^{\star}_{\mathrm{ML}}(E[t]|B[t])\right\} \quad (1a)$$

$$\text{subject to} \quad B[T+1] \geq Q_B \quad (1b)$$

$$B[t+1] = \min\left\{B[t] + G[t] - (E[t] - \tau P_r)^{+}, B_{\max}\right\}, \quad \forall t \quad (1c)$$

$$E[t] \leq \tau P_r + B[t], \quad \forall t \quad (1d)$$



The Middle Layer: a Knapsack Problem

$$\underset{\mathbf{S}}{\text{maximize}} \quad R_{\mathrm{ML}}(\mathbf{S}) = W \cdot \sum_{u=1}^{U} \rho_u \sum_{k=1}^{K} \sum_{d=1}^{D} r_d \cdot s_{u,k,d} \tag{2a}$$

$$\text{subject to} \quad \sum_{u=1}^{U} \sum_{k=1}^{K} \sum_{d=1}^{D} s_{u,k,d} \cdot \varphi^{\star}_{u,k,d} \leq E \leq \tau P_r + B \tag{2b}$$

$$\sum_{u=1}^{U} \sum_{d=1}^{D} s_{u,k,d} \leq 1, \quad \forall k, \quad s_{u,k,d} \in \{0,1\}, \quad \forall u,k,d \tag{2c}$$

Method of Lagrangian Multiplier with μ for the energy constraint:
Step 1 Solve K convex optimization problems:

$$\underset{\mathbf{S}_k}{\text{maximize}} \quad L_k\left(\mathbf{S}_k, \mu\right) \quad \text{subject to} \quad \sum_{u=1}^{U} \sum_{d=1}^{D} s_{u,k,d} \leq 1$$

$$\text{where} \quad L_k(\mathbf{S}_k, \mu) = \sum_{u=1}^{U} \sum_{d=1}^{D} s_{u,k,d} \cdot \left(W \cdot \rho_u r_d - \mu \cdot \varphi^{\star}_{u,k,d}\right)$$

The Middle Layer Solution

Step 2 For a given μ, compute

$$g(\mu) = \underset{\mathbf{S}}{\text{maximize}} \quad L(\mathbf{S}, \mu) = \sum_{k=1}^{K} L_k(\mathbf{S}_k, \mu) + \mu \cdot E$$

Step 3 Solve for

$$\underset{\mu \geq 0}{\text{minimize}} \quad g(\mu)$$

Since $g(\mu)$ is piecewise linear in μ $\rightarrow$ Step 3 is solved by subgradient-aided bisection search.

The gradient is computed at $g(\mu)$ at $\mu = (\mu_l + \mu_h)/2$ as

$$\nabla_\mu g(\mu) = E - \sum_{u=1}^{U} \sum_{k=1}^{K} \sum_{d=1}^{D} s_{u,k,d}^{(\mu)} \cdot \varphi_{u,k,d}^{\star}$$

where $s_{u,k,d}^{(\mu)}$: Optimal solution in Step 1 with a given μ.

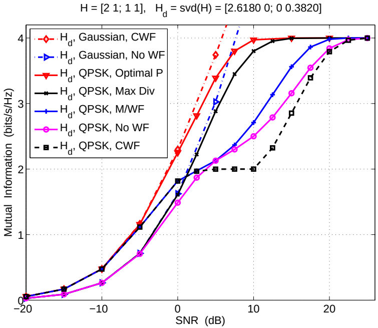
The Lower Layer: Power Allocation

Rate maximization:

$$\underset{\mathbf{P}}{\text{maximize}} \quad \mathcal{I}(\mathbf{P} | \mathbf{\Omega}_{u,k})$$
$$\text{subject to} \quad \tau \cdot \text{Tr}\left(\mathbf{P} \cdot \mathbf{P}^{\mathsf{H}}\right) \leq E$$

Two Optimization Methods:

- Convex optimization w.r.t Gram Matrix $(\mathbf{HP})^{\dagger}\mathbf{HP}$;
- Maximize a Tight Upper Bound of Mutual Information.



The dual problem:

$$\varphi^{\star}_{u,k,d} = \underset{\mathbf{P}}{\text{minimize}} \quad \varphi(\mathbf{P}) = \tau \cdot \text{Tr}\left(\mathbf{P} \cdot \mathbf{P}^{\mathsf{H}}\right) \tag{4a}$$
$$\text{subject to} \quad \mathcal{I}(\mathbf{P} | \mathbf{\Omega}_{u,k}) \geq r_d, \quad \forall u, k, d. \tag{4b}$$

C. Xiao, **Y. R. Zheng**, and Z. Ding, "Globally Optimal Linear Precoders for Finite Alphabet Signals over Complex Vector Gaussian Channels," *IEEE Trans. SP*, July 2011.
W. Zeng, C. Xiao, M. Wang, and J. Lu, "Linear precoding for finite alphabet inputs over MIMO fading channels with statistical CSI," *IEEE Trans. SP*, June 2012.

The Lower Layer Solution

Step 1: SVD channel correlation matrix $\mathbf{\Phi}_{u,k} = \mathbf{U}_{u,k}\Sigma_{u,k}\mathbf{V}_{u,k}$

Let $\mathbf{P} = \mathbf{U} \cdot \sqrt{\mathrm{Diag}(\boldsymbol{\lambda})} \cdot \mathbf{V}$. Set $\mathbf{U} = \mathbf{U}_{u,k}$.

Set $\mathbf{V}$ as the modulation diversity matrix.

Step 2: Approximate the average mutual information by

$$\mathcal{I}_{\mathrm{A}}(\boldsymbol{\lambda}|\mathbf{\Omega}_{u,k}) = N_t \log M_d - \frac{1}{M_d^{N_t}} \sum_{m=1}^{M_d^{N_t}} \log \sum_{n=1}^{M_d^{N_t}} \prod_q \left(1 + \frac{\alpha_{u,k}\psi_{u,k}^{(q)}}{2\sigma^2} \tilde{\mathbf{e}}_{mn}^{(d)\mathrm{H}} \mathrm{Diag}(\boldsymbol{\lambda}) \mathbf{\Sigma}_{u,k} \tilde{\mathbf{e}}_{mn}^{(d)}\right)^{-1}.$$

where $\psi_{u,k}^{(q)}$ is the q-th eigenvalue of the receive correlation matrix $\mathbf{\Psi}_{u,k}$.

Step 3: Solve for the power allocation $\boldsymbol{\lambda}$ by

$$\begin{aligned} \underset{\boldsymbol{\lambda}}{\text{minimize}} \quad & \tau \cdot \mathbf{1}^{\mathrm{T}}\boldsymbol{\lambda} \\ \text{subject to} \quad & \mathcal{I}_{\mathrm{A}}\left(\boldsymbol{\lambda}\middle|\mathbf{\Omega}_{u,k}\right) \geq r_d \quad \text{and} \quad \boldsymbol{\lambda} \geq \mathbf{0} \end{aligned}$$

With the interior-point algorithm with the log-barrier function.

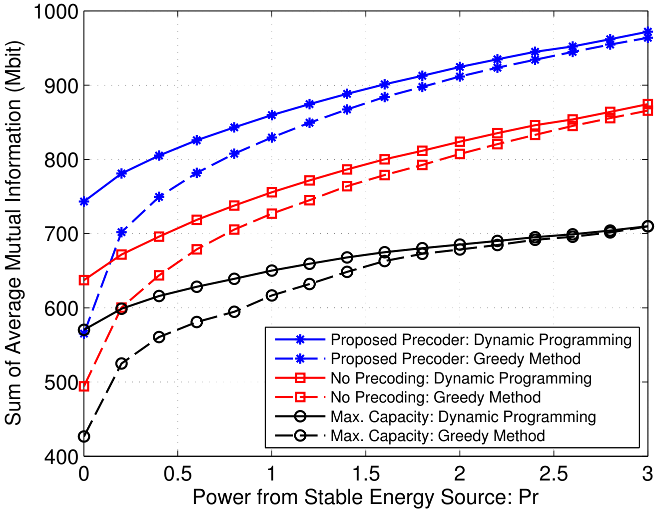
Simulation Setup

- Network Resource Profile
  - # of subchannels $K = 128$, # of MCR schemes $D = 12$;
  - # of Users $U = 9$, user priority weights $\rho_u = 1$ for all u ;
- MIMO Fading Channel Profile
  - COST231-Hata propagation model with 6 multipath taps
  - Per User: 2×2 MIMO
  - transmit correlation matrix $\mathbf{\Psi_t} = \mathbf{\Psi}(0.95)$
  - receive correlation matrix $\mathbf{\Psi_r} = \mathbf{\Psi}(0.5)$
  - the (i, j)-th element of $\mathbf{\Psi}(\rho)$ follows the exponential model

$$[\mathbf{\Psi}(\rho)]_{i,j} = \rho^{|i-j|}, \quad \rho \in [0, 1) \text{ and } i, j = 1, \cdots, N_t \text{ or } N_r$$

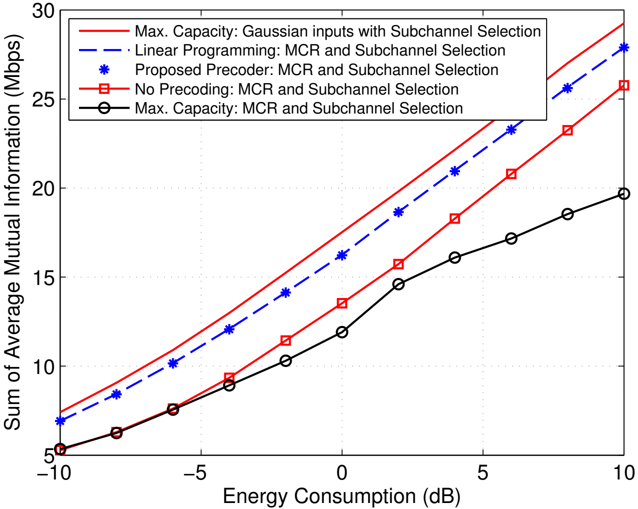
- Energy Harvesting Profile
  - initial battery state: 1 unit; battery capacity $B_{\max} = 10$
  - harvested energy E_k, $\forall k$, takes its value in set $\{0,1,2\}$ with equal probability; the length of time slot $\tau = 1$.
  - battery state set and power choice set are discretized as $\{0, 1, \cdots, B_{\max}\}$

Performance: Upper Layer



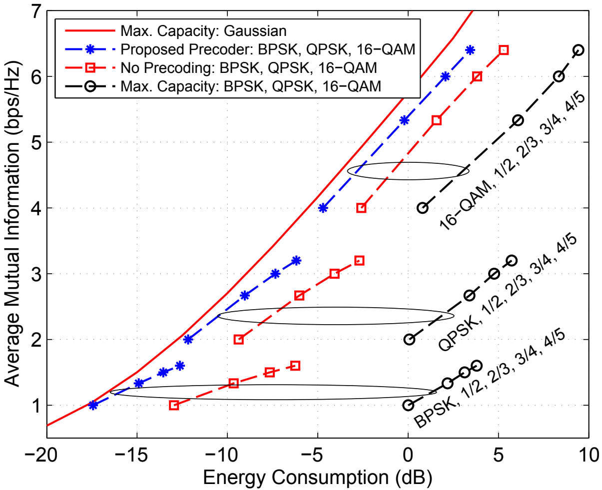
Achievable AMI vs. regular power source P_r with $T = 40$ and $B_{\max} = 10$.

Performance: Middle Layer



The proposed algorithm approaches the Gaussian input upper bound.

Performance: Lower Layer



Energy consumption vs. AMI for 2×2 correlated MIMO channels.

Concluding Remarks

- Online resource allocation for energy harvesting wireless networks: multi-dimensional SDP problem.
- An innovative layered decomposition approach solves the SDP optimization with preserved optimality
- Computational complexity with KUD subchannels, MJN battery/energy levels, and $2N_t^2KUD$ real-valued precoder coefficients:

Direct computation of SDP:

$$\mathcal{O}\left(TMJ \cdot (N^{2N_t^2KUD} + 2^{KUD}) \cdot f_{\mathrm{O}}\right)$$

f_{O} = complexity for computing the MIMO Average Mutual Information.

The Proposed 3-layer algorithm:

$$\mathcal{O}\left(TMJN + N \cdot KUD + \sqrt{N_t} \cdot KUD \cdot f_{\mathrm{A}}\right).$$

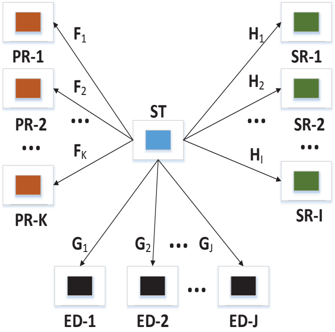
$f_{\mathrm{A}} = 7.2 \times 10^{-8} f_{\mathrm{O}}$ for QPSK
$f_{\mathrm{A}} = 3.2 \times 10^{-6} f_{\mathrm{O}}$ for BPSK.

Security in Cognitive Radio Networks

$$\underset{\mathbf{P}}{\text{maximize}} \quad R_{sc}(\mathbf{P}|\mathbf{\Omega}) = \sum_{i=1}^{I} \min_{1\leq j\leq J} \left[\mathrm{E}_{\mathbf{H}_i}\mathcal{I}(\mathbf{x}_i;\mathbf{y}_i) - \mathrm{E}_{\mathbf{G}_j}\mathcal{I}(\mathbf{x}_i;\mathbf{z}_j) \right]^{+}$$

$$\text{subject to} \quad \mathrm{Tr}(\mathbf{P}^\dagger\mathbf{P}) \leq \gamma_0 \quad \& \quad \mathrm{Tr}(\mathbf{P}^\dagger\mathbf{\Theta}_{f_k}\mathbf{P}) \leq \gamma_k, \ \forall k.$$

$$\text{where} \quad \mathbf{\Omega} = \{\mathbf{\Theta}_{h_i}, \mathbf{\Theta}_{g_j}, \mathbf{\Theta}_{f_k}\}, \ \forall i,j,k.$$



ST – secondary transmitter to design precoder $\mathbf{P}$;
PR-k – kth primary user with channel $\mathbf{F}_k$;
SR-i – ith secondary receiver with channel $\mathbf{H}_i$;
ED-j – jth eavesdropper with channel $\mathbf{G}_j$;

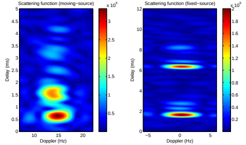
Algorithms:

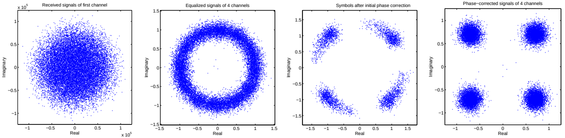
- ST utilizes channel statistical information $\mathbf{\Omega}$;
- Difference of Convex Functions in terms of $\mathbf{W} = \mathbf{P}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{P}$;
- Generalized Quadratic Matrix Programming (GQMP).

J. Jin, Y.R. Zheng, W. Chen, and C. Xiao, "Generalized Quadratic Matrix Programming: A Unified Framework for Linear Precoding With Arbitrary Input Distributions," IEEE Trans. Signal Processing. Accepted June 2017.

High Data-Rate Underwater Acoustic Communications

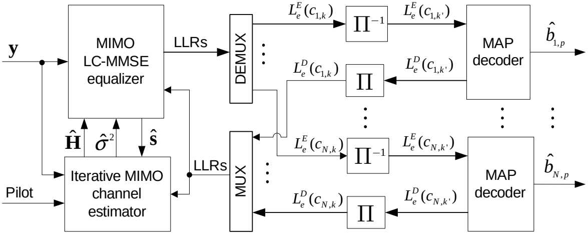
Severe Multipath and Doppler Fading Channels Cause Phase Rotation.





Algorithms:

- Multiple-Input Multiple-Output (MIMO);
- TD and FD Turbo Equalization;
- Adaptive Channel Estimation or Direct Adaptation.

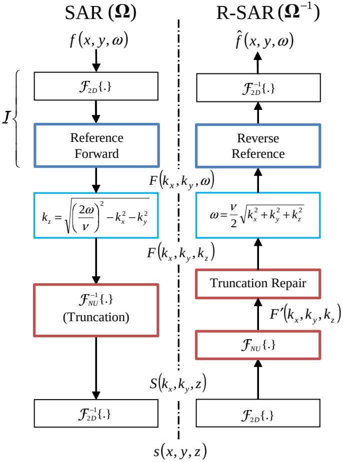


Y.R.Zheng, J. Wu, and C. Xiao, "Turbo Equalization for Underwater Acoustic Communications," IEEE Commun. Mag., Vol. 53, No. 11, pp. 79-87, Nov. 2015.

Y.R.Zheng and Weimin Duan, "Improved Turbo Receivers for underwater acoustic communications," US Patent Application #62483358. Pending. April 9, 2017.

Compressive Sensing Using L1 Optimization

Near-Field SAR Image Reconstruction



3D SAR Transform by NUFFT:

$$s(x, y, z) = \mathcal{F}_{\mathrm{2D}}^{-1}\left\{\mathcal{F}_{\mathrm{NU}}^{-1}\left\{F(k_x, k_y, \omega)\right\}\right\}$$

with $F(k_x, k_y, \omega) = \mathcal{F}_{\mathrm{2D}}\left\{f(x, y, \omega)\right\}\exp(-jz_0 k_z)$ and wavenumber $k_z = \sqrt{4k^2 - k_x^2 - k_y^2}$

Matrix form CS-SAR by minimizating:

$$\mathcal{J}(\mathbf{s}) = \lambda_\ell \left\|\mathbf{\Psi s}\right\|_1 + \lambda_t \mathrm{TV}(\mathbf{s}) + \left\|\mathbf{f} - \mathbf{\Phi}\mathbf{\Omega}^{-1}\mathbf{s}\right\|_2^2$$

where $\mathbf{\Psi}$ – sparsifying domain; $\mathbf{\Phi}$ – measurement matrix; $\mathbf{\Omega}$ – SAR transform.

Algorithms:

- Adaptive Bases Selection within CS Iterations (i.e. OMP, ADM, CG);
- Split-Bregman Algorithm for 3D CS-SAR w/ Fast Convergence;
- Adaptive Measurement via Non-reference Image Quality Assessment.

D. Bi and L. Ma and X. Xie and Y. Xie and X. Li and **Y. R. Zheng**, "A Splitting Bregman-Based Compressed Sensing Approach for Radial UTE MRI," IEEE Transactions on Applied Superconductivity, Vol. 26, No. 7, pp. 1-5. Oct. 2016.
X. Yang, **Y.R. Zheng**, M. T. Ghasr, and K. Donnell Hilgedick, "Microwave Imaging from Sparse Measurements for Nearfield Synthetic Aperture Radar (SAR)," IEEE Trans. Instrumentation Measurement, In press Mar. 2017.