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Downside risk and the performance of volatility-managed portfolios

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ABSTRACT

Recent studies find mixed evidence on the performance of volatility-managed portfolios. We show that strategies scaled by downside volatility exhibit significantly better performance than strategies scaled by total volatility. The improved performance is evident in spanning regressions, direct Sharpe-ratio comparisons, and real-time trading strategies. A decomposition analysis indicates that the *enhanced* performance of downside volatility-managed portfolios is primarily due to return timing, i.e., downside volatility negatively predicts future returns. We find that employing fixed-weight strategies significantly improves the performance of volatility-managed portfolios for real-time investors. Our results hold for nine equity factors and a broad sample of 94 anomaly portfolios.

1. Introduction

Volatility-managed strategies have been the subject of considerable research during the past few years. These strategies are characterized by conservative positions in the underlying factors when volatility was recently high and more aggressively levered positions when volatility was recently low. Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) show that volatility-managed momentum strategies virtually eliminate momentum crashes and nearly double the Sharpe ratio of the original momentum strategy. Moreira and Muir (2017) extend the analysis to nine equity factors and find that volatility-scaled factors produce significantly positive alphas relative to their unscaled counterparts. However, Cederburg et al. (2020) show that the trading strategies implied by the spanning regressions of Moreira and Muir (2017)'s are not implementable in real time and reasonable out-of-sample versions do not outperform simple investments in the original, unmanaged portfolios.[2]

Previous studies of volatility-managed strategies focus exclusively on total volatility. In this paper, we examine downside volatility-managed strategies. The motivation for our focus on downside volatility is twofold. First, there is a long-standing literature contending that downside risk is a more appropriate measure of risk because investors typically associate risk with downside losses rather than upside gains. Markowitz (1959), for example, advocates the use of semivariance as a measure of risk. Second, there is considerable evidence that downside volatility contains valuable information about future volatility and returns (e.g., Barndorff-Nielsen et al., 2010; Feunou et al., 2013; Patton and Sheppard, 2015; Bollerslev et al., 2020; Atilgan et al., 2020). If downside volatility is persistent and negatively predicts future returns, then downside volatility-managed strategies should exhibit superior performance because taking less risk when downside volatility was recently high not only avoids high future volatility but also avoids poor future returns.

We estimate downside volatility from negative returns by following the approach of Patton and Sheppard (2015) and Bollerslev et al. (2020). We then construct downside volatility-managed portfolios similarly to total volatility-managed portfolios except that we scale returns by lagged downside volatility instead of lagged total volatility. For ease of comparison, we examine the same nine equity factors studied by Moreira and Muir (2017), namely, *MKT*, *SMB*, and *HML* from the Fama and French (1993) three-factor model, *MOM* from the Carhart (1997) four-factor model, *RMW* and *CWA* from the

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2 Barroso and Maio (2018) and Eisdorfer and Misirli (2020) find that volatility-scaled betting-against-beta and financial distress strategies significantly outperform their corresponding unscaled strategies. Liu et al. (2019) argue that the volatility-managed strategies of Moreira and Muir (2017) contain a look-ahead-bias and cannot be implemented in real time. Barroso and Detzel (2021) examine whether volatility-managed strategies survive trading cost. Much of this recent literature follows from Fleming et al. (2001) and Fleming et al. (2003), who document large economic gains from volatility timing for short-term investors across several asset classes.

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Fama and French (2015) five-factor model, *ROE* and *IA* from Hou et al. (2015)'s *q*-factor model, and lastly the *BAB* factor of Frazzini and Pedersen (2014).[3] We also examine the 94 anomaly portfolios considered by Cederburg et al. (2020) in order to draw more general conclusions. We follow the previous literature and evaluate the performance of volatility-managed portfolios by using three approaches: Spanning regressions, real-time trading strategies, and direct Sharpe ratio comparisons. Our general finding is that downside volatility-managed portfolios exhibit significantly better performance than total volatility-managed portfolios. The improved performance in out-of-sample real-time trading strategies is especially noteworthy in light of the recent controversy about the real-time performance of volatility-managed portfolios (Cederburg et al., 2020).

Our first approach to evaluating the performance of volatility-managed portfolios is to estimate the spanning regressions of Moreira and Muir (2017), i.e., regressing volatility-managed factors on their corresponding unmanaged factors. We confirm the findings of Moreira and Muir (2017) and find significantly positive spanning regression alphas for volatility-managed *MKT*, *HML*, *MOM*, *RMW*, *ROE*, *IA*, and *BAB* and insignificant alphas for volatility-managed *SMB* and *CMA*. In comparison, downside volatility-managed factors exhibit positive and significant spanning regression alphas across all nine factors examined by Moreira and Muir (2017). The two factors for which Moreira and Muir (2017) find insignificant alphas now generate positive alphas that are statistically significant at the 10% level. This performance improvement extends to the sample of 94 anomalies. Looking at total volatility-managed portfolios, we find that about two thirds of the anomalies (62 out of 94 anomalies) exhibit positive spanning regression alphas. This finding is consistent with Moreira and Muir (2017) and Cederburg et al. (2020). In comparison, nearly 95% of the anomalies (89 out of 94 anomalies) exhibit positive alphas for downside volatility-managed portfolios. Overall, our results indicate that downside volatility-managed portfolios perform significantly better than total volatility-managed portfolios in spanning regressions.

To explore the sources of the performance of volatility-managed portfolios, we decompose the spanning regression alpha into two components, volatility timing and return timing. The volatility timing component is positive if lagged volatility is positively related to future volatility. The return timing component is positive if lagged volatility is negatively related to future returns. Volatility clustering is one of the most robust stylized facts in finance, so the volatility timing component is likely to be positive. However, the literature is ambiguous about the volatility-return relation (e.g., French et al., 1987; Glosten et al., 1993; Brandt and Kang, 2004).[4] If the conditional expected return is positively related to lagged volatility, then the benefit of volatility timing is likely to be offset by the cost of negative return timing and, as a result, volatility-managed strategies will not work. If the conditional expected return is uncorrelated or even negatively correlated with lagged volatility, then volatility-managed strategies are likely to perform well because they take advantage of the attractive risk-return trade-off when volatility is low and avoids the poor risk-return trade-off when volatility is high.

Our decomposition results indicate that the positive alphas of total volatility-managed portfolios stem primarily from volatility timing. The large contribution from volatility timing is unsurprising because volatility is highly persistent. The small, and sometimes even negative contribution from return timing suggests that total volatility is largely unrelated to future returns. Volatility timing also plays a major role in explaining the superior performance of downside volatility-managed strategies. However, the enhanced performance of downside volatility-managed strategies relative to total volatility-managed portfolios is almost entirely attributable to the return-timing component. For total volatility-managed strategies, the return-timing component is positive among just two of the nine equity factors and 42 of the 94 anomalies. In contrast, eight of the nine equity factors and 71 of the 94 anomalies exhibit a positive return-timing component for downside volatility-managed strategies. The positive return-timing component associated with downside volatility-managed strategies suggests that high downside volatility tends to be associated with low future returns. In summary, we find that the superior performance of downside volatility-managed factors is a result of both volatility timing and return timing, but the improvement over total volatility-managed portfolios is attributed to return timing.

Cederburg et al. (2020) point out that the trading strategies implied by the spanning regressions, i.e., combining the volatility-managed portfolio and the unmanaged portfolio using ex post optimal weights, are not implementable in real time because the optimal weights for the volatility-managed portfolio and the unmanaged portfolio depend on full-sample return moments, which are not known to real-time investors. Therefore, in our second approach we evaluate the real-time (i.e., out-of-sample) performance of volatility-managed strategies. We follow Cederburg et al. (2020) and compare the performance of two real-time strategies: the combination strategy and the original, unmanaged strategy. Consistent with Cederburg et al. (2020), we find little evidence that managing total volatility is systematically advantageous for real-time investors—the combination strategy that incorporates total volatility-managed portfolios outperforms the unmanaged strategy in 50 of the 103 equity factors and anomaly portfolios, while underperforming in the remaining 53. Managing downside volatility, however, significantly improves the performance of the combination strategy. Specifically, the combination strategy that incorporates downside volatility-managed portfolios outperforms the original, unmanaged strategy in 70 of the 103 equity factors and anomalies. A simple binomial test indicates that the null hypothesis of equal performance between the combination strategy and the unmanaged strategy is rejected at the 1 percent level.

The relatively poor out-of-sample performance of the real-time combination strategies is primarily due to parameter instability and estimation risk (Cederburg et al., 2020). A potential remedy for this issue, therefore, is to examine combination strategies that use fixed portfolio weights. These fixed weights, e.g., 50% in the volatility-managed portfolio and 50% in the original portfolio, are unlikely to be optimal ex post, but employing them removes the need to estimate "optimal" weights in real time and therefore may improve performance. We find that fixed-weight strategies indeed perform better than standard real-time strategies. Depending on the specific weight, we show that the combination strategy that incorporates total volatility-managed portfolios outperforms the original, unmanaged strategy in 64–68 (out of 103) equity factors and anomalies. Recall that the corresponding number is only 50 for standard real-time strategies. For downside volatility-managed portfolios, the combination strategy with fixed weights outperforms the original, unmanaged strategy in 80–81 equity factors and anomalies (compared to 70 for standard real-time strategies). In summary, we find that fixed-weight strategies outperform standard real-time strategies. Moreover, we continue to find that downside volatility-managed portfolios significantly outperform the performance of total volatility-managed portfolios.

[3] Moreira and Muir (2017) also examine a currency carry trade factor. Similar to Cederburg et al. (2020), we focus on their nine equity factors.

[4] Barroso and Maio (2019) is the first study on the risk-return trade-off of long-short equity factors. They find the trade-offs to be weak or nonexistent for most factors.

Most prior studies (e.g., Barroso and Santa-Clara, 2015; Daniel and Moskowitz, 2016; Barroso and Maio, 2018; Cederburg et al., 2020; Eisdorfer and Misirli, 2020) assess the value of volatility management by directly comparing the Sharpe ratios of volatility-managed portfolios with the Sharpe ratios of unmanaged portfolios. We follow these studies and employ direct Sharpe ratio comparison as our third approach to evaluating the performance of volatility-managed portfolios. Our findings are similar to those for the spanning regressions and real-time trading strategies. That is, downside volatility-managed portfolios perform significantly better than the total volatility-managed portfolios. Specifically, total volatility-managed portfolios exhibit higher Sharpe ratios than original, unmanaged portfolios among 63 (out of 103) equity factors and anomalies. The corresponding number increases sharply to 93 for downside volatility-managed portfolios, suggesting that, as stand-alone investment, downside volatility-managed portfolios are also beneficial to investors. A direct comparison between downside and total volatility-managed portfolios indicates that downside volatility-managed portfolios exhibit higher Sharpe ratios in 8 out of 9 equity factors and 80 out of 94 anomalies.

Our paper makes several contributions to the growing literature on volatility-managed strategies. First, we show that managing downside volatility instead of total volatility significantly improves the performance of volatility-managed portfolios. This finding is important in light of the recent controversy on whether total volatility-managed portfolios are systematically beneficial to investors. In contrast to the inconsistent, and sometimes mediocre performance of total volatility-managed portfolios, we find that downside volatility-managed portfolios exhibit superior performance across all methodologies, i.e., spanning regressions, real-time trading strategies, and direct Sharpe ratio comparisons.[5]

Second, we provide a first analysis of the sources of the performance of volatility-managed portfolios. We find that the positive spanning regression alphas of total volatility-managed portfolios are driven entirely by volatility timing, whereas the superior performance of downside volatility-managed portfolios are due to both return timing and volatility timing. Moreover, the enhanced performance of downside volatility-managed portfolios relative to total volatility-managed portfolios is due to return timing, i.e., downside volatility negatively predicts future returns.

Third, we propose an approach to improving the poor out-of-sample performance of real-time volatility-managed strategies. Specifically, we show that fixed-weight strategies significantly outperform standard strategies that estimate portfolio weights in real time. Fixed-weight strategies remove the need to estimate portfolio weights in real time and therefore mitigates the parameter instability and estimation risk concerns. Our approach is general (i.e., not specific to volatility-managed portfolios) and can be applied to other settings that involve real-time trading strategies.

The remainder of the paper is organized as follows. Section 2 describes the data and empirical methods to evaluate the performance of volatility managed strategies. Section 3 presents the empirical results. Section 4 concludes.

[5] Qiao et al. (2020) also find that downside volatility-managed portfolios expand the mean-variance frontiers constructed using the original portfolios and the total volatility-managed portfolios. Our paper differs from Qiao et al. (2020) in several important ways. First, in addition to spanning regressions, we also evaluate the performance of downside volatility-managed portfolios using real-time trading strategies and direct Sharpe ratio comparisons. Second, in addition to equity factors, we also examine 94 anomaly portfolios. Third, we explore the sources of the superior performance of downside volatility-managed portfolios. Fourth, we perform a trading cost analysis for downside volatility-managed strategies. Finally, we show that using fixed-weights significantly improves the real-time performance of volatility-managed strategies.

2. Data and methodology

2.1. Data

We use two sets of test assets. The first group consists of the nine equity factors considered by Moreira and Muir (2017), i.e., the market (*MKT*), size (*SMB*), and value (*HML*) factors from the Fama and French (1993) three-factor model, the momentum (*MOM*) factor from Carhart (1997)' 4-factor model, the profitability (*RMW*) and investment (*CMA*) factors from the Fama and French (2015) five-factor model, the profitability (*ROE*) and investment (*IA*) from Hou et al. (2015)'s *q*-factor model, and the betting-against-beta factor (*BAB*) from Frazzini and Pedersen (2014). We obtain daily and monthly excess returns for the above factors from Kenneth French's website, Andrea Frazzini's website, and Lu Zhang.[6] The sample period starts in August 1926 for *MKT*, *SMB*, and *HML*; January 1927 for *MOM*; August 1963 for *RMW* and *CMA*; February 1967 for *ROE* and *IA*; and February 1931 for *BAB*. The sample periods end in December 2018.

The second group of test assets includes 94 stock market anomalies. Although the nine equity factors examined by Moreira and Muir (2017) provide a reasonable representation of factors in leading asset pricing models, recent studies suggest that more characteristics are needed to summarize the cross-section of stock returns (e.g., Kelly et al., 2019; Kozak et al., 2020). We therefore follow Cederburg et al. (2020) and augment the nine equity factors with a comprehensive sample of stock market anomalies from Hou et al. (2015) and McLean and Pontiff (2016). We restrict our sample to anomaly variables that are continuous (rather than an indicator variable) and can be constructed using the CRSP, COMPUSTAT, and I/B/E/S data. We also exclude anomalies that are based on industry-level variables. Table A.1 in the appendix contains the detailed list of the 94 anomaly variables along with their definitions, sources, and sample periods. Many anomalies are based on related characteristics. We follow Hou et al. (2015) and group them into seven major categories, including accrual ($N = 10$), intangibles ($N = 10$), investment ($N = 9$), momentum ($N = 8$), profitability ($N = 20$), trading ($N = 19$), and value ($N = 18$).

We construct the anomaly variables following the descriptions in Hou et al. (2015), McLean and Pontiff (2016), and Cederburg et al. (2020). We begin with all NYSE, AMEX, and NASDAQ common stocks (with a CRSP share code of 10 or 11) during the period from 1926 to 2018 with data necessary to compute anomaly variables and subsequent stock returns. We exclude financial stocks and stocks with a price lower than $5 at the portfolio formation date. We also remove stocks whose market capitalization is ranked in the lowest NYSE decile at the portfolio formation date. We remove low-priced and micro-cap stocks to ensure that our results are not driven by small, illiquid stocks that comprise a tiny fraction of the market. We sort all sample stocks into deciles based on each anomaly variable and then construct value-weighted portfolios. The hedge strategy goes long on stocks in the top decile and short those stocks in the bottom decile, where the top (bottom) decile includes the stocks that are expected to outperform (underperform) based on prior literature.

2.2. Construction of volatility-managed portfolios

We follow prior literature (Barroso and Santa-Clara, 2015) and construct the volatility-managed portfolio as a scaled version of

[6] Data on *MKT*, *SMB*, *HML*, *MOM*, *RMW*, and *CMA* are from Kenneth French's website at http://www.mba.tuck.dartmouth.edu/pages/faculty/ken.french/. Data on *BAB* are from Andrea Frazzini's website at http://www.people.stern.nyu.edu/afrazzin/. We thank Kenneth French and Andrea Frazzini for making these data available. We thank Lu Zhang for sharing the data on *ROE* and *IA*.

Table 1
Spalnning regressions for the 9 equity factors. This table reports results from spanning regressions of volatility-managed factor returns on the corresponding original factor returns. The spanning regressions are given by $f_{\sigma,t} = \alpha + \beta f_t + \epsilon_t$, where $f_{\sigma,t}$ is the monthly return for volatility-managed factor, and f_t is the monthly return for the original factor. Panel A reports results for total volatility-managed strategies, and Panel B reports those for the downside volatility-managed strategies. In addition to univariate spanning regressions, we also control for the Fama and French (1993) three factors. The reported alphas are in annualized, percentage terms. The appraisal ratio is α/σ_ϵ, where σ_ϵ is the root mean square error. *MKT*, *SMB* and *HML* are obtained from Fama and French (1993), *MOM* is from Carhart (1997), *RMW* and *CMA* are from Fama and French (2015), *ROE* and *IA* are from Hou et al. (2015), and *BAB* is from Frazzini and Pedersen (2014). Numbers in parentheses are *t*-statistics based on White (1980) standard errors.

	MKT	*SMB*	*HML*	*MOM*	*RMW*	*CMA*	*ROE*	*IA*	*BAB*
Panel A: Total volatility-managed strategy									
Panel A.1: Univariate regressions									
Alpha, α	3.34	0.44	1.48	9.36	1.35	0.08	3.32	0.75	3.99
	(3.39)	(0.78)	(2.21)	(6.74)	(2.26)	(0.20)	(4.64)	(1.93)	(5.96)
R^2	0.72	0.72	0.65	0.62	0.67	0.78	0.76	0.80	0.71
Panel A.2: Controlling for Fama and French (1993) three factors									
Alpha, α	3.99	0.24	2.14	7.31	1.95	0.21	3.68	0.44	3.81
	(4.06)	(0.44)	(3.15)	(6.30)	(3.25)	(0.52)	(5.13)	(1.10)	(5.46)
R^2	0.73	0.73	0.67	0.65	0.73	0.78	0.78	0.80	0.72
Panel B: Downside volatility-managed strategy									
Panel B.1: Univariate regressions									
Alpha, α	4.83	1.11	3.47	8.32	2.83	0.88	4.41	1.70	6.16
	(4.10)	(1.66)	(4.83)	(5.33)	(4.14)	(1.73)	(5.18)	(3.76)	(8.25)
R^2	0.62	0.60	0.60	0.41	0.53	0.67	0.52	0.70	0.51
Panel B.2: Controlling for Fama and French (1993) three factors									
Alpha, α	5.27	1.37	4.11	6.57	3.49	0.56	4.52	1.50	5.95
	(4.50)	(2.02)	(5.63)	(4.97)	(4.80)	(1.50)	(5.66)	(3.15)	(7.86)
R^2	0.62	0.60	0.62	0.43	0.57	0.67	0.53	0.70	0.51

the original portfolio, with the investment position proportional to the inverse of lagged realized volatility:[7]

$$f_{\sigma,t} = \frac{c^*}{\sigma_{t-1}} f_t, \quad (1)$$

where f_t is the monthly excess return for the original portfolio, σ_{t-1} is the realized volatility of the original portfolio in month $t-1$ computed from daily returns, and c^* is a constant chosen such that f_t and $f_{\sigma,t}$ have the same full-sample volatility. We note that c^* is not known to investors in real time, but some performance measures such as Sharpe ratios and appraisal ratios are invariant to the choice of this parameter. We also note that f_t is the excess return of a zero-cost portfolio. Therefore, the dynamic investment position in the original portfolio, c^*/σ_{t-1}, is a measure of leverage.

For a given asset pricing factor or stock market anomaly, we construct two versions of volatility-managed portfolios following Eq. (1), one scaled by total volatility and the other scaled by downside volatility. We first compute realized total volatility and downside volatility in month t as follows:

$$\sigma_{Total,t} = \sqrt{\sum_{j=1}^{N_t} f_j^2}, \quad (2)$$

$$\sigma_{Down,t} = \sqrt{\sum_{j=1}^{N_t} f_j^2 I_{[f_j<0]}}, \quad (3)$$

where f_j represents the return on day j in month t, and N_t is the number of daily returns in month t. That is, we compute total volatility using all daily returns in month t and compute downside volatility using only negative daily returns in month t. If the number of negative daily returns is less than three in month t, then $\sigma_{Down,t}$ is measured using negative daily returns over both month t and month $t-1$.

We then construct total volatility- and downside volatility-managed portfolios as follows:

$$f_{\sigma,t}^{Total} = \frac{c^*}{\sigma_{Total,t-1}} f_t, \quad (4)$$

$$f_{\sigma,t}^{Down} = \frac{\tilde{c}^*}{\sigma_{Down,t-1}} f_t, \quad (5)$$

To understand the relation between total volatility- and downside volatility-managed portfolios, we can express $f_{\sigma,t}^{Down}$ as a function of $f_{\sigma,t}^{Total}$:

$$f_{\sigma,t}^{Down} = \frac{c^\dagger}{\left(\frac{\sigma_{Down,t-1}}{\sigma_{Total,t-1}}\right)} f_{\sigma,t}^{Total}, \quad (6)$$

where $c^\dagger = \tilde{c}^*/c^*$.

Essentially, one can think of $f_{\sigma,t}^{Down}$ as a managed portfolio of $f_{\sigma,t}^{Total}$, taking a larger position in $f_{\sigma,t}^{Total}$ when downside volatility is relatively low and vice versa. Eq. (6) suggests that, if the total volatility-managed portfolio tends to perform better when downside volatility is relatively low, then downside volatility-managed portfolio will tend to outperform total volatility-managed portfolio.[8]

3. Empirical results

3.1. Spanning regressions

Our first approach to evaluating the performance of volatility-managed portfolios is to estimate the spanning regressions of Moreira and Muir (2017), i.e., regressing volatility-managed portfolio returns on their corresponding unmanaged portfolio returns as follows:

$$f_{\sigma,t} = \alpha + \beta f_t + \epsilon_t. \quad (7)$$

We extend Moreira and Muir (2017) by estimating Eq. (7) for both total volatility-managed portfolios and downside volatility-managed portfolios.

[7] Moreira and Muir (2017) scale factor returns by lagged realized variance. We decide to use lagged realized volatility primarily because it leads to less extreme investment weights and hence lower turnover and trading cost. Our results are slightly weaker if we use realized variance instead of realized volatility, but the main conclusions are qualitatively unchanged.

[8] We thank an anonymous referee for suggesting this connection between total volatility- and downside volatility- managed portfolios.

3.1.1. Baseline results

Table 1 presents the annualized alphas from the spanning regressions for the nine equity factors. Panel A reports the results for total volatility-managed factors. Consistent with Moreira and Muir (2017), we find that volatility-managed factors often produce positive and significant alphas relative to their corresponding unmanaged factors. Specifically, the spanning regression alpha is positive and statistically significant at the 5% level for volatility-managed *MKT*, *HML*, *MOM*, *RMW*, *ROE*, and *BAB*, and is positive and significant at the 10% level for volatility-managed *IA*. The volatility-managed *CMA* and *SMB* exhibit insignificant alphas.

Panel B of Table 1 presents the spanning regression results for downside volatility-managed factors. We find that downside volatility-managed factors perform significantly better than total volatility-managed factors. In particular, all nine equity factors exhibit positive and significant spanning regression alphas in Panel B. The two factors for which Moreira and Muir (2017) find insignificant spanning regression alphas now generate positive alphas that are statistically significant at the 10% level. Specifically, downside volatility-managed *SMB* has an alpha of 1.11% per year (t-statistic = 1.66), compared to-0.44% (t-statistic = 0.78) for the total volatility-managed *SMB*. Similarly, downside volatility-managed *CMA* has an alpha of 0.88% per year (t-statistic = 1.73), compared to 0.08% (t-statistic = 0.20) for total volatility-managed *CMA*. Moreover, among six of the remaining seven factors, downside volatility-managed factors exhibit larger alphas and higher t-statistics than total volatility-managed factors. For example, total volatility-managed *MKT* has an alpha of 3.34% per year with a t-statistic of 3.39, while downside volatility-managed *MKT* exhibits an alpha of 4.83% with a t-statistic of 4.10.[9]

We follow Moreira and Muir (2017) and also control for the Fama and French (1993) three factors in the spanning regressions. From an economic perspective, including the Fama and French factors as controls likely provides a better characterization of the investment opportunity set for investors sophisticated enough to consider volatility-managed strategies. Our results indicate that downside volatility-managed portfolios continue to outperform total volatility-managed portfolios in spanning regressions when we include Fama-French three factors as controls.[10]

We also extend the analyses to the sample of 94 anomalies in order to draw broader conclusions. The results are summarized in Table 2. To conserve space, we report the total number of positive and negative alphas, as well as the number of significant alphas across the 94 anomalies instead of detailed anomaly-by-anomaly results. Looking at total volatility-managed portfolios, we find that two thirds of the anomalies (62 out of 94 anomalies) exhibit positive spanning regression alphas, with 15 of them statistically significant at the 5% level. The number of negative alphas is 32, with only 2 being statistically significant. This evidence is consistent with Cederburg et al. (2020) and supports the finding of Moreira and Muir (2017). In comparison, when we examine downside volatility-managed portfolios, nearly 95% of the anomalies (89 out of 94 anomalies) exhibit positive alphas, and 34 of them are statistically significant at the 5% level. Among the five anomalies with negative alphas, none is statistically significant. This broad sample evidence confirms our previous finding from the nine equity factors that downside volatility-managed portfolios exhibit significantly higher spanning regression alphas than total volatility-managed portfolios.

To further demonstrate that downside volatility-managed portfolios outperform total volatility-managed portfolios, we estimate an alternative spanning regression in which we regress the return of the downside volatility-managed portfolio on the return of the total volatility-managed portfolio. In essence, we are trying to gauge whether downside volatility-managed portfolios are spanned by total volatility-managed portfolios. We present the results of this analysis in Table 3. Panel A presents the results for the nine equity factors, and Panel B presents the results for 94 anomalies. Our results are overwhelmingly in favor of downside volatility-managed portfolios. Specifically, we find that the spanning regression alpha is significantly positive among eight of the nine equity factors in Panel A. The only exception is the momentum factor, for which the alpha is insignificant. Among the 94 anomalies, we find that the alpha is positive in 84 anomalies, with 43 statistically significant. Among the 10 negative alphas, none is statistically significant. These results suggest that downside volatility-managed portfolios are not spanned by total volatility-managed portfolios and that they provide significant incremental benefits to investors beyond those offered by total volatility-managed portfolios.

3.1.2. Transaction costs

Implementing volatility-managed investment strategies requires significant amount of trading. Therefore, an important question is whether the significant spanning regression alphas of volatility-managed portfolios are robust to transaction costs. We note that it is beyond the scope of this paper to provide detailed transaction cost estimates associated with the construction of the equity factors and anomaly portfolios by using stock-level data. Instead, we consider several reasonable estimates of trading cost. Specifically, we follow Moreira and Muir (2017) and consider the trading costs of 1 basis point, 10 basis points, and 14 basis points. The 1 basis cost is from Fleming et al. (2003) and is a reasonable trading cost only for the market factor. The 10 and 14 basis points are motivated by Frazzini et al. (2015) and represent a reasonable trading cost for sophisticated institutional investors who are able to time their trades to minimize liquidity demands and associated costs. In addition, we also consider 25 and 50 basis points, which are more relevant for regular liquidity-demanding investors. These larger trading cost estimates are consistent with those documented by Hasbrouck (2009), Novy-Marx and Velikov (2016), and Barroso and Detzel (2021).

We report before- as well as after-cost spanning regression alphas of both total- and downside-volatility managed portfolios for the nine equity factors in Table 4. We also compute the break-even transaction costs that render the spanning regression alpha zero. In addition, we report the average absolute change in investment weights, which is an estimate of turnover in the equity factors.[11]

Panel A presents the results for total volatility-managed portfolios. We find that most of the spanning regression alphas remain positive for low-level transaction costs, i.e., 1, 10, and 14 basis points. However, at 25 and 50 basis points, most of the equity factors exhibit negative spanning regression alphas. This latter finding is consistent with Barroso and Detzel (2021), who find that only the volatility-managed market factor survives trading cost.

Panel B presents the results for downside volatility-managed factors. Here, we again find evidence that the spanning regression alphas are robust to low levels of trading cost. At higher levels of trading cost, some of the alphas turn negative. Comparing between Panel A and Panel B, we find that downside

[9] The betting-against-beta factor (*BAB*) from Frazzini and Pedersen (2014) is beta-rank-weighted. Novy-Marx and Velikov (2018) show that the value-weighted BAB factor exhibits insignificant average returns. We are able to confirm this finding. Moreover, we find that the spanning regression alpha of the value-weighted BAB factor is positive but statistically insignificant when scaled by total volatility and is positive and marginally significant when scaled by downside volatility.

[10] In Table IA.1 in the Internet Appendix, we show that our results are robust to including Fama-French five factors (Fama and French, 2015) or six factors (Fama and French, 2018) as controls.

[11] This turnover estimate does not account for the stock-level turnover of the equity factors themselves.

Table 2
Spanning regressions for the 94 anomalies. This table summarizes results from spanning regressions for the 94 stock market anomalies. The spanning regressions are given by $f_{\sigma,t} = \alpha + \beta f_t + \epsilon_t$, where $f_{\sigma,t}$ is the monthly return for volatility-managed anomaly returns, and f_t is the monthly return for the original strategy. Panel A reports results for total volatility-managed strategies, and Panel B reports those for the downside volatility-managed strategies. The results in columns (3) and (4) correspond to univariate spanning regressions, and those in columns (5) and (6) are for regressions that add the Fama and French (1993) three factors as controls. The table reports the number of alphas that are positive, positive and significant at the 5% level, negative, and negative and significant at the 5% level. Statistical significance of the alpha estimates is based on White (1980) standard errors.

		Univariate regressions		Controlling for Fama and French (1993) factors	
		$\alpha > 0$ [Signif.]	$\alpha < 0$ [Signif.]	$\alpha > 0$ [Signif.]	$\alpha < 0$ [Signif.]
(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Total volatility-managed strategy					
All	94	62[15]	32[2]	60[14]	34[2]
Accruals	10	7[2]	3[0]	6[1]	4[0]
Intangibles	10	4[1]	6[0]	4[0]	6[0]
Investment	9	4[0]	5[0]	4[0]	5[0]
Momentum	8	8[7]	0[0]	8[7]	0[0]
Profitability	20	17[0]	3[0]	17[2]	3[0]
Trading	19	13[3]	6[1]	13[3]	6[2]
Value	18	9[2]	9[1]	8[1]	10[0]
Panel B: Downside volatility-managed strategy					
All	94	89[34]	5[0]	84[37]	10[0]
Accruals	10	10[3]	0[0]	10[5]	0[0]
Intangibles	10	10[2]	0[0]	10[2]	0[0]
Investment	9	8[2]	1[0]	7[2]	2[0]
Momentum	8	8[6]	0[0]	8[6]	0[0]
Profitability	20	19[8]	1[0]	19[11]	1[0]
Trading	19	17[7]	2[0]	13[5]	6[0]
Value	18	17[6]	1[0]	17[6]	1[0]

Table 3
Spanning regression of downside volatility-managed strategies on total volatility-managed strategies. This table reports results from spanning regressions of downside volatility-managed portfolio returns on the corresponding total volatility-managed returns. The spanning regressions are given by $f_{\sigma,t}^{Down} = \alpha + \beta f_{\sigma,t}^{Total} + \epsilon_t$ or $f_{\sigma,t}^{Down} = \alpha + \beta f_{\sigma,t}^{Total} + f_t + \epsilon_t$, where $f_{\sigma,t}^{Total}(f_{\sigma,t}^{Down})$ is the monthly return for total volatility-managed (downside volatility-managed) portfolio returns and f_t is the monthly return for the original factor. Panel A reports results from spanning regressions for the nine equity factors. The reported alphas are in annualized, percentage terms. Numbers in parentheses are t-statistics based on White (1980) standard errors. The appraisal ratio is α/σ_ϵ. Panel B presents summary results of the number of alphas that are positive, positive and significant at the 5% level, negative, and negative and significant at the 5% level for the 94 anomaly portfolios.

Panel A: Factors									
	MKT	*SMB*	*HML*	*MOM*	*RMW*	*CMA*	*ROE*	*IA*	*BAB*
Panel A.1: Results from $f_{\sigma,t}^{Down} = \alpha + \beta f_{\sigma,t}^{Total} + \epsilon_t$									
Alpha, α	1.53	1.52	2.22	0.30	1.55	0.78	1.33	0.96	2.51
	(2.53)	(3.17)	(5.45)	(0.40)	(4.64)	(2.40)	(3.22)	(3.74)	(5.77)
R^2	0.90	0.82	0.86	0.77	0.83	0.87	0.75	0.90	0.75
Panel A.2: Results from $f_{\sigma,t}^{Down} = \alpha + \beta f_{\sigma,t}^{Total} + f_t + \epsilon_t$									
Alpha, α	1.49	1.50	2.19	0.88	1.55	0.80	1.16	0.97	2.45
	(2.48)	(3.04)	(5.5)	(1.19)	(4.67)	(2.42)	(2.98)	(3.77)	(5.58)
R^2	0.90	0.82	0.86	0.78	0.83	0.87	0.75	0.90	0.76

Panel B: Anomalies			
	Total	$\alpha > 0$ [Signif.]	$\alpha < 0$ [Signif.]
Panel B.1: Results from $f_{\sigma,t}^{Down} = \alpha + \beta f_{\sigma,t}^{Total} + \epsilon_t$			
All	94	84[43]	10[0]
Accruals	10	10[4]	0[0]
Intangibles	10	10[7]	0[0]
Investment	9	9[3]	0[0]
Momentum	8	5[0]	3[0]
Profitability	20	20[12]	0[0]
Trading	19	13[6]	6[0]
Value	18	17[11]	1[0]
Panel B.2: Results from $f_{\sigma,t}^{Down} = \alpha + \beta f_{\sigma,t}^{Total} + f_t + \epsilon_t$			
All	94	83[43]	11[0]
Accruals	10	10[4]	0[0]
Intangibles	10	10[7]	0[0]
Investment	9	9[3]	0[0]
Momentum	8	4[0]	4[0]
Profitability	20	20[12]	0[0]
Trading	19	13[6]	6[0]
Value	18	17[11]	1[0]

volatility-managed portfolios exhibit higher turnover rates than total volatility-managed portfolios. In addition, downside volatility-managed portfolios tend to have higher alphas than total volatility-managed portfolios at lower levels of trading costs, but perform similarly to total volatility-managed portfolios at higher levels of trading costs. This finding, along with the turnover result, suggests that the superior before-cost performance of downside volatility-managed portfolios may be due to limits to arbitrage.

We also implement the above analysis for the 94 anomaly portfolios. For brevity, we report the results in Table IA.2 in the Internet Appendix. We find that total volatility-managed portfolios tend to exhibit positive alphas at trading costs up to 14 basis points,

Table 4

Transaction costs of volatility managed factors. This table reports the alphas of volatility managed factors after accounting for transaction costs. $|\Delta w|$ is the average absolute change in monthly weights. We consider five levels of transaction costs: 1 bps, 10 bps, 14 bps, 25 bps, and 50 bps. $\alpha_{Break-even}$ is the implied transaction costs needed to drive alphas to zero. All results are in annualized, percentage terms. *MKT*, *SMB* and *HML* are obtained from Fama and French (1993), *MOM* is from Carhart (1997), *RMW* and *CMA* are from Fama and French (2015), *ROE* and *IA* are from Hou et al. (2015), and *BAB* is from Frazzini and Pedersen (2014).

	MKT	*SMB*	*HML*	*MOM*	*RMW*	*CMA*	*ROE*	*IA*	*BAB*
Panel A: Total volatility-managed strategy									
α	3.34	0.44	1.48	9.36	1.35	0.08	3.32	0.75	3.99
$\|\Delta w\|$	0.36	0.31	0.38	0.49	0.33	0.28	0.29	0.27	0.33
α_{1bps}	3.30	0.48	1.43	9.30	1.31	0.05	3.29	0.72	3.95
α_{10bps}	2.91	0.81	1.03	8.77	0.96	0.26	2.97	0.42	3.59
α_{14bps}	2.74	0.96	0.85	8.54	0.80	0.39	2.84	0.29	3.43
α_{25bps}	2.27	1.37	0.35	7.90	0.37	0.77	2.45	0.07	2.99
α_{50bps}	1.21	2.30	0.77	6.43	0.61	1.61	1.59	0.89	2.00
$\alpha_{Break-even}$	0.78	0.12	0.33	1.60	0.34	0.02	0.96	0.23	1.00
Panel B: Downside volatility-managed strategy									
α	4.83	1.11	3.47	8.32	2.83	0.88	4.41	1.70	6.16
$\|\Delta w\|$	0.69	0.49	0.53	0.75	0.46	0.45	0.48	0.42	0.51
α_{1bps}	4.75	1.05	3.41	8.23	2.77	0.83	4.35	1.65	6.10
α_{10bps}	4.00	0.53	2.83	7.42	2.28	0.33	3.84	1.19	5.55
α_{14bps}	3.67	0.29	2.58	7.05	2.05	0.12	3.61	0.99	5.31
α_{25bps}	2.76	0.35	1.88	6.06	1.45	0.48	2.98	0.43	4.64
α_{50bps}	0.69	1.81	0.29	3.80	0.06	1.85	1.55	0.83	3.12
$\alpha_{Break-even}$	0.58	0.19	0.55	0.92	0.51	0.16	0.77	0.34	1.01

while downside volatility-managed portfolios tend to exhibit positive alphas at trading costs up to 25 basis points. Both total and downside volatility-managed portfolios tend to exhibit negative alphas at the trading cost of 50 basis points.

Overall, our results suggest that some investors, particularly trading cost savy institutional investors, may be able to implement volatility-managed strategies profitably. However, for investors facing high trading cost and for anomaly portfolios that are expensive to construct and trade, the volatility-managed trading strategies are unlikely to be profitable. Finally, downside volatility-managed portfolios tend to outperform total volatility-managed portfolios at low levels of trading costs, but the outperformance evaporates at high levels of trading costs.

3.2. Decomposition

To understand the sources of the superior performance of downside volatility-managed portfolios, consider an investor who allocates between a risky asset and a risk-free asset. To maximize the unconditional Sharpe ratio of the investor's portfolio, the optimal weight placed on the risky asset should be proportional to the ratio between the conditional expected return and the conditional variance (Daniel and Moskowitz, 2016; Moreira and Muir, 2019). Volatility-managed strategies, i.e., increasing (decreasing) the investment position when volatility was recently low (high), are therefore consistent with Sharpe ratio maximization if (i) lagged volatility is positively related to future volatility (volatility timing), and (ii) lagged volatility is not strongly and positively related to future returns (return timing). Volatility clustering is one of the most robust stylized facts in finance, so (i) is likely to be true. The literature is ambiguous about the volatility-return relation (e.g., French et al., 1987; Glosten et al., 1993; Brandt and Kang, 2004), so (ii) is uncertain. If the conditional expected return is positively related to lagged volatility, then the benefit of volatility timing is likely to be offset by the cost of negative return timing and, as a result, volatility-managed strategies will not work. If the conditional expected return is uncorrelated or even negatively correlated with lagged volatility, then volatility-managed strategies are likely to perform well because they take advantage of the attractive risk-return trade-off when volatility is low and avoid the poor risk-return trade-off when volatility is high.

We formalize the above idea by building on prior work on conditional asset pricing models (Lewellen and Nagel, 2006; Boguth et al., 2011; Cederburg and O'Doherty, 2016) and decomposing the spanning regression alpha of volatility managed strategies into return-timing and volatility-timing components. The return-timing component reflects the relation between lagged volatility and the conditional returns, and the volatility-timing component reflects the relation between lagged volatility and future volatility.

We begin with the definition of the volatility-managed portfolio in Eq. (1), $f_{\sigma,t} = w_t f_t$, where $w_t = c^*/\sigma_{t-1}$. Taking unconditional expectations, we obtain

$$\mathrm{E}(f_{\sigma,t}) = \mathrm{E}(w_t)\mathrm{E}(f_t) + \mathrm{cov}(w_t, f_t). \tag{8}$$

The spanning regression alpha of $f_{\sigma,t}$ relative to f_t is given by

$$\hat{\alpha} = \mathrm{E}(f_{\sigma,t}) - \hat{\beta}\mathrm{E}(f_t) \tag{9}$$

$$= \mathrm{E}(f_t)[\mathrm{E}(w_t) - \hat{\beta}] + \mathrm{cov}(w_t, f_t). \tag{10}$$

Let $w_t = \mathrm{E}(w_t) + e_t$, where e_t is the time-varying component of the investment position in the original portfolio, f_t. The unconditional beta is

$$\hat{\beta} = \frac{\mathrm{cov}(f_{\sigma,t}, f_t)}{\mathrm{Var}(f_t)} \tag{11}$$

$$= \frac{\mathrm{cov}[(f_t(\mathrm{E}(w_t) + e_t), f_t]}{\mathrm{Var}(f_t)} \tag{12}$$

$$= \frac{\mathrm{E}(w_t)\mathrm{Var}(f_t) + \mathrm{cov}(e_t, f_t^2) - \mathrm{cov}(e_t, f_t)\mathrm{E}(f_t)}{\mathrm{Var}(f_t)} \tag{13}$$

$$= \mathrm{E}(w_t) - \frac{\mathrm{E}(f_t)}{\mathrm{Var}(f_t)}\mathrm{cov}(w_t, f_t) + \frac{\mathrm{cov}(w_t, f_t^2)}{\mathrm{Var}(f_t)} \tag{14}$$

Substituting Eq. (14) into Eq. (10), we obtain

$$\hat{\alpha} = \left(1 + \frac{\mathrm{E}^2(f)}{\mathrm{Var}(f_t)}\right)\mathrm{cov}(w_t, f_t) - \frac{\mathrm{E}(f_t)}{\mathrm{Var}(f_t)}\mathrm{cov}(w_t, f_t^2). \tag{15}$$

Eq. (15) shows that the spanning regression alpha can be decomposed into return-timing and volatility-timing components, $\hat{\alpha} = \mathrm{RT} + \mathrm{VT}$, where $\mathrm{RT} = (1 + \frac{\mathrm{E}^2(f)}{\mathrm{Var}(f_t)})\mathrm{cov}(w_t, f_t)$ and $\mathrm{VT} = -\frac{\mathrm{E}(f_t)}{Var(f_t)}\mathrm{cov}(w_t, f_t^2)$. The return-timing component depends on the covariance between the investment weight and portfolio returns,

Table 5
Decomposition for the 9 equity factors. This table provides alpha decomposition of volatility-managed factors attributable to return timing and volatility timing. The return-timing effect is estimated as $(1+\frac{E^2(f_t)}{Var(f_t)})cov(\frac{c^*}{\sigma_{t-1}}, f_t)$, and the volatility-timing effect is estimated as $-\frac{E(f_t)}{Var(f_t)}cov(\frac{c^*}{\sigma_{t-1}}, f_t^2)$, where σ_{t-1} is a volatility measure from month $t-1$, and f_t is the monthly return for the original factor, and c^* is a constant chosen such that the original strategy and the volatility-managed strategy have the same full-sample volatility. $E(f_t)$ and $Var(f_t)$ are the expected return and variance of the original factor returns. Panel A reports results for total-volatility managed factors, and Panel B provides results for downside volatility-managed factors. All results are converted to annualized, percentage terms.

	MKT	*SMB*	*HML*	*MOM*	*RMW*	*CMA*	*ROE*	*IA*	*BAB*
Panel A: Total volatility-managed strategy									
Return Timing	0.09	1.35	1.16	2.88	0.28	1.07	0.87	0.60	0.51
Volatility Timing	3.43	0.91	2.65	6.49	1.64	1.15	2.45	1.35	4.50
Total	3.34	0.44	1.48	9.36	1.35	0.08	3.32	0.75	3.99
Panel B: Downside volatility-managed strategy									
Return Timing	0.79	0.29	1.13	2.93	1.34	0.09	2.01	0.49	2.33
Volatility Timing	4.04	0.83	2.34	5.39	1.49	0.97	2.40	1.22	3.83
Total	4.83	1.11	3.47	8.32	2.83	0.88	4.41	1.70	6.16

and the volatility-timing component is determined by the covariance between the investment weight and the second moment of the portfolio returns. Given that $w_t = c^*/\sigma_{t-1}$, the return-timing component will be positive when lagged volatility is negatively related to current factor return, and the volatility-timing component will be positive when lagged volatility is positively related to current factor volatility.

A positive spanning regression alpha can arise either from return timing or volatility timing, or both. To assess the relative contribution of volatility timing and return timing to the performance of volatility-managed portfolios, we perform a decomposition according to Eq. (15). We present the results for the nine equity factors in Table 5. Panel A reports the alpha decomposition for total volatility-managed factors. We find that all nine volatility-managed factors have positive volatility-timing components, consistent with volatility persistence. However, the return-timing component is negative among seven of the nine equity factors. In the case of *SMB*, the negative return timing effect (1.35%) is large enough to offset the positive volatility timing effect (0.91%), resulting in a negative spanning regression alpha of-0.44% per year. Overall, we find that the positive spanning regression alphas of total volatility-managed factors are primarily due to volatility timing, and the return-timing component is often negative. It is worth noting that volatility-managed *MOM* has large and positive volatility timing as well as return timing components. This explains why the volatility-managed momentum strategies perform so well in Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016).

Panel B of Table 5 presents the decomposition results for downside volatility-managed factors. We find that, similar to total volatility-managed factors, the volatility-timing component is positive across all nine factors. In addition, the magnitudes of the volatility timing component are similar between total volatility-managed factors and downside volatility-managed factors. For example, total volatility-managed *ROE* has a volatility-timing component of 2.45% per year, while the downside volatility-managed *ROE* has a volatility-timing component of 2.40% per year. In contrast to the results for total volatility-managed factors, we find that the return-timing component for downside volatility-managed factors is positive for eight of the nine equity factors (the only exception is *CMA*). Recall that the return-timing component is negative among seven of the nine factors for total volatility-managed factors. Therefore, although volatility timing contributes significantly to the superior performance of downside volatility-managed factors, the return-timing component is the reason why downside volatility-managed factors outperform total volatility-managed factors. For example, the return-timing component for total volatility-managed *SMB* is-1.35%, but it increases to 0.29% for downside volatility-managed *SMB*. Given that volatility-timing components are similar, the spanning regression alpha increases from-0.44% for total volatility-managed *SMB* to 1.11% for downside volatility-managed *SMB*. This pattern of negative return timing changing to positive return timing also applies to five other equity factors, i.e., *MKT*, *HML*, *RMW*, *IA* and *BAB*. The improvement in the return timing component is also quite large for *ROE* (from 0.87% to 2.01%) and *CMA* (from-1.07% to-0.09%). For the remaining factor *MOM*, downside volatility-managed factor has a similar return-timing component to the total volatility-managed factor.

Table 6
Decomposition for the 94 anomalies. This table summarizes results from alpha decomposition of volatility-managed factors attributable to return timing and volatility timing for the 94 anomalies. The return-timing effect is estimated as $(1+\frac{E^2(f_t)}{Var(f_t)})cov(\frac{c^*}{\sigma_{t-1}}, f_t)$, and the volatility-timing effect is estimated as $-\frac{E(f_t)}{Var(f_t)}cov(\frac{c^*}{\sigma_{t-1}}, f_t^2)$, where σ_{t-1}, where σ_{t-1} is a volatility measure from month $t-1$, and f_t is the monthly return for the original portfolio. c^* is a constant chosen such that the original strategy and the volatility-managed strategy have the same full-sample volatility. $E(f_t)$ and $Var(f_t)$ are the expected return and variance of the original portfolio returns. Panel A reports results for total volatility-managed anomalies, and Panel B provides results for downside volatility-managed anomalies. The table reports the number of return-(volatility-) timing that are positive and negative.

		Return timing		Volatility timing	
		Positive	Negative	Positive	Negative
Panel A: Total volatility-managed strategy					
All	94	42	52	80	14
Accruals	10	3	7	10	0
Intangibles	10	2	8	7	3
Investment	9	3	6	9	0
Momentum	8	8	0	7	1
Profitability	20	10	10	16	4
Trading	19	11	8	14	5
Value	18	5	13	17	1
Panel B: Downside volatility-managed strategy					
All	94	71	23	80	14
Accruals	10	7	3	10	0
Intangibles	10	8	2	7	3
Investment	9	5	4	9	0
Momentum	8	7	1	7	1
Profitability	20	19	1	16	4
Trading	19	12	7	14	5
Value	18	13	5	17	1

Table 6 summarizes alpha decomposition results for the 94 anomalies. We find qualitatively similar results to those in Table 5. For both total and downside volatility-managed portfolios, we find that the volatility-timing component is positive among 80 out of 94 anomalies. There is, however, a large difference in the return timing component between total volatility and downside volatility-managed portfolios. For total volatility-managed portfolios, the return-timing component is positive in just 42 of the 94 anomalies. In contrast, 71 of the 94 anomalies exhibit a positive return-timing component for downside volatility-managed strategies. The positive return-timing component associated with downside volatility-managed strategies suggests that downside volatility tends to negatively predict future returns. At first glance, this result appears to be at odds with prior finding (e.g., Kelly and Jiang, 2014) that downside risk or tail risk commands a return premium. We note that a critical difference between our analysis and prior studies is that we focus on the downside risk of equity factors and anomaly portfolios rather than the downside risk of individual stock returns.

To summarize, our decomposition analysis in this section indicates that volatility timing, i.e., volatility persistence, is an important reason for the positive spanning regression alphas for both total and downside volatility-managed portfolios. The volatility-timing component, however, is similar across total volatility and downside volatility managed portfolios. The enhanced performance of managing downside volatility relative to managing total volatility, therefore, primarily arises from the return-timing component. Although downside volatility and total volatility are highly correlated with each other, they differ significantly in their predictive content for future returns. It is this difference that explains the superior performance of downside volatility-managed portfolios.

3.3. Real-time strategies

The spanning regression results indicate substantial in-sample benefits of volatility management. Cederburg et al. (2020), however, point out that the trading strategies implied by the spanning regressions are not implementable in real time because they require investors to combine the volatility-managed portfolio and the unmanaged portfolio using ex post optimal weights, which are not known to real-time investors. A natural question is whether real-time investors can capture the economic gains implied by the spanning regression. Therefore, in our second approach we evaluate the out-of-sample performance of real-time trading strategies implied by the spanning regressions. Prior literature suggests that estimation risk and parameter instability are key factors in the out-of-sample, mean-variance portfolio choice problem, making real-time portfolios often underperform relative to their in-sample optimal counterparts.

3.3.1. Methodology

As in Cederburg et al. (2020), our out-of-sample tests focus on quantifying the impact of incorporating a volatility-managed portfolio in the investment opportunity set. We follow Cederburg et al. (2020) and compare the performance of two real-time strategies: (1) a strategy that allocates between a given volatility-managed portfolio, its corresponding original, unmanaged portfolio, and a risk-free asset; and (2) a strategy constrained to invest only in the original portfolio and the risk-free asset. For ease of exposition, we refer to the first strategy as the “combination strategy” and the second one as the “unmanaged strategy”.

For each asset pricing factor and stock market anomaly, we start with T monthly excess return observations. We use the first K months as the training period to estimate the return moments to decide the weights to construct the combination strategy and the unmanaged strategy, respectively. We evaluate the portfolio performance over the out-of-sample period of $T-K$ months. Following Cederburg et al. (2020), we set our initial training period as $K=120$ months, and employ an expanding-window approach to estimate the relevant parameters. At the beginning of each month t in the out-of-sample period, we first estimate the real-time scaling parameter, c_t^*, as the constant that allows the original and volatility-managed portfolios to have the same volatility over the training period preceding month t.

To determine the portfolio weights for the combination strategy in month t, consider an investor with mean-variance utility who is allocating between volatility-managed portfolio ($f_{\sigma,t}$), and unmanaged portfolio (f_t). The optimal allocation to $f_{\sigma,t}$ and f_t is the solution to the following problem:

$$\max_{w_t} U(w_t) = w_t^\top \hat{\mu}_t - \frac{\gamma}{2} w_t^\top \hat{\Sigma}_t w_t, \tag{16}$$

where $\hat{\mu}_t = [\bar{f}_{\sigma,t}, \bar{f}_t]^\top$ is the vector of mean excess returns and $\hat{\Sigma}_t$ is the variance-covariance matrix over the training period before month t, and γ is the investor's risk aversion parameter.[12] The vector of optimal weights on $f_{\sigma,t}$ and f_t for month t is

$$w_t = \begin{bmatrix} x_{\sigma,t} \\ x_t \end{bmatrix} = \frac{1}{\gamma} \hat{\Sigma}_t^{-1} \hat{\mu}_t. \tag{17}$$

The setup implicitly allows the investor to have access to a risk-free asset. The investor's optimal policy allocates a weight of $x_{\sigma,t}$ to the volatility-managed portfolio and a weight of x_t to the unmanaged portfolio. Given the definition of the volatility-managed portfolio, i.e., $f_{\sigma,t} = \frac{c_t^*}{\sigma_{t-1}} f_t$, the combination strategy can be considered based on a dynamic investment rule on the unmanaged portfolio, with the weight (w_t^c) of $x_t + \frac{c_t^*}{\sigma_{t-1}} x_{\sigma,t}$. Therefore, the excess return of the combination strategy for month t is $w_t^c f_t$. Similarly, for the unmanaged strategy the optimal weight (w_t^u) on f_t is simply $\frac{1}{\gamma}\frac{\bar{\mu}}{\hat{\sigma}^2}$, where $\bar{\mu}$ and $\hat{\sigma}^2$ are the mean and variance of f_t over the training period preceding month t. Accordingly, we construct the portfolio excess return for the unmanaged strategy as $w_t^u f_t$.

The magnitude of w_t^c and w_t^u is essentially a measure of leverage. Extreme leverage may occur in out-of-sample analysis for two reasons. First, volatility-managed portfolios, by definition, call for substantial leverage following periods of low volatility. Second, mean-variance optimization often leads to extreme values of portfolio weights. Following Cederburg et al. (2020), we impose a leverage constraint of $|w_t^c|(|w_t^u|) \le 5$. The above out-of-sample real-time trading strategy results in a time series of T - K monthly excess returns for the combination strategy and the unmanaged strategy, respectively. We compute the Sharpe ratio for each strategy and the difference in Sharpe ratio between the two strategies. We assess whether the Sharpe ratio difference is statistically significant following the approach of Kirby and Ostdiek (2012).[13]

3.3.2. Results

Table 7 reports results for the out-of-sample performance of the combination strategy and the unmanaged strategy for the nine equity factors. We consider two combination strategies, one based on total volatility-managed factors and the other based on downside

[12] We follow Cederburg et al. (2020) and assume γ is equal to 5. Our results are robust to alternative risk aversion values.

[13] Let $\hat{\mu}_i$ and $\hat{\sigma}_i$ be the mean and standard deviation of excess returns for portfolio i over a period of length T. Similarly, $\hat{\mu}_j$ and $\hat{\sigma}_j$ are the mean and standard deviation for portfolio j, and $\hat{\sigma}_{i,j}$ is the covariance between excess returns for the two portfolios. $\hat{\lambda}_i$ and $\hat{\lambda}_j$ denote the estimated Sharpe ratios for portfolios i and j. To test the null hypothesis of equal Sharpe ratios for portfolios i and j, we compute the test statistic, which is asymptotically distributed as a standard normal:

$$\hat{z} = \sqrt{T}\left(\frac{\hat{\lambda}_j - \hat{\lambda}_i}{\sqrt{\hat{V}_\lambda}}\right),$$

To estimate $\hat{V}_\lambda$, we follow Kirby and Ostdiek (2012) and use the generalized method of moments to construct the following estimator. Let

$$e_t(\hat{\theta}) = \begin{pmatrix} r_i - \hat{\sigma}_i\hat{\lambda}_i \\ r_j - \hat{\sigma}_j\hat{\lambda}_j \\ (r_i - \hat{\sigma}_i\hat{\lambda}_i)^2 - \hat{\sigma}_i^2 \\ (r_j - \hat{\sigma}_j\hat{\lambda}_j)^2 - \hat{\sigma}_j^2 \end{pmatrix}$$

where $\hat{\theta} = (\hat{\lambda}_i, \hat{\lambda}_j, \hat{\sigma}_i^2, \hat{\sigma}_j^2)'$. $\sqrt{T}(\hat{\theta} - \theta) \overset{a}{\sim} N(0, \hat{D}^{-1}\hat{S}\hat{D}^{-1'})$, where $\hat{D} = (1/T)\sum_{t=L+1}^{T+L} \partial e_t(\hat{\theta})\partial\hat{\theta}'$ and $\hat{S} = \hat{\Gamma}_0 + \sum_{l=1}^{m}(1 - l/(m+1))(\hat{\Gamma}_l + \hat{\Gamma}_l')$ with $\hat{\Gamma}_l = (1/T)\sum_{t=L+l+1}^{T+L} e_t(\hat{\theta})e_{t-l}(\hat{\theta})'$. We follow Kirby and Ostdiek (2012) and set $m=5$. $\hat{V}_\lambda = \hat{V}_{22} - 2\hat{V}_{21} + \hat{V}_{11}$, where $\hat{V} \equiv \hat{D}^{-1}\hat{S}\hat{D}^{-1'}$.

Table 7

Real time performance for the 9 equity factors. The table reports results for real-time strategies that combine original factors and volatility-managed factors. The initial training period length (K) is 120 months. We use an expanding-window design for the out-of-sample tests, and the out-of-sample period runs from month K+1 to month T, where T is the total number of sample months for a given factor. The "unmanaged strategy" ([S1]) results correspond to the real-time combination of the original factor and the risk-free asset, and the "combination strategy" results correspond to the real-time combination of the original factor, the volatility-managed factor, and the risk-free asset. [S2] refers to the combination strategy based on total volatility-managed factor, and [S3] refers to the combination strategy based on downside volatility-managed factor. For each strategy, the table shows the annualized Sharpe ratio in percentage per year over the out-of-sample period. The numbers in brackets are p-values for the Sharpe ratio differences. The p-values are computed following the approach in Kirby and Ostdiek (2012). We use a risk aversion parameter of $\gamma = 5$ and impose a leverage constraint that the sum of absolute weights on the risky factors is less than or equal to five.

	MKT	*SMB*	*HML*	*MOM*	*RMW*	*CMA*	*ROE*	*IA*	*BAB*
[S1] Unmanaged strategy	0.46	0.18	0.39	0.49	0.34	0.51	0.78	0.63	0.83
[S2] Combination Strategy - Total volatility	0.48	0.12	0.36	0.97	0.43	0.46	1.17	0.63	1.09
[S3] Combination Strategy - Down volatility	0.51	0.22	0.58	0.75	0.65	0.50	1.13	0.71	1.22
[S2]-[S1]	0.02	0.06	0.03	0.48	0.09	0.05	0.39	0.00	0.27
	[0.76]	[0.31]	[0.63]	[0.00]	[0.65]	[0.05]	[0.00]	[0.97]	[0.00]
[S3]-[S1]	0.05	0.04	0.19	0.26	0.31	0.00	0.34	0.08	0.39
	[0.63]	[0.62]	[0.01]	[0.03]	[0.13]	[0.94]	[0.00]	[0.24]	[0.00]
[S3]-[S2]	0.02	0.10	0.23	0.22	0.22	0.04	0.04	0.08	0.12
	[0.65]	[0.21]	[0.00]	[0.00]	[0.00]	[0.30]	[0.46]	[0.02]	[0.00]

volatility-managed factors. We present the Sharpe ratio of the unmanaged strategy as well as the Sharpe ratios of two combination strategies over the evaluation period from month $K+1$ to month T. In addition, we report three pairwise Sharpe ratio differences among the three strategies.

The results indicate that, the combination strategy that incorporates total volatility-managed factors significantly outperforms the unmanaged strategy among three of the nine factors, i.e., *MOM*, *ROE*, and *BAB*. That is, the difference in Sharpe ratio between the combination strategy and the unmanaged strategy is positive and statistically significant for these three factors. Across the remaining six factors, the difference in Sharpe ratio is not statistically significant. Half of them (*MKT*, *RMW*, and *IA*) show positive differences, and the other half (*SMB*, *HML*, and *CMA*) show negative differences.

Incorporating downside volatility-managed factors improves the real-time performance of the combination strategy. Specifically, the difference in Sharpe ratio between the combination strategy and the unmanaged strategy is positive for eight equity factors, and is statistically significant at the 5% level for four of them (*HML*, *MOM*, *ROE*, and *BAB*). For *MKT*, *SMB*, *RMW*, and *IA*, the combination strategy outperforms the unmanaged strategy, but the difference is not statistically significant. For the remaining factor, *CMA*, the combination strategy slightly underperforms the unmanaged strategy (0.50 versus 0.51).

To assess the relative merit of downside volatility management versus total volatility management, we can also directly compare the performance of the combination strategy that incorporates downside volatility-managed factors with the combination strategy that incorporates total volatility-managed factors. This direct comparison indicates that the Sharpe ratio of the combination strategy based on downside volatility-managed factors is higher than that based on total volatility-managed factors across seven equity factors (*MKT*, *SMB*, *HML*, *RMW*, *CMA*, *IA*, and *BAB*). The performance improvement ranges from 0.02 to 0.22, and statistically significant among four factors. For example, the Sharpe ratio of the combination strategy is 0.36 for total volatility-managed *HML*, and is 0.58 for downside-volatility-managed *HML*. For the remaining two factors (*MOM* and *ROE*), the combination strategy based on total volatility-managed factors performs better than the combination strategy based on downside volatility-managed factors.

We repeat the above real-time analyses for the broader sample of 94 anomalies. Table 8 summarizes the results. Specifically, the table shows the number of positive and negative Sharpe ratio differences between the combination strategy and the unmanaged strategy. Panel A presents the results for total volatility-managed portfolios. We find that the combination strategy outperforms the unmanaged strategy among only 44 of the 94 anomalies. For the remaining 50 anomalies, the combination strategy underperforms the unmanaged strategy. In contrast, we find in Panel B that the combination strategy that incorporates downside volatility-managed portfolios outperforms the original, unmanaged strategy in 62 of the 94 anomalies. A simple binomial test of the null hypothesis that the combination strategy performs the same as the unmanaged strategy is rejected with a two-sided p-value of 0.002. In Panel C, we find that downside volatility-based combination strategy outperforms total volatility-based combination strategy among 69 out of 94 anomalies, while underperforming among 25 anomalies. Our findings in this section are consistent with Cederburg et al. (2020) that managing total volatility is not systematically advantageous for real-time investors. However, managing downside volatility significantly improves the performance of the combination strategy and is beneficial for real-time investors.

3.3.3. Fixed-weight strategies

Cederburg et al. (2020) point out that the relatively poor out-of-sample performance of the real-time combination strategy is primarily due to parameter instability and estimation risk. DeMiguel et al. (2009) note that optimal portfolios constructed from sample moments often exhibit extreme weights that fluctuate dramatically over time. They further demonstrate that such strategies often underperform simpler approaches to portfolio formation including a naïve rule of equally weighting the assets under consideration. Prior literature (e.g., Jobson (1979) and Chopra and Ziemba (1993)) also shows that the global minimum variance (GMV) portfolio often performs better than other mean-variance efficient portfolios because we can estimate its weights without estimating expected returns, which alleviates a large part of the estimation risk.[14] With two assets, the GMV portfolio has an estimated weight on the first asset of the form

$$\hat{w}_1 = \frac{\hat{\sigma}_2^2 - \hat{\sigma}_1\hat{\sigma}_2\hat{\rho}_{12}}{\hat{\sigma}_1^2 + \hat{\sigma}_2^2 - 2\hat{\sigma}_1\hat{\sigma}_2\hat{\rho}_{12}}, \tag{18}$$

where $\hat{\rho}_{12}$ is the estimated correlation between the returns for the two assets. Now suppose the two assets under consideration are the original, unmanaged factor (f_t) and the volatility-managed factor ($f_{\sigma,t}$). Because $f_{\sigma,t}$ is constructed such that it has the same estimated variance as f_t, the estimated weights of the GMV portfolio are simply $\hat{w}_1 = 1/2$ and $\hat{w}_2 = 1/2$ for all values of $\hat{\rho}_{12}$. This provides a rationale for looking at a naïve diversification strategy.

Motivated by the above arguments, we next examine combination strategies that assign fixed relative weights to the volatility-

[14] Merton (1980) shows that expected returns are particularly difficult to estimate precisely.

Table 8

Real time performance for the 94 anomalies. The table summarizes results for real-time portfolio strategies that combine original portfolios and volatility-managed portfolios for the 94 anomalies. The initial training period length (K) is 120 months. We use an expanding-window design for the out-of-sample tests, and the out-of-sample period runs from month $K+1$ to month T, where T is the total number of sample months for a given anomaly. The "unmanaged strategy" is based on the real-time combination of the original factor and the risk-free asset, and the "combination strategy" corresponds to the real-time combination of the original factor, the volatility-managed factor, and the risk-free asset. For each anomaly, we compute the difference between two strategies. The table reports the number of Sharpe ratio differences that are positive, positive and significant at the 5% level, negative, and negative and significant at the 5% level. Statistical significance of the Sharpe ratio is based on the approach in Kirby and Ostdiek (2012). We use a risk aversion parameter of $\gamma = 5$ and impose a leverage constraint that the sum of absolute weights on the risky factors is less than or equal to five.

		Sharpe ratio difference	
		Positive [Signif.]	Negative [Signif.]
Panel A: Combination strategy (Total volatility)-Unmanaged strategy			
All	94	44[6]	50[4]
Accruals	10	3[0]	7[1]
Intangibles	10	5[0]	5[0]
Investment	9	3[0]	6[1]
Momentum	8	8[3]	0[0]
Profitability	20	8[1]	12[0]
Trading	19	10[0]	9[1]
Value	18	7[2]	11[1]
Panel B: Combination strategy (Downside volatility)-Unmanaged strategy			
All	94	62[8]	32[1]
Accruals	10	6[0]	4[0]
Intangibles	10	7[2]	3[0]
Investment	9	4[0]	5[0]
Momentum	8	6[3]	2[0]
Profitability	20	17[1]	3[0]
Trading	19	11[0]	8[1]
Value	18	11[2]	7[0]
Panel C: Combination strategy (Downside volatility)-Combination strategy (Total volatility)			
All	94	69[11]	25[4]
Accruals	10	8[1]	2[0]
Intangibles	10	8[1]	2[0]
Investment	9	8[0]	1[0]
Momentum	8	1[0]	7[2]
Profitability	20	19[6]	1[0]
Trading	19	12[3]	7[1]
Value	18	13[0]	5[1]

managed and original portfolios. In addition to the naïvely diversified portfolio (i.e., $w = 50\%$), we also consider the following relative weights in the volatility-managed portfolios: 10%, 25%, 75%, and 90%. These fixed weights are unlikely to be optimal ex post, but employing them removes the need to estimate portfolio weights in real time and therefore may improve performance.

The real-time portfolio construction is similar to that outlined in Section 3.3.1. We set an initial training sample of $K = 120$ months paired with an expanding estimation window. At the beginning of month t in the out-of-sample period, we first compute the scaling parameter for the volatility-managed portfolio, c_t^*, and then form a combination of the volatility-scaled and unscaled portfolios using the specified static weight vector, for example, 50% invested in the volatility-managed portfolio and 50% in the original strategy. Finally, the investor optimally allocates between the fixed-weight risky portfolio and the risk-free asset. We benchmark this strategy by comparing it with a portfolio that allocates between the original, unmanaged factor and the risk-free asset in real time. The positive in-sample spanning regression intercepts for many volatility-scaled factors and anomaly portfolios suggest that we should expect some static combination to perform well in each case. The more interesting question is whether a specific fixed weight leads to consistent and economically large gains across the broad set of strategies under consideration.

Table 9 presents the results for the nine equity factors. Panel A compares the combination strategy that incorporates total volatility-managed factors with the unmanaged strategy. For brevity, we only report the Sharpe ratio difference between these two strategies. A positive (negative) number suggests that the combination strategy outperforms (underperforms) the unmanaged strategy. We find that real-time strategies with fixed weights generally produce better out-of-sample performance than that for the standard real-time strategies. For example, when the relative weight for the volatility-managed portfolio is fixed at 25%, the out-of-sample performance for the combination strategy is better than that of the unmanaged strategy in eight out of the nine equity factors. The only exception is *SMB*, where the combination strategy underperforms by 0.02 with a p-value of 0.39. Across all fixed weights we examine, the combination strategy outperforms the unmanaged strategy for at least seven of the nine equity factors. Recall that in the previous section we document that standard combination strategy underperforms the unmanaged strategy among three of the nine factors.

Panel B of Table 9 shows results for the fixed-weight strategies that incorporate downside volatility-managed factors. The results are striking. We find that, for each of the fixed weights we consider, the combination strategy outperforms the unmanaged strategy across all nine equity factors. The Sharpe ratio difference between the combination strategy and the unmanaged strategy is statistically significant in most cases. For example, for the fixed weight of 25% in the downside volatility-managed factor, seven of the nine Sharpe ratio differences are statistically significant at the 5% level and one at the 10% level. These results are stronger than the standard real-time strategies we examined in the previous section, where only four of the Sharpe ratio differences are statistically significant at the 5% level and one Sharpe ratio difference is actually negative. The results are also significantly stronger than those reported in Panel A, and continue to suggest that downside volatility-managed strategies tend to outperform total volatility-managed strategies.

Table 9
Fixed-weight real time analysis for the 9 equity factors. The table reports results for real-time portfolio strategies combining original factors and volatility-managed factors with fixed relative weights. For each factor and out-of-sample design, we present the difference between the Sharpe ratio of the strategy that combines the original factor, the volatility-managed factor, and the risk-free asset (with fixed relative weights on the two risky assets) and that of the strategy that combines the original factor and the risk-free asset. The initial training period length (K) is 120 months. We use an expanding-window design for the out-of-sample tests, and the out-of-sample period runs from month K+1 to month T, where T is the total number of sample months for a given anomaly. Panel A reports results for total volatility-managed strategies and Panel B reports those for the downside volatility-managed strategies. The numbers in brackets are p-values for the Sharpe ratio differences, following the approach in Kirby and Ostdiek (2012).

$(w_{\sigma,t}, w_t)$	*MKT*	*SMB*	*HML*	*MOM*	*RMW*	*CMA*	*ROE*	*IA*	*BAB*
Panel A: Total volatility									
(0.10, 0.90)	0.01	0.01	0.01	0.07	0.02	0.00	0.03	0.01	0.04
	[0.10]	[0.46]	[0.18]	[0.00]	[0.00]	[0.86]	[0.00]	[0.13]	[0.00]
(0.25, 0.75)	0.03	0.02	0.02	0.16	0.05	0.00	0.09	0.02	0.10
	[0.14]	[0.39]	[0.25]	[0.00]	[0.00]	[0.97]	[0.00]	[0.17]	[0.00]
(0.50, 0.50)	0.05	0.04	0.03	0.29	0.10	0.01	0.18	0.03	0.18
	[0.24]	[0.29]	[0.41]	[0.00]	[0.00]	[0.84]	[0.00]	[0.35]	[0.00]
(0.75, 0.25)	0.05	0.07	0.03	0.38	0.15	0.02	0.28	0.03	0.25
	[0.35]	[0.22]	[0.59]	[0.00]	[0.01]	[0.66]	[0.00]	[0.52]	[0.00]
(0.90, 0.10)	0.05	0.09	0.02	0.42	0.18	0.03	0.33	0.03	0.28
	[0.41]	[0.20]	[0.69]	[0.00]	[0.02]	[0.55]	[0.00]	[0.62]	[0.00]
Panel B: Downside volatility									
(0.10, 0.90)	0.02	0.01	0.04	0.06	0.04	0.01	0.04	0.02	0.06
	[0.02]	[0.18]	[0.00]	[0.00]	[0.00]	[0.07]	[0.00]	[0.01]	[0.00]
(0.25, 0.75)	0.05	0.03	0.08	0.14	0.10	0.03	0.11	0.05	0.15
	[0.03]	[0.21]	[0.00]	[0.00]	[0.00]	[0.08]	[0.00]	[0.01]	[0.00]
(0.50, 0.50)	0.08	0.05	0.14	0.23	0.20	0.06	0.22	0.07	0.28
	[0.08]	[0.27]	[0.00]	[0.00]	[0.00]	[0.14]	[0.00]	[0.04]	[0.00]
(0.75, 0.25)	0.09	0.06	0.17	0.27	0.29	0.06	0.34	0.09	0.37
	[0.15]	[0.34]	[0.00]	[0.00]	[0.00]	[0.26]	[0.00]	[0.07]	[0.00]
(0.90, 0.10)	0.09	0.06	0.19	0.28	0.35	0.06	0.41	0.10	0.41
	[0.21]	[0.39]	[0.00]	[0.01]	[0.00]	[0.38]	[0.00]	[0.10]	[0.00]

Table 10
Fixed-weight real time analysis for the 94 anomalies. The table reports results for portfolio strategies combining original anomalies and volatility-managed anomalies with fixed relative weights. For each anomaly and out-of-sample design, we present the difference between the Sharpe ratio of the strategy that combines the original portfolio, the volatility-managed portfolio, and the risk-free asset (with fixed relative weights on the two risky assets) and that of the strategy that combines the original portfolio and the risk-free asset. The initial training period length (K) is 120 months. We use an expanding-window design for the out-of-sample tests. Panel A reports results for total volatility-managed strategies, and Panel B reports those for the downside volatility-managed strategies. This table presents summary results of the number of Sharpe ratio differences that are positive, positive and significant at the 5% level, negative, and negative and significant at the 5% level for the 94 anomaly portfolios. The p-values are computed following the approach in Kirby and Ostdiek (2012).

		Sharpe ratio difference	
		Positive [Signif.]	Negative [Signif.]
Panel A: Total volatility			
(0.10, 0.90)	94	60[13]	34[1]
(0.25, 0.75)	94	60[12]	34[1]
(0.50, 0.50)	94	57[10]	37[2]
(0.75, 0.25)	94	57[8]	37[3]
(0.90, 0.10)	94	57[7]	37[3]
Panel B: Downside volatility			
(0.10, 0.90)	94	72[31]	22[2]
(0.25, 0.75)	94	72[29]	22[2]
(0.50, 0.50)	94	72[27]	22[1]
(0.75, 0.25)	94	72[23]	22[1]
(0.90, 0.10)	94	71[19]	23[1]

We repeat the fixed weight real-time analysis for the extended sample of 94 anomaly portfolios and report the results in Table 10. We consider the same set of fixed weights as in Table 9. As in Table 8, we present the number of positive and negative Sharpe ratio differences between the combination strategy and the unmanaged strategy across the 94 anomaly portfolios. The main takeaways are in line with those from Table 9. In Panel A, we find that the combination strategy that incorporates total volatility-managed portfolios tends to outperform the unmanaged strategy. Depending on the specific weight, between 57 and 60 (out of 94) stock market anomalies exhibit positive Sharpe ratio differences. Recall that the corresponding number is only 44 for standard real-time strategies. In Panel B, for downside volatility-managed portfolios, the combination strategy with fixed weights outperforms the original, unmanaged strategy in 71–72 anomalies. These numbers are significantly higher than the 57–60 for total volatility-managed strategies reported in Panel A. It is also significantly higher than the corresponding number for the standard real-time strategies (i.e., 62) reported in Table 8. Overall, we continue to find that downside volatility-managed portfolios dominate the performance of total volatility-managed portfolios. Moreover, using fixed portfolio weights leads to significant improvements in out-of-sample performance of the combination strategy.

3.4. Direct performance comparisons

Most prior studies (e.g., Barroso and Santa-Clara, 2015; Daniel and Moskowitz, 2016; Barroso and Maio, 2018; Cederburg et al., 2020; Eisdorfer and Misirli, 2020) assess the value of volatility management by directly comparing the Sharpe ratio of volatility-managed portfolios with the Sharpe ratio of original, unmanaged portfolios. For example, Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) demonstrate that volatility-managed momentum factor exhibits significantly higher Sharpe ratios than the original, unmanaged momentum factor. We follow these studies and employ direct performance comparison as our third approach to evaluating the performance of volatility-managed portfolios. In addition to Sharpe ratio, we also examine an alternate performance measure—Sortino ratio, which is defined as the average excess return divided by downside volatility. Sortino ratio is similar to Sharpe ratio in that it captures a reward-to-volatility ratio. Instead of using total volatility, the Sortino ratio scales the average excess return by downside volatility. This measure is appropriate for us because of our focus on downside risk.

Table 11
Direct comparisons for the 9 equity factors. The table reports the Sharpe ratio and the Sortino ratio for original, total volatility-managed and downside volatility-managed factors. Sharpe ratios and Sortino ratios are annualized. Panel A reports results for Sharpe ratio, and Panel B provides those for Sortino ratio. The table also reports the difference between the Sharpe ratio (Sortino ratio) of the total (downside) volatility-managed factor and that of the original factor, as well as the difference between total volatility-managed strategy and downside volatility-managed strategy. The numbers in brackets are *p*-values for the Sharpe ratio (Sortino ratio) differences and are computed following the approach in Kirby and Ostdiek (2012).

	MKT	*SMB*	*HML*	*MOM*	*RMW*	*CMA*	*ROE*	*IA*	*BAB*
Panel A: Sharpe ratio									
[S1] Original strategy	0.42	0.22	0.37	0.49	0.41	0.49	0.75	0.70	0.81
[S2] Total strategy	0.54	0.15	0.42	0.96	0.52	0.45	1.04	0.74	1.05
[S3] Downside strategy	0.59	0.27	0.57	0.82	0.68	0.53	1.05	0.85	1.14
[S2]-[S1]	0.12	0.07	0.05	0.47	0.11	0.05	0.28	0.04	0.24
	[0.05]	[0.18]	[0.40]	[0.00]	[0.19]	[0.43]	[0.00]	[0.48]	[0.00]
[S3]-[S1]	0.17	0.05	0.20	0.34	0.27	0.04	0.30	0.15	0.33
	[0.01]	[0.43]	[0.00]	[0.00]	[0.00]	[0.62]	[0.00]	[0.02]	[0.00]
[S3]-[S2]	0.05	0.12	0.15	0.14	0.16	0.08	0.01	0.11	0.09
	[0.09]	[0.01]	[0.00]	[0.01]	[0.00]	[0.15]	[0.84]	[0.01]	[0.05]
Panel B: Sortino ratio									
[S1] Original strategy	0.57	0.40	0.64	0.46	0.55	0.84	0.92	1.20	0.94
[S2] Total strategy	0.77	0.23	0.73	1.35	0.86	0.83	1.66	1.37	1.43
[S3] Downside strategy	0.88	0.44	1.17	1.13	1.45	0.94	2.06	1.70	2.13
[S2]-[S1]	0.20	0.17	0.08	0.90	0.32	0.01	0.74	0.17	0.48
	[0.04]	[0.49]	[0.01]	[0.00]	[0.01]	[0.00]	[0.00]	[0.04]	[0.00]
[S3]-[S1]	0.31	0.05	0.53	0.68	0.90	0.10	1.14	0.50	1.19
	[0.14]	[0.01]	[0.00]	[0.11]	[0.00]	[0.97]	[0.01]	[0.28]	[0.00]
[S3]-[S2]	0.11	0.21	0.45	0.22	0.58	0.11	0.40	0.33	0.70
	[0.41]	[0.05]	[0.68]	[0.00]	[0.04]	[0.01]	[0.43]	[0.16]	[0.27]

Table 11 presents results for the nine equity factors. Panel A presents the results for Sharpe ratio, while Panel B presents the results for Sortino ratio. In each panel, we report the results for the original factor, total-volatility-managed factor, and downside volatility-managed factor. We also report the three pairwise differences among these three versions of factors. We follow the approach of Kirby and Ostdiek (2012) to determine whether each difference is statistically significant.

In Panel A, we find that total volatility-managed *MKT*, *MOM*, *ROE* and *BAB* achieve statistically significant Sharpe ratio gains compared to the original, unmanaged factor. The remaining five factors exhibit differences in Sharpe ratios that are statistically insignificant. We also find that downside volatility-managed factors exhibit higher Sharpe ratios than both the original factors and total volatility-managed factors. The Sharpe ratio difference between the downside volatility-managed factor and the original factor is positive across all nine equity factors. Seven of these differences are statistically significant at the 5% level. Moreover, downside volatility-managed factors achieve higher Sharpe ratios than their total volatility-managed counterparts among eight of the nine factors, and statistically significant for four of these eight factors. For example, total volatility-managed *HML* exhibits a Sharpe ratio of 0.42, while downside volatility-managed *HML* produces a Sharpe ratio of 0.57. The momentum factor, again, is the only one for which the downside volatility-managed factor does not outperform the total volatility-managed factor.

The results for the Sortino ratio presented in Panel B are largely the same as those based on the Sharpe ratio. We find that downside volatility-managed factors outperform the original factors across all nine factors, and significantly so for five factors. Downside volatility-managed factors also outperform the total volatility-managed factors among eight of nine factors, and significantly so for four of them. Overall, we find that downside volatility-managed factors outperform their total volatility-managed counterparts based on direct Sharpe ratio and Sortino ratio comparisons.

Table 12 summarizes the Sharpe ratio and Sortino ratio differences among the volatility-managed and original portfolios for 94 anomalies. Panel A compares total volatility-managed portfolios with the original portfolios. Panel B compares downside volatility-managed portfolios with the original portfolios. Panel C compares total volatility-managed portfolios with downside volatility-managed portfolios.

In particular, each panel presents the number of Sharpe ratio or Sortino ratio differences that are positive or negative and the number of these differences that are statistically significant at the 5% level. In Panel A, we find that total volatility-managed portfolios exhibit higher Sharpe ratios than the original, unmanaged portfolios among 56 anomalies. In Panel B, the corresponding number increases sharply to 84 for downside volatility-managed portfolios. Panel C reveals that downside volatility-managed portfolios have higher Sharpe ratios than total volatility-managed portfolios among 80 of the 94 anomaly portfolios. The results for Sortino ratio are qualitatively similar to those for Sharpe ratio.

Therefore, the findings from the direct performance comparison are similar to those for the first two approaches. That is, we find that downside volatility-managed portfolios perform significantly better than the total volatility-managed portfolios. Overall, across all three approaches—spanning regressions, real-time trading strategies, and direct Sharpe ratio comparisons—we find consistent evidence that downside volatility-managed portfolios exhibit significant improvement in performance relative to total volatility-managed portfolios.

3.5. Robustness tests and additional analyses

In this section, we discuss the results of a number of robustness tests and additional analyses. For brevity, we present the detailed results of these analyses in the Internet Appendix.

3.5.1. Average return decomposition

In Section 3.2, we decompose the spanning regression alphas into a return timing component and a volatility timing component. We find that the superior performance of downside volatility-managed portfolios relative to the total volatility-managed portfolios stems primarily from the return timing component. In this section, we present an alternative decomposition.

Recall that each volatility-managed portfolio is constructed as

$$f_{\sigma,t} = \frac{c^*}{\sigma_{t-1}} f_t = w_t f_t, \tag{19}$$

Table 12
Direct comparisons for the 94 anomalies. Panel A (Panel B) summarizes results for the differences between the Sharpe ratio (Sortino ratio) of the total (downside) volatility-managed strategy and that of the original strategy for the 94 anomalies. Panel C summarizes results for the differences between the Sharpe ratio (Sortino ratio) of downside volatility-managed strategy and that of total volatility-managed strategy. The table reports the number of differences that are positive, positive and significant at the 5% level, negative, and negative and significant at the 5% level. Statistical significance is based on the approach in Kirby and Ostdiek (2012).

		Sharpe ratio difference		Sortino ratio difference	
		Positive [Signif.]	Negative [Signif.]	Positive [Signif.]	Negative [Signif.]
Panel A: Total volatility-managed strategy-Original strategy					
All	94	56 [11]	38 [3]	57[27]	37[10]
Accruals	10	5 [0]	5 [0]	6[1]	4[1]
Intangibles	10	3 [0]	7 [0]	3[1]	7[3]
Investment	9	4 [0]	5 [1]	4[2]	5[1]
Momentum	8	8 [8]	0 [0]	8[7]	0[0]
Profitability	20	16 [0]	4 [0]	16[4]	4[1]
Trading	19	12 [2]	7 [1]	12[8]	7[3]
Value	18	8 [1]	10 [1]	8[4]	10[1]
Panel B: Downside volatility-managed strategy-Original strategy					
All	94	84 [18]	10 [0]	83[15]	11[2]
Accruals	10	8 [1]	2 [0]	8[1]	2[0]
Intangibles	10	10 [2]	0 [0]	10[2]	0[0]
Investment	9	7 [0]	2 [0]	7[2]	2[0]
Momentum	8	8 [3]	0 [0]	8[3]	0[0]
Profitability	20	19 [5]	1 [0]	19[2]	1[0]
Trading	19	16 [5]	3 [0]	15[5]	4[1]
Value	18	16 [2]	2 [0]	16[0]	2[1]
Panel C: Downside volatility-managed strategy-Total volatility-managed strategy					
All	94	80[33]	14[3]	77[25]	17[10]
Accruals	10	10[2]	0[0]	9[2]	1[1]
Intangibles	10	10[5]	0[0]	10[2]	0[0]
Investment	9	9[2]	0[0]	8[3]	1[1]
Momentum	8	1[0]	7[2]	1[1]	7[4]
Profitability	20	20[10]	0[0]	19[6]	1[1]
Trading	19	13[5]	6[1]	13[5]	6[2]
Value	18	17[9]	1[0]	17[6]	1[1]

where c^* is a constant, and σ_{t-1} is the realized volatility in month $t-1$. Because f_t and $f_{\sigma,t}$ are constructed to have the same unconditional standard deviation, comparing f_t and $f_{\sigma,t}$ based on Sharpe ratio is equivalent to comparing them based on average return. We can decompose the average return difference as

$$\bar{f}_{\sigma,t} - \bar{f}_t = \text{cov}(w_t, f_t) + \bar{f}_t(\bar{w}_t - 1), \tag{20}$$

where $\bar{w}_t$ is the volatility-managed portfolio's average investment position in the unmanaged portfolio. For a strategy with positive average return ($\bar{f}_t > 0$), Eq. (20) shows that volatility management enhances average return and Sharpe ratio if the investment weight w_t positively predicts the unscaled portfolio's return (the return forecast component), and/or the scaled portfolio takes a levered position (i.e., $\bar{w}_t > 1$) in the unscaled portfolio on average (the leverage component). The return forecast component here is analogous to the return timing in our decomposition in Section 3.2.

Panel A of Table IA.3 presents the results for the nine equity factors, while Panel B summarizes the results for the 94 anomalies. Across all nine equity factors, we find that the return forecast component of downside volatility-managed portfolios is higher than that for the total volatility-managed portfolios. Across 94 anomalies, we also find that the return forecast component is more likely to be positive in downside volatility-managed portfolios than in the total volatility-managed portfolios. These results confirm our finding in Section 3.2 that the enhanced performance of downside volatility-managed portfolios relative to the total volatility-managed portfolios stems primarily from the ability of downside volatility to predict future returns.

3.5.2. Volatility-managed strategies based on past two-month and three-month volatility

In our main analysis, we follow Moreira and Muir (2017) and estimate volatility based on past one-month of daily returns. In this robustness test, we estimate volatility based on past two or three months of daily returns and then re-estimate spanning regression alphas. We present the detailed results in Tables IA.4 and IA.5 in Internet Appendix. Overall, our results are qualitatively and quantitatively similar to our main results.

3.5.3. Volatility-managed strategies based on expected and unexpected volatility

In this section, we decompose the realized volatility into expected and unexpected volatility and examine which component is driving our results. Specifically, we decompose the realized volatility into expected and unexpected volatility components as $\sigma_{t+1} = \mu_t z_{t+1}$, where $\mu_t = \text{E}_t(\sigma_{t+1})$ and $z_{t+1} \geq 0$ satisfies $\text{E}_t(z_{t+1}) = 1$. To estimate μ_t, we use an exponential smoothing model by finding the value of λ that minimizes $\sum_{t=1}^{T}(\sigma_t - \mu_{t-1})^2$, where $\mu_t = \mu_{t-1} + \lambda(\sigma_t - \mu_{t-1})$ with $\mu_0 = (1/T)\sum_{t=1}^{T}\sigma_t$. The expected volatility-managed portfolio is constructed as $f_{\mu,t} = \frac{c^*}{\mu_{t-1}} f_t$, and the unexpected volatility-managed strategy is given by $f_{z,t} = \frac{\hat{c}^*}{z_{t-1}} f_t$, where c^* and $\hat{c}^*$ are constants.

We then construct volatility-managed portfolios separately for expected and unexpected volatility and then estimate spanning regression alphas. Tables IA.6 and IA.7 in the Internet Appendix present the detailed results. Our main results can be summarized as follows. First, the expected volatility component is important for the performance of both total volatility- and downside volatility-managed portfolios. Second, the unexpected volatility component does not contribute to the performance of total volatility-managed portfolios, but plays a positive role in the performance of downside volatility-managed portfolios. Third, the performance difference between downside volatility-managed portfolios and total volatility-managed portfolios is primarily attributed to the unexpected volatility component.

3.5.4. Daily volatility-managed strategies

Existing literature has examined volatility-managed strategies at the monthly frequency. In this robustness test, we examine the performance of daily volatility-managed strategies. We estimate the daily volatility, σ_j, based on the squared daily returns, f_j^2. We obtain the exponential smoothing estimator of $\sigma_j^2 = \mathrm{E}_j(f_{j+1}^2)$ by minimizing $\sum_{j=1}^{N}(f_j^2 - \sigma_{j-1}^2)^2$ over a length of period N, where $\sigma_j^2 = \sigma_{j-1}^2 + \lambda(f_j^2 - \sigma_{j-1}^2)$ with $\sigma_0^2 = (1/N)\sum_{j=1}^{N} f_j^2$. The volatility-managed portfolio based on daily returns is constructed as

$$f_{\sigma,j} = \frac{c^*}{\sigma_{j-1}} f_j, \tag{21}$$

where c^* is a constant chosen such that f_j and $f_{\sigma,j}$ have the same full-sample volatility.

To construct the daily downside volatility, we note that the exponential smoothing estimator can be expressed as

$$\sigma_j^2 = \lambda\left(\sum_{y=0}^{j-1}(1-\lambda)^y f_{j-y}^2\right) + (1-\lambda)^j (1/N)\sum_{k=1}^{N} f_k^2,$$

which can also be written as $\sigma_{Total,j}^2 = \sigma_{Down,j}^2 + \sigma_{Up,j}^2$. The estimated daily downside volatility is given by

$$\sigma_{Down,j}^2 = \lambda\left(\sum_{y=0}^{j-1}(1-\lambda)^y I_{[f_{j-y}<0]} f_{j-y}^2\right) + (1-\lambda)^j (1/N)\sum_{k=1}^{N} I_{[f_k} < 0] f_k^2,$$

The downside volatility-managed portfolio based on daily returns is

$$f_{\sigma,j}^{Down} = \frac{c^*}{\sigma_{Down,j-1}} f_j. \tag{22}$$

We present the results in Table IA.8 in the Internet Appendix. The qualitative results are similar to those for monthly volatility-managed portfolios. We find that both total and downside volatility-managed portfolios exhibit positive alphas, but downside volatility-managed portfolios outperform total volatility-managed portfolios.

3.5.5. Volatility-managed strategies based on realized variance

Previous studies scale factor returns by either realized volatility or realized variance. In this paper we use realized volatility because it leads to less extreme investment weights. As a robustness test, we re-estimate spanning regressions by using volatility-managed portfolios scaled by realized variance. Table IA.9 presents the results. Overall, our results are slightly weaker than those for realized volatility-scaled portfolios, but the main conclusions are qualitatively unchanged.

3.5.6. Upside volatility

The innovation of our paper is to focus on downside volatility instead of total volatility. For completeness, we also estimate spanning regressions for upside volatility-managed portfolios. We present the results in Table IA.10 in the Internet Appendix. Overall, we find that the performance of upside volatility-managed portfolios is significantly worse than that of downside volatility-managed portfolios.

4. Conclusions

The recent literature shows mixed evidence on the performance of volatility-managed portfolios (Barroso and Santa-Clara, 2015; Daniel and Moskowitz, 2016; Moreira and Muir, 2017; Cederburg et al., 2020). We document that volatility-managed strategies that scale portfolio excess returns by prior downside volatility generates significantly better performance than strategies scaled by total volatility. In contrast to the inconsistent, and sometimes mediocre performance of total volatility-managed portfolios, we find that downside volatility-managed portfolios exhibit superior performance in spanning regressions, direct Sharpe ratio comparisons, and real-time trading strategies. The superior performance of managing downside volatility is confirmed across nine equity factors and a broad sample of market anomalies. We find that the positive spanning regression alphas of total volatility-managed portfolios are driven entirely by volatility timing, whereas the superior performance of downside volatility-managed portfolios are due to both return timing and volatility timing. Moreover, the enhanced performance of downside volatility-managed portfolios relative to total volatility-managed portfolios is due to return timing, i.e., downside volatility negatively predicts future returns. We find that downside volatility-managed portfolios tend to outperform total volatility-managed portfolios at lower levels of trading costs, but the outperformance evaporates at higher levels of trading costs. We also present evidence that real-time strategies with fixed weights perform significantly better than standard real-time strategies. This finding is particularly important in light of the controversy surrounding the real-time performance of volatility-managed portfolios. A promising area of future research is to look into why high downside volatility predicts low future returns. One might also study whether the performance of downside volatility-managed portfolios varies with macroeconomic conditions in order to better understand the underlying economics.

Appendix A

Table A1
List of anomalies. The table summarizes the firm characteristics used to construct the long-short anomaly decile portfolios in the paper. The panels of the table are organized by anomaly type (i.e., accruals, intangibles, investment, momentum, profitability, trading, and value). For each characteristic, we provide a symbol and brief description and note the original study documenting the corresponding anomaly. We construct the anomaly variables following the descriptions provided by Hou et al. (2015) and McLean and Pontiff (2016), and the relevant source (i.e., "HXZ" or "MP") for a given anomaly is listed in the final column of the table.

Anomaly	Description	Original study	Source	Sample period
Panel A: Accruals				
IvC	Inventory changes	Thomas and Zhang (2002)	HXZ	1963:08–2018:12
IvG	Inventory growth	Belo and Lin (2012)	HXZ	1963:08–2018:12
NOA	Net operating assets	Hirshleifer et al. (2004)	HXZ	1963:08–2018:12
OA	Operating accruals	Sloan (1996)	HXZ	1963:08–2018:12
POA	Percent operating accruals	Hafzalla et al. (2011)	HXZ	1963:08–2018:12
PTA	Percent total accruals	Hafzalla et al. (2011)	HXZ	1963:08–2018:12
TA	Total accruals	Richardson et al. (2005)	HXZ	1963:08–2018:12
ΔNCO	Changes in net noncurrent operating assets	Soliman (2008)	MP	1963:08–2018:12
ΔNWC	Changes in net non-cash working capital	Soliman (2008)	MP	1963:08–2018:12
NoaG	Growth in net operating assets minus accruals	Fairfield et al. (2003)	MP	1963:08–2018:12
Panel B: Intangibles				
AccQ	Accrual quality	Francis et al. (2005)	HXZ	1966:08–2018:12
AD/M	Advertisement expense-to-market	Chan et al. (2001)	HXZ	1974:08–2018:12
BC/A	Brand capital-to-assets	Belo et al. (2014b)	HXZ	1980:08–2018:12

(*continued on next page*)

Table A1 (*continued*)

H/N	Hiring rate	Belo et al. (2014a)	HXZ	1963:08–2018:12
OC/A	Organizational capital-to-assets	Eisfeldt and Papanikolaou (2013)	HXZ	1963:08–2018:12
OL	Operating leverage	Novy-Marx (2011)	HXZ	1963:08–2018:12
RC/A	R&D capital-to-assets	Li (2011)	HXZ	1980:08–2018:12
RD/M	R&D-to-market	Chan et al. (2001)	HXZ	1976:08–2018:12
RD/S	R&D-to-sales	Chan et al. (2001)	HXZ	1976:08–2018:12
Age	Firm age	Barry and Brown (1984)	MP	1963:08–2018:12
Panel C: Investment				
ΔPI/A	Changes in PP&E plus changes in inventory	Lyandres et al. (2008)	HXZ	1963:08–2018:12
ACI	Abnormal corporate investment	Titman et al. (2004)	HXZ	1966:08–2018:12
CEI	Composite issuance	Daniel and Titman (2006)	HXZ	1931:08–2018:12
I/A	Investment-to-assets	Cooper et al. (2008)	HXZ	1963:08–2018:12
IG	Investment growth	Xing (2008)	HXZ	1963:08–2018:12
NSI	Net stock issues	Pontiff and Woodgate (2008)	HXZ	1963:08–2018:12
NXF	Net external financing	Bradshaw et al. (2006)	HXZ	1974:08–2018:12
BeG	Growth in book equity	Lockwood and Prombutr (2010)	MP	1963:08–2018:12
I-ADJ	Industry-adjusted growth in investment	Abarbanell and Bushee (1998)	MP	1965:08–2018:12
Panel D: Momentum				
Abr-1	Cumulative abnormal stock returns around earnings announcements	Chan et al. (1996)	HXZ	1974:08–2018:12
R11-1	Price momentum (11-month prior returns)	Fama and French (1996)	HXZ	1927:08–2018:12
R6-1	Price momentum (6-month prior returns)	Jegadeesh and Titman (1993)	HXZ	1926:09–2018:12
RE-1	Revisions in analysts' earnings forecasts	Chan et al. (1996)	HXZ	1976:08-2018:12
SUE-1	Earnings surprise	Foster et al. (1984)	HXZ	1976:08–2018:12
R6-Lag	Lagged momentum	Novy-Marx (2012)	MP	1927:08–2018:12
Season	Seasonality	Heston and Sadka (2008)	MP	1946:08–2018:12
W52	52-week high	George and Hwang (2004)	MP	1927:08–2018:12
Panel E: Profitability				
ATO	Asset turnover	Soliman (2008)	HXZ	1963:08–2018:12
CTO	Capital turnover	Haugen and Baker (1996)	HXZ	1963:08–2018:12
F	*F*-score	Piotroski (2000)	HXZ	1974:08–2018:12
FP	Failure probability	Campbell et al. (2008)	HXZ	1976:08–2018:12
GP/A	Gross profitability-to-assets	Novy-Marx (2013)	HXZ	1963:08–2018:12
O	*O*-score	Dichev (1998)	HXZ	1963:08–2018:12
PM	Profit margin	Soliman (2008)	HXZ	1963:08–2018:12
RNA	Return on net operating assets	Soliman (2008)	HXZ	1963:08–2018:12
ROA	Return on assets	Balakrishnan et al. (2010)	HXZ	1974:08–2018:12
ROE	Return on equity	Haugen and Baker (1996)	HXZ	1974:08–2018:12
RS	Revenue surprise	Jegadeesh and Livnat (2006)	HXZ	1976:08–2018:12
TES	Tax expense surprise	Thomas and Zhang (2011)	HXZ	1976:08–2018:12
TI/BI	Taxable income-to-book income	Green et al. (2017)	HXZ	1963:08–2018:12
ΔATO	Change in asset turnover	Soliman (2008)	MP	1963:08–2018:12
ΔPM	Change in profit margin	Soliman (2008)	MP	1963:08–2018:12
E-con	Earnings consistency	Alwathainani (2009)	MP	1971:08–2018:12
S/IV	Change in sales minus change in inventory	Abarbanell and Bushee (1998)	MP	1963:08–2018:12
S/P	Sales-to-price	Barbee et al. (1996)	MP	1963:08–2018:12
S/SG&A	Change in sales minus change in SG&A	Abarbanell and Bushee (1998)	MP	1963:08–2018:12
Z	*Z*-score	Dichev (1998)	MP	1963:08–2018:12
Panel F: Trading				
β-D	Dimson's beta (daily data)	Dimson (1979)	HXZ	1926:09–2018:12
β-FP	Frazzini and Pedersen's beta	Frazzini and Pedersen (2014)	HXZ	1931:08–2018:12
1/P	1/share price	Miller and Scholes (1982)	HXZ	1926:08–2018:12
Disp	Dispersion of analysts' earnings forecasts	Diether et al. (2002)	HXZ	1976:08–2018:12
Dvol	Dollar trading volume	Brennan et al. (1998)	HXZ	1926:08–2018:12
Illiq	Illiquidity as absolute return-to-volume	Amihud (2002)	HXZ	1926:08–2018:12
Ivol	Idiosyncratic volatility	Ang et al. (2006)	HXZ	1926:09–2018:12
MDR	Maximum daily return	Bali et al. (2011)	HXZ	1926:09–2018:12
ME	Market equity	Banz (1981)	HXZ	1926:08–2018:12
S-Rev	Short-term reversal	Jegadeesh (1990)	HXZ	1926:08–2018:12
Svol	Systematic volatility	Ang et al. (2006)	HXZ	1986:08–2018:12
Turn	Share turnover	Datar et al. (1998)	HXZ	1926:08–2018:12
Tvol	Total volatility	Ang et al. (2006)	HXZ	1926:09–2018:12
β-M	Fama and MacBeth's beta (monthly data)	Fama and MacBeth (1973)	MP	1931:08–2018:12
σ(Dvol)	Dollar volume volatility	Chordia et al. (2001)	MP	1929:08–2018:12
B-A	Bid-ask spread	Amihud and Mendelson (1986)	MP	1963:08–2018:12
Short	Short interest	Dechow et al. (2001)	MP	1973:08–2018:12
Skew	Coskewness	Harvey and Siddique (2000)	MP	1931:08–2018:12
Vol-T	Volume trend	Haugen and Baker (1996)	MP	1931:08–2018:12
Panel G: Value				
A/ME	Market leverage	Bhandari (1988)	HXZ	1963:08–2018:12
B/M	Book-to-market equity	Rosenberg et al. (1985)	HXZ	1963:08–2018:12
CF/P	Cash flow-to-price	Lakonishok et al. (1994)	HXZ	1963:08–2018:12
D/P	Dividend yield	Litzenberger and Ramaswamy (1979)	HXZ	1927:08–2018:12
Dur	Equity duration	Dechow et al. (2004)	HXZ	1963:08–2018:12
E/P	Earnings-to-price	Basu (1983)	HXZ	1963:08–2018:12
EF/P	Analysts' earnings forecasts-to-price	Elgers et al. (2001)	HXZ	1976:08–2018:12
LTG	Long-term growth forecasts of analysts	La Porta (1996)	HXZ	1982:08–2018:12
NO/P	Net payout yield	Boudoukh et al. (2007)	HXZ	1974:08–2018:12
O/P	Payout yield	Boudoukh et al. (2007)	HXZ	1974:08–2018:12
Rev	Long-term reversal	De Bondt and Thaler (1985)	HXZ	1931:08–2018:12
SG	Sales growth	Lakonishok et al. (1994)	HXZ	1967:08–2018:12
An-V	Analyst value	Frankel and Lee (1998)	MP	1976:08–2018:12
σ(CF)	Cash flow variance	Haugen and Baker (1996)	MP	1978:08–2018:12
B/P-E	Enterprise component of book-to-price	Penman et al. (2007)	MP	1984:08–2018:12
B/P-Lev	Leverage component of book-to-price	Penman et al. (2007)	MP	1984:08–2018:12
Enter	Enterprise multiple	Loughran and Wellman (2012)	MP	1963:08–2018:12
Pension	Pension funding status	Franzoni and Marin (2006)	MP	1981:08–2018:12

Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.jbankfin.2021.106198

CRediT authorship contribution statement

Feifei Wang: Data curation, Methodology, Formal analysis, Writing. **Xuemin Sterling Yan:** Conceptualization, Writing.

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