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Expected Stock Market Returns and Volatility: Three Decades Later

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ABSTRACT

We replicate the findings of French, Schwert, and Stambaugh (FSS, 1987) almost exactly. Consistent with FSS, we find modest evidence of a positive relation between market risk premium and the expected market volatility and strong evidence of a negative relation between market excess returns and the unexpected change in market volatility during 1928 to 1984. These results persist during 1985 to 2018 and are robust to alternative data and model specifications. We extend the analysis to 23 developed countries and find qualitatively similar results. We show that the risk-return tradeoff is stronger during expansions than during recessions and does not vary significantly with investor sentiment.

Keywords: Expected market return, Volatility, Risk-return tradeoff, EGARCH

JEL Codes: G11, G12, G14, G15

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1 Introduction

The risk-return tradeoff is one of the most fundamental issues in finance. In static asset pricing models such as the CAPM, the unconditional equity risk premium varies proportionally with unconditional stock market volatility. Dynamic models such as Merton's (1973) intertemporal capital asset pricing model (ICAPM) predict that the conditional expected excess return on the stock market should vary positively with the conditional variance of market returns. The empirical relationship between expected market risk premium and market volatility has been the subject of considerable research during the past several decades. The evidence, however, is mixed and inconclusive. For example, Harvey (1989), Campbell and Hentschel (1992), Harrison and Zhang (1999), Ghysels *et al.* (2005), and Ludvigson and Ng (2007) find a positive risk-return relation, while Campbell (1987), Breen *et al.* (1989), Nelson (1991), Glosten *et al.* (1993), and Brandt and Kang (2004) find a negative relation.

One of the earliest and most influential studies in this literature is French, Schwert, and Stambaugh (FSS, 1987). FSS investigate the relation between expected market risk premium and stock market volatility by using two approaches: the realized volatility approach and the GARCH-in-mean approach. Using the first approach, they find little evidence of a significant relation between market risk premium and the predictable volatility of market returns. However, they find strong evidence that market excess returns are negatively related to the unexpected change in the volatility of stock market returns. FSS interpret this negative relation as indirect evidence of a positive relation between expected risk premiums and expected volatility.[1] Using the second approach, i.e., a GARCH-in-mean model, FSS find evidence of a positive relation between expected market returns and conditional market volatility. The purpose of our paper is to replicate and extend the results of FSS. Specifically, we organize our study into five sections: (1) Replication; (2) Out-of-sample tests; (3) Robustness tests; (4) International evidence; and (5) Additional analyses.

We begin with replicating the results of FSS by using their original data and methodology. As in FSS, we use two statistical approaches to investigate the relation between expected market risk premium and market volatility. In the first approach, we compute monthly realized volatility from daily market returns. We fit an ARIMA model to the logarithm of monthly realized volatility and then decompose the total volatility into predictable and unpredictable components. Consistent with FSS, we find little evidence of a positive relation between expected risk premium and the predictable component (i.e., *ex ante*) of market volatility.

[1] Because volatility is persistent, an unexpected increase in market volatility leads to an increase in expected future market volatility, which in turn leads to an increase in expected market return if there is a positive relation between expected market return and expected market volatility. Everything else equal, an increase in expected market return will lead to a lower price today. Therefore, unexpected changes in market volatility will be negatively associated with contemporaneous market returns.

There is a strong negative relation, however, between market excess returns and the unpredictable component of market volatility. Our replication results are almost identical to FSS. For example, the estimate of the regression coefficient on unexpected market volatility is −1.010 (SE = 0.111) in FSS and −0.999 (SE = 0.108) in our replication. In the second approach, we use daily market returns and a GARCH-in-mean model to estimate the relation between *ex ante* measures of volatility and market risk premium. Consistent with FSS, we find a reliably positive relation between expected risk premium and conditional market volatility. Our results are once again nearly identical to those in FSS. For example, the coefficient estimate on the GARCH-in-mean term is 0.073 (SE = 0.023) in FSS and 0.073 (SE = 0.024) in our replication.

After successfully replicating the results of FSS in sample, we examine whether the main results of FSS extend to the out-of-sample period 1985 to 2018. We find that they do. Using the realized volatility approach, we continue to find little evidence of a significant relation between the predictable component volatility and expected risk premium and strong evidence of a negative relation between market excess returns and the unpredictable component of volatility. The coefficient estimate on the unexpected market volatility is slightly lower than that for the 1928 to 1984 period, but remain economically and statistically significant [i.e., −0.849 (SE = 0.103)]. Moreover, we continue to find a positive return-volatility relation using the GARCH-in-mean model. Here, the coefficient estimate on the GARCH-in-mean term, i.e., 0.097 (SE = 0.033), is actually larger than that for the 1928 to 1984 period. Overall, we find that the main findings of FSS continue to hold during the 1985 to 2018 period.

Next, we perform a number of robustness tests. First, FSS use S&P 500 index daily returns (ex-dividend) to estimate market volatility because the CRSP daily stock returns were not available prior to 1963 when FSS carried out their study. FSS also use the NYSE portfolio returns as their estimate of market expected returns. In the first robustness test, we use (by now the standard) CRSP value-weighted index returns for both the estimation of market volatility and market expected returns. We find that our results regarding the risk-return relation are unchanged using these alternative data. Second, FSS use a Weighted Least Squares (WLS) approach, with the reciprocal of the predicted variance as the weight, to estimate the relation between monthly market excess returns and the expected (or unexpected) market volatility. We examine whether their results are robust to the alternative Ordinary Least Squares (OLS) procedure. We find that the OLS results are qualitatively similar to the WLS results. Specifically, we continue to find an insignificant relation between market excess return and the predicted market volatility and strong evidence of a significantly negative relation between market excess return and the unpredicted market volatility. Third, we examine whether the positive market return-volatility relation documented in the GARCH-in-mean model shows up in an EGARCH-in-mean model.[2] We find that it does. In our final robustness test, we control for several popular market return predictors in the regression model. Campbell (1987) and Scruggs (1998) point out that, according to Merton (1973), if changes in the investment opportunity set are captured by state variables in addition to the conditional variance itself, then those variables must be included in the regression equation of expected returns. We include business cycle variables including the T-bill rate, dividend yield, term spread, and default spread in the return-volatility equation and find that the risk-return relation is virtually unchanged.

Having shown that FSS's results are robust to a variety of alternative data and model specifications, we perform two extensions. First, we examine whether the main findings of FSS extend to international stock markets. We obtain daily and monthly stock market returns of 23 developed countries for the period 1985 to 2018 and repeat the main analyses of FSS for these markets. Similar to the results for U.S., we find that the relation between market excess returns and the predicted market volatility is largely statistically insignificant. Moreover, we find strong evidence of a negative relation between market excess returns and the unexpected market volatility. Specifically, 21 of the 23 countries exhibit a statistically significant negative relation. We also find an overall positive relation between market excess return and conditional market volatility in a GARCH-in-mean model for international markets.

In our second extension, we dig deeper into the risk-return relation by linking it to business conditions and investor sentiment. We find that the negative relation between market excess return and the unexpected market volatility is robust across recessions and expansions. However, we find that the positive relation between market excess return and the predicted market volatility is stronger during expansions than during recessions. This finding is somewhat puzzling because one might expect risk aversion, and hence the risk-return tradeoff, to be more pronounced during recessions. We do not find that the return-volatility relation differs significantly across high- and low-sentiment periods.

To summarize, we are able to replicate the findings of FSS by using their original data, sample, and methodology. We show that the main findings of FSS are robust to alternative data and model specifications. More importantly, FSS's results persist out of sample and hold in international markets. Finally, we find some evidence of a time-varying risk-return relation across business cycles.

The relation between expected market return and conditional market volatility has been an important and active research area since French *et al.* (1987). Glosten *et al.* (1993) extend FSS's GARCH-in-mean model to allow nominal interest rates to predict conditional volatility and find a negative relation between conditional market return and conditional volatility. Whitelaw (1994) estimates the relation

[2] The GARCH-in-mean model and the EGARCH-in-mean model both capture volatility persistence while allowing the expected return to vary with the conditional volatility or variance. The EGARCH-in-mean model can also accommodate volatility asymmetry.

between market returns and volatility by using conditioning variables such as dividend yield and default spread to estimate conditional volatility and returns simultaneously. Scruggs (1998) includes long-term bond returns in the conditional return equation and finds a positive intertemporal relation between market return and market volatility. Guo and Whitelaw (2006) also uncovers a positive risk-return relation by controlling for the hedging component in an ICAPM setting. Ludvigson and Ng (2007) employ a large number of conditioning variables and a factor analysis approach to estimate conditional market return and volatility and find a positive risk-return relation. Brandt and Kang (2004) use a latent variable approach to model conditional mean and variance of the market returns and find a negative risk-return relation. Ghysels *et al.* (2005) use a mixed data sampling (MIDAS) approach to model conditional market volatility and document a positive relation between market excess return and expected market volatility. Bali and Peng (2006) use intra-daily data to estimate conditional market volatility and find a positive risk-return relation. Pástor *et al.* (2008) use implied cost of capital as a measure of expected market return and document a positive risk-return tradeoff.

Much of the above literature focuses on the direct relation between expected market risk premium and predicted market volatility rather than the indirect relation between market return and the unexpected market volatility. As noted by FSS, although the observed strong negative relation between excess holding returns and unexpected volatility is consistent with a positive *ex ante* relation between risk premiums and volatility, it could also arise because of the leverage effect, i.e., negative shocks to stock prices raise financial or operating leverage and return volatility.

The research into the empirical risk-return relation will likely continue to be an active area of research for at least two reasons. First, the risk-return tradeoff is one of the most fundamental issues in finance. Second, the existing evidence has been mixed despite decades of research by some top scholars. Merton (1980) points out that estimating expected market risk premium precisely is difficult and requires long sample periods. In comparison, conditional market volatility is relatively easier to estimate precisely, particularly with high-frequency data. As such, the main challenge to identify the risk-return relation is to pin down the expected market risk premium. Given the relatively short sample period and the time-varying nature of the market risk premium, employing conditioning information to estimate expected market returns is likely to be a fruitful approach. In particular, Lettau and Ludvigson (2010) argue that using a large amount of conditioning information to model the conditional mean and conditional volatility of excess stock market returns can be particularly helpful. We agree with Lettau and Ludvigson, but would also like to caution that researchers need to guard against data mining because theory offers little guidance as to what conditioning information to use to model risk and return. Harvey (2001), for example, shows that the risk-return relation is sensitive to the set of conditioning information used to estimate conditional market returns and volatility. In addition, we argue that the risk-return relation is likely to be time-varying and non-monotonic (Backus and Gregory, 1993 and Rossi and Timmermann, 2010). Research into the shape and the dynamic nature of the risk-return relation is likely to yield important insights.

Another channel through which to gain fresh insights into the risk-return relation is by examining the performance of managed strategies. In particular, recent research into the performance of volatility-managed portfolios has generated a considerable amount of insights into the risk-return tradeoff for asset pricing factors including the market portfolio. Barroso and Santa-Clara (2015) and Daniel and Moskowitz (2016) show that volatility-managed momentum strategies nearly double the Sharpe ratio of the original momentum strategy. Moreira and Muir (2017) extend their analysis to nine equity factors and show that volatility-scaled factors produce significantly positive alphas relative to their unscaled counterparts.[3] Because volatility is persistent, volatility-managed strategies, i.e., increasing (decreasing) the investment position when volatility was recently low (high), are consistent with Sharpe ratio maximization as long as conditional return is not highly positively related to conditional volatility. If the risk-return relation is positive, then the benefit of volatility timing is likely to be offset by the cost of negative return timing and, as a result, volatility-managed strategies will not work. However, if the conditional expected return is uncorrelated or even negatively correlated with conditional volatility, then volatility-managed strategies are likely to perform well because they take advantage of the attractive risk-return tradeoff when volatility is low and avoid the poor risk-return tradeoff when volatility is high. The evidence of superior performance of volatility-managed portfolios, therefore, implies that the risk-return tradeoff is weak or nonexistent.

The rest of the paper proceeds as follows. Section 2 describes the data and methodology. Section 3 presents the empirical results. Section 4 concludes.

2 Data and Methodology

2.1 Data and Sample

We follow French *et al.* (1987) and use their original data and methodology in our replication. We obtain S&P 500 index daily returns excluding dividends and NYSE value-weighted monthly returns from the Center for Research for Security Prices (CRSP). We also obtain daily and monthly CRSP value-weighted index returns from CRSP and use them in a robustness test. We obtain the risk-free rate from

[3] Cederburg *et al.* (2020) confirm Moreira and Muir's (2017) spanning regression result across a sample of 103 equity factors and anomalies, but also show that volatility-managed portfolios do not systematically outperform their corresponding unmanaged portfolios in direct Sharpe ratio comparisons.

Kenneth French's website, https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. We obtain the above data for the period from 1928 to 2018. We use data from 1928 to 1984 in our replication of FSS. We then perform out-of-sample tests using data from 1985 to 2018.

FSS focus exclusively on the U.S. market. We extend their analyses to the following 23 developed markets: Australia, Austria, Belgium, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, United Kingdom, Greece, Hong Kong, Ireland, Israel, Italy, Japan, Netherlands, Norway, New Zealand, Portugal, Singapore, and Sweden. We obtain daily and monthly stock market excess returns (denominated in U.S. dollars and in excess of the U.S. Treasury bill rate) for these 23 developed markets from AQR for the period from 1985 to 2018.[4] AQR compiles the data from the Compustat/XpressFeed Global database using all available common stocks. Companies are assigned to a market based on the location of the primary exchange when they are traded on multiple countries.

We obtain the three-month T-bill rates, dividend yield, term spread, and default spread for the period 1928:01 to 2018:12 from Amit Goyal's website, https://sites.google.com/view/agoyal145. We control for these popular market return predictors in one of our robustness tests. We obtain recession dates for the U.S. from NBER's website, https://www.nber.org/cycles.html. Finally, we obtain investor sentiment data from Jeffery Wurgler's website, http://people.stern.nyu.edu/jwurgler/.

2.2 Estimating Market Volatility

The primary objective of FSS is to investigate relations of the form

$$E\left(R_{mt} - R_{ft} | \hat{\sigma}^{\mathrm{p}}_{mt}\right) = \alpha + \beta \hat{\sigma}^{\mathrm{p}}_{mt}, \quad p = 1,2 \tag{1}$$

where R_{mt} is the return on the stock market portfolio, R_{ft} is the risk-free rate, $\hat{\sigma}^{p}_{mt}$ is an *ex ante* measure of the stock market portfolio's risk, i.e., $\hat{\sigma}_{mt}$ for standard deviation, and $\hat{\sigma}^{2}_{mt}$ for variance. In order to estimate the relation in Eq. (1), we need *ex ante* measures of market volatility and variance. We follow FSS and use three different methods to estimate market volatility: the realized volatility approach, the ARCH model, and the GARCH model. For ease of comparison, we use the same equation numbers as in FSS whenever possible.

2.2.1 Realized Volatility

First, we estimate monthly realized volatility and variance using daily returns based on Eq. (2) of FSS:

$$\sigma^2_{mt} = \sum_{i=1}^{N_t} r^2_{it} + 2 \sum_{i=1}^{N_t - 1} r_{it} r_{i+1,t}. \tag{2}$$

[4] https://www.aqr.com/Insights/Datasets/Betting-Against-Beta-Equity-Factors-Monthly.

The first term is the sum of squared daily returns within month t, while the second term accounts for non-synchronous trading (Scholes and Williams, 1977).

Realized market volatility exhibits positive skewness and high persistence. We follow FSS and take the logarithm of monthly realized market volatility to remove skewness and then estimate a third-order ARIMA model to control for persistence. Specifically,

$$(1-L)\ln\sigma_{mt} = \theta_0 + \left(1-\theta_1 L - \theta_2 L^2 - \theta_3 L^3\right)u_t. \tag{3}$$

We then construct predicted market volatility and variance according to Eqs. (4a) and (4b) of FSS. Specifically, we estimate conditional volatility as

$$\hat{\sigma}_{mt} = \exp\left[\widehat{\ln\sigma_{mt}} + 0.5V\left(u_t\right)\right], \tag{4a}$$

and conditional variance as

$$\hat{\sigma}^2_{mt} = \exp\left[2\widehat{\ln\sigma_{mt}} + 2V(u_t)\right]. \tag{4b}$$

2.2.2 ARCH Model

The second measure of stock market volatility is estimated from the following ARCH model:

$$R_{mt} - R_{ft} = \alpha + \varepsilon_t - \theta\varepsilon_{t-1}, \tag{5c}$$

$$\sigma^2_t = a + b\left(\sum_{i=1}^{22}\frac{\varepsilon^2_{t-1}}{22}\right), \tag{5d}$$

where $R_{mt} - R_{ft}$ is the daily market excess return and σ^2_t is the conditional variance of ε_t. A positive coefficient estimate b would indicate that volatility is persistent.[5]

2.2.3 GARCH Model

The third measure of stock market volatility is estimated from the following GARCH model:

$$R_{mt} - R_{ft} = \alpha + \varepsilon_t - \theta\varepsilon_{t-1}, \tag{5c}$$

$$\sigma^2_t = a + b\sigma^2_{t-1} + c_1\varepsilon^2_{t-1} + c_2\varepsilon^2_{t-2}, \tag{5e}$$

where $R_{mt} - R_{ft}$ is the daily market excess return and σ^2_t is the conditional variance of ε_t. A positive coefficient estimate b would indicate that volatility is persistent.

[5] We use the SAS procedure AUTOREG to estimate ARCH, GARCH, and EGARCH models in this paper.

3 Empirical Results

This section presents our empirical results. First, we perform an in-sample study and replicate the findings of FSS using data from 1928 to 1984. Second, we conduct an out of sample analysis using data from 1985 to 2018. Third, we perform a number of robustness tests using alternative data and model specifications. Fourth, we extend the analysis to 23 developed markets. Finally, we investigate whether the risk-return tradeoff varies with business cycle and investor sentiment.

3.1 Replication of FSS

3.1.1 Tables 1–3: Summary Statistics

We begin our empirical analysis by replicating Table 1 through Table 3 of FSS, which contain primarily descriptive statistics and results of diagnostic tests. It is important to reproduce these results in order to replicate the main findings of FSS. For ease of exposition and comparison, we present our replication of FSS's tables in identical table numbers in our paper. For example, we present the replication results of Table 1 of FSS in our Table 1. In each table, FSS consider three sample periods, 1928 to 1984, 1928 to 1952, and 1953 to 1984. We present replication results for all three sample periods, but for brevity, we focus our discussion on their full sample period 1928 to 1984.

Table 1 reports the time-series properties of the realized monthly market volatility estimated from daily market returns. More specifically, Panel A presents the mean, standard deviation, skewness, and various autocorrelation coefficients of the monthly realized volatility. Panel B presents the same set of statistics for changes in monthly realized volatility. Panel C presents estimates of the ARIMA model of realized volatility. Overall, we find that our results are nearly identical to those of FSS. For example, the mean, standard deviation, skewness, and the first-order autocorrelation coefficient for the monthly realized market volatility are 0.0474, 0.0325, 2.80, and 0.71, respectively in FSS. The corresponding numbers in our replication are 0.0480, 0.0333, 2.87, and 0.71. Similarly, estimates of the moving average coefficients in the ARIMA model are 0.524, 0.158, and 0.090 in FSS, and are 0.535, 0.149, and 0.087 in our replication. Consistent with FSS, our replication results indicate that the volatility estimate is positively skewed and has high autocorrelations that decay slowly, which are indicative of a possibly non-stationary process.

In Table 2, we present our replication results for the estimates of ARCH and GARCH models. Again we are able to replicate FSS's results very closely. For example, in the ARCH model, i.e., Eq. (5d), the coefficient estimate of b is 0.938 (SE = 0.012) in FSS, and is 0.976 (SE = 0.013) in our replication. In the GARCH model, i.e., Eq. (5e), the estimate of the GARCH term is 0.919 (SE = 0.002) in

Period	Mean	Std. Dev.	Skewness	Autocorrelation at Lags											
				1	2	3	4	5	6	7	8	9	10	11	12
Panel A: Monthly Standard Deviation of S&P Composite Returns Estimated from Daily Data															
1928–84	0.0480	0.0333	2.8652	0.71	0.58	0.55	0.54	0.50	0.51	0.54	0.54	0.49	0.50	0.49	0.45
1928–52	0.0615	0.0428	2.1345	0.68	0.52	0.50	0.49	0.44	0.46	0.50	0.51	0.44	0.45	0.44	0.40
1953–84	0.0374	0.0171	1.7678	0.62	0.49	0.37	0.33	0.30	0.27	0.23	0.22	0.24	0.25	0.20	0.17
Panel B: Percent Changes of Monthly Standard Deviation of S&P Composite Returns Estimated from Daily Data															
1928–84	0.0001	0.4002	0.2126	−0.34	−0.08	−0.08	0.05	−0.03	−0.03	0.00	0.05	−0.05	0.03	−0.02	0.08
1928–52	−0.0013	0.4467	0.3187	−0.32	−0.14	−0.06	0.08	−0.07	−0.05	0.01	0.13	−0.08	0.02	−0.02	0.14
1953–84	0.0011	0.3604	0.0527	−0.36	0.00	−0.11	0.00	0.02	0.00	−0.02	−0.04	−0.01	0.04	−0.03	0.01

Table 1: Time Series Properties of Estimates of the Standard Deviation of the Return to the S&P Composite.

Panel C: ARIMA Models for the Logarithm of the Monthly Standard Deviation of S&P Composite Returns Estimated from Daily Data

	θ_0	θ_1	θ_2	θ_3
1928–84	0.0001	0.5346	0.1485	0.0874
	(0.0031)	(0.0382)	(0.0430)	(0.0382)
1928–52	−0.0010	0.5568	0.1950	0.0187
	(0.0052)	(0.0583)	(0.0657)	(0.0583)
1953–84	0.0009	0.5226	0.0794	0.1713
	(0.0037)	(0.0506)	(0.0572)	(0.0506)

Table 1: *Continued.*

Description: Replication of Table I of French *et al.* (1987). The monthly standard deviation is estimated as the sum of the squared daily S&P portfolio returns plus twice the sum of the products of adjacent returns. Panel A presents mean, standard deviation, skewness and autocorrelations of the monthly standard deviations. Panel B reports the same statistics for the logarithm of the monthly standard deviation. Panel C presents the ARIMA results assuming that the logarithm of the monthly standard deviation follows a third-order moving average process. Standard errors are reported in parentheses. The sample period is from January 1928 to December 1984.

Interpretation: The results are nearly identical to those of French *et al.* (1987) and indicate that the volatility estimate is positively skewed and has high autocorrelations that decay slowly, which are indicative of a possibly non-stationary process.

	$a \times 10^3$	$a \times 10^5$	b	c_1	c_2	θ
Panel A: January 1928 to December 1984						
ARCH	0.254	0.919	0.976			−0.134
(5c), (5d)	(0.062)	(0.048)	(0.013)			(0.007)
GARCH	0.313	0.059	0.917	0.121	−0.040	−0.146
(5c), (5e)	(0.064)	(0.006)	(0.003)	(0.007)	(0.007)	(0.008)
Panel B: January 1928 to December 1952						
ARCH	0.407	1.521	0.981			−0.076
(5c), (5d)	(0.114)	(0.098)	(0.017)			(0.011)
GARCH	0.495	0.148	0.895	0.109	−0.008	−0.084
(5c), (5e)	(0.114)	(0.014)	(0.004)	(0.009)	(0.009)	(0.012)
Panel C: January 1953 to December 1984						
ARCH	0.207	0.895	0.885			−0.182
(5c), (5d)	(0.079)	(0.066)	(0.023)			(0.010)
GARCH	0.245	0.049	0.922	0.129	−0.056	−0.196
(5c), (5e)	(0.083)	(0.008)	(0.004)	(0.010)	(0.011)	(0.012)

Table 2: ARCH and GARCH Models for Daily Excess Returns to the S&P Composite Portfolio.

Description: Replication of Table 2 of French *et al.* (1987). This table reports the results of ARCH and GARCH models for daily excess returns to the S&P composite portfolio. $R_{mt} - R_{ft}$ is the daily excess return to S&P composite portfolio. In Panels A, B, and C, respectively, the sample periods are January 1928 to December 1984, January 1928 to December 1952 and January 1953 to December 1984. Standard errors are reported in parentheses.

$$R_{mt} - R_{ft} = \alpha + \varepsilon_t - \theta \varepsilon_{t-1}, \tag{5c}$$

$$\sigma_t^2 = a + b \left(\sum_{i=1}^{22} \frac{\varepsilon_{t-1}^2}{22} \right), \tag{5d}$$

$$\sigma_t^2 = a + b\sigma_{t-1}^2 + c_1 \varepsilon_{t-1}^2 + c_2 \varepsilon_{t-2}^2. \tag{5e}$$

Interpretation: The results are very close to those in French *et al.* (1987). Consistent with FSS, there is strong evidence of persistence in the volatility of stock market returns.

FSS, and is 0.917 (SE = 0.003) in our replication. Therefore, consistent with FSS, we find strong evidence of persistence in the volatility of stock market returns.

In Table 3, we replicate the summary statistics for the market returns as proxied by the NYSE value-weighted excess returns. The table shows average returns (equal weighted averages and WLS averages), standard deviations, and skewness of the NYSE value-weighted excess returns. Our results are essentially the same as those reported in FSS. The mean, standard deviation, skewness, and two WLS means of the market returns are 0.0061, 0.0579, 0.44, 0.0116, and 0.0055, respectively

	Mean	**WLS Mean**[a]	**WLS Mean**[b]	**Std. Dev.**	**Skewness**
1928–84	0.0060	0.0116	0.0053	0.0586	0.49
	(2.67)	(9.35)	(3.34)		
1928–52	0.0075	0.0160	0.0085	0.0753	0.49
	(1.73)	(7.02)	(2.78)		
1953–84	0.0048	0.0099	0.0040	0.0411	−0.05
	(2.29)	(6.65)	(2.22)		

Table 3: Summary Statistics of the Monthly NYSE Value-Weighted Market Excess Returns.

Description: Replication of Table 3 of French *et al.* (1987). This table reports the mean, standard deviation, and skewness of the monthly market excess returns. The one-month T-bill rate is subtracted from the monthly value-weighted returns of all NYSE stocks to create the monthly market excess returns. WLS mean[a] is the sample mean estimated by weighted least squares, where the variance estimated using S&P composite daily returns is used as weights. WLS mean[b] is the sample mean estimated by weighted least squares, where the predicted variance of the S&P composite portfolio estimated from the ARIMA model is used as weights. Standard errors are reported in parentheses. The sample period is from January 1928 to December 1984.

Interpretation: The results are essentially the same as those reported in French *et al.* (1987).

in FSS. The corresponding numbers in our replication are 0.0060, 0.0586, 0.49, 0.0116, and 0.0053.[6] In summary, we successfully replicate the summary statistics and time series properties of the volatility estimates and stock market returns reported in Tables 1–3 of French *et al.* (1987).

3.1.2 Table 4: Weighted Least Squares Regression

In this subsection, we replicate the main results of FSS for the realized volatility approach reported in their Table 4. Specifically, we follow FSS and estimate weighted least squares regressions of monthly market excess returns on the predictable ($\hat{\sigma}^{\mathrm{p}}_{mt}$) and unpredictable components ($\hat{\sigma}^{\mathrm{pu}}_{mt}$) of standard deviations or variance of market returns ($p = 1,2$), where $\hat{\sigma}^{\mathrm{p}}_{mt}$ is estimated from ARIMA model using Eqs. (4a) or (4b), and the $\hat{\sigma}^{\mathrm{pu}}_{mt} = \sigma^{p}_{mt} - \hat{\sigma}^{\mathrm{p}}_{mt}$. We follow FSS and estimate the following regressions:

$$R_{mt} - R_{ft} = \alpha + \beta\hat{\sigma}^{\mathrm{p}}_{mt} + \varepsilon_t, \tag{6}$$

$$R_{mt} - R_{ft} = \alpha + \beta\hat{\sigma}^{\mathrm{p}}_{mt} + \Upsilon\hat{\sigma}^{\mathrm{pu}}_{mt} + \varepsilon_t. \tag{7}$$

[6]There is a minor typo in French *et al.* (1987) in their description of the calculation of the weighted average market returns (WLS mean[b] and WLS mean[c]) in Table 3. Page 12 of FSS states that the weight for each observation to compute WLS mean[b] is the reciprocal of monthly *standard deviation* estimated from daily returns. Our results indicate that FSS actually use the reciprocal of monthly realized *variance* as the weight. Similarly, FSS state that the weight to calculate WLS mean[c] is the predicted standard deviation from ARIMA model, but they actually use the predicted variance. The results are qualitatively the same whether we use standard deviation or variance as the weight.

	Eq. (6)		Eq. (7)		
	α	β	α	β	γ
Panel A: January 1928 to December 1984					
σ_{mt}	0.0042	0.0321	0.0073	−0.0412	−0.9994
	(0.0044)	(0.1164)	(0.0040)	(0.1076)	(0.0909)
	[0.0042]	[0.1108]	[0.0039]	[0.1064]	[0.1080]
σ^2_{mt}	0.0047	0.3887	0.0054	0.1695	−4.2744
	(0.0021)	(0.9216)	(0.0020)	(0.8733)	(0.4811)
	[0.0022]	[0.8833]	[0.0021]	[0.9216]	[0.8279]
Panel B: January 1928 to December 1952					
σ_{mt}	0.0147	−0.1404	0.0203	−0.2340	−0.9974
	(0.0087)	(0.1830)	(0.0078)	(0.1640)	(0.1148)
	[0.0088]	[0.1792]	[0.0084]	[0.1785]	[0.1278]
σ^2_{mt}	0.0093	−0.3074	0.0114	−0.6139	−3.8168
	(0.0042)	(1.1220)	(0.0039)	(1.0297)	(0.5049)
	[0.0043]	[1.0842]	[0.0040]	[1.1277]	[0.6487]
Panel C: January 1953 to December 1984					
σ_{mt}	0.0017	0.0763	0.0061	−0.0551	−1.0332
	(0.0060)	(0.1817)	(0.0057)	(0.1724)	(0.1482)
	[0.0056]	[0.1660]	[0.0052]	[0.1613]	[0.1974]
σ^2_{mt}	0.0026	1.2264	0.0042	−0.1900	−8.8049
	(0.0032)	(2.1597)	(0.0031)	(2.0838)	(1.4915)
	[0.0032]	[1.9756]	[0.0031]	[2.1342]	[2.1769]

Table 4: Weighted Least Squares Regressions at Monthly Frequency.

Description: Replication of Table 4 of French *et al.* (1987). This table presents results from weighted least squares regressions of monthly market excess returns on predictable ($\hat{\sigma}^{\mathrm{p}}_{mt}$) and unpredictable components ($\hat{\sigma}^{\mathrm{pu}}_{mt}$) of standard deviations or variance of market returns. $R_{mt} - R_{ft}$ is the monthly excess return to S&P composite portfolio. The predicted variance of S&P composite portfolio, $\hat{\sigma}^2_{mt}$, is used as the weight to standardize each observation. In Panel A, B, and C, respectively, the sample periods are January 1928 to December 1984, January 1928 to December 1952 and January 1953 to December 1984. Standard errors are reported in parentheses. The numbers in brackets are standard errors based on **White**'s (1980) consistent heteroskedasticity correction.

$$R_{mt} - R_{ft} = \alpha + \beta \hat{\sigma}^{\mathrm{p}}_{mt} + \varepsilon_t, \tag{6}$$

$$R_{mt} - R_{ft} = \alpha + \beta \hat{\sigma}^{\mathrm{p}}_{mt} + \Upsilon \hat{\sigma}^{\mathrm{pu}}_{mt} + \varepsilon_t. \tag{7}$$

Interpretation: The results are very close to those in French *et al.* (1987). There is an insignificant relation between predicted volatility (or variance) and market excess returns, but a strong negative relation between the unexpected market volatility and market excess returns. The negative relation between unexpected market volatility and market excess returns implies an indirect evidence for a positive relation between expected market volatility and expected risk premiums.

$R_{mt} - R_{ft}$ is the monthly excess return to S&P composite portfolio. As in FSS, we estimate regressions Eqs. (6) and (7) by using WLS, where the predicted variance of the S&P composite portfolio, $\hat{\sigma}^2_{mt}$, is used as the weight to standardize each observation.

We follow FSS and present the results for three sample periods, 1928 to 1984, 1928 to 1854, and 1955 to 1984. However, we focus our discussion on their full sample period 1928 to 1984 for brevity. Table 4 reports qualitatively and quantitatively similar results to those in Table 4 of FSS. Specifically, the results indicate little evidence of a significant relation between the predictable volatility or variance and market excess returns [i.e., Eq. (6)]. For example, Panel A shows a point estimate of $\beta = 0.032$ with a standard error of 0.111 when using standard deviation $\hat{\sigma}_{mt}$ as a proxy for risk, and a point estimate of $\beta = 0.389$ with a standard error of 0.883 when using variance $\hat{\sigma}^2_{mt}$ as a proxy for risk. Both estimates are indistinguishable from zero.

Next, we include both the predictable ($\hat{\sigma}^{\mathrm{p}}_{mt}$) and unpredictable components ($\hat{\sigma}^{\mathrm{pu}}_{mt}$) of standard deviations or variance of market returns ($p = 1, 2$) in the regression of market excess returns [i.e., Eq. (7)]. We note that the right-hand side variable in Eq. (7) $\hat{\sigma}^{\mathrm{p}}_{mt}$ and $\hat{\sigma}^{\mathrm{pu}}_{mt}$ are uncorrelated with each other by construction, so the parameter estimates are not affected by including them in the same regression. Consistent with FSS, we continue to find an insignificant relation between predicted volatility (or variance) and market excess returns. More importantly, we find strong evidence of a negative relation between the unexpected market volatility (or variance) and market excess returns. For example, the coefficient estimate on $\hat{\sigma}^{\mathrm{pu}}_{mt}$ is -0.999 (SE $= 0.108$) in the volatility specification and is -4.274 (SE $= 0.828$) in the variance specification. Both of these estimates are highly statistically significant. Moreover, they are similar to those obtained by FSS. Specifically, the corresponding numbers in FSS are -1.010 (SE $= 0.111$) and -4.438 (SE $= 0.886$).

FSS note that a negative relation between the unpredictable component $\hat{\sigma}^{\mathrm{pu}}_{mt}$ of risk (standard deviation or variance) and returns implies a positive relation between the predictable component $\hat{\sigma}^{\mathrm{p}}_{mt}$ of risk (standard deviation or variance) and returns. The intuition is as follows. Because volatility is persistent, an unexpected increase in market volatility leads to an increase in expected future market volatility, which in turn leads to an increase in expected market return if there is a positive relation between expected market return and expected market volatility. Everything else equal, an increase in expected market return will lead to a lower market price today. Therefore, unexpected changes in market volatility will be negatively associated with contemporaneous market returns.

Although the observed strong negative relation between excess holding returns and unexpected volatility is consistent with a positive *ex ante* relation between risk premiums and volatility, Christie (1982) suggest another interpretation. They note that leverage can induce a negative *ex post* relation between returns and volatility. Specifically, negative shocks to stock prices raise financial or operating leverage. As a result, the stock becomes riskier and more volatile, thus leading to a negative relation between *ex-post* return and return volatility.

3.1.3 Tables 5 and 6: Risk-Return Tradeoff using ARCH-type Models

In this subsection, we replicate FSS's results in Tables 5 and 6, i.e., those of the GARCH-in-mean model. Specifically, we estimate the following GARCH-in-Mean models [standard deviation in Eq. (8a) or variance Eq. (8b)] in Table 5:

$$R_{mt} - R_{ft} = \alpha + \beta\sigma_t + \varepsilon_t - \theta\varepsilon_{t-1}, \tag{8a}$$

$$R_{mt} - R_{ft} = \alpha + \beta\sigma_t^2 + \varepsilon_t - \theta\varepsilon_{t-1}, \tag{8b}$$

$$\sigma_t^2 = a + b\sigma_{t-1}^2 + c_1\varepsilon_{t-1}^2 + c_2\varepsilon_{t-2}^2. \tag{5e}$$

The GARCH-in-mean model builds on the standard GARCH model while allowing the expected excess return to be a function of conditional volatility [Eq. (8a)] or conditional variance [Eq. (8b)]. A positive coefficient β would indicate that expected market excess return is positively related to conditional volatility or conditional variance. We estimate the above models using daily market returns.

Panel A of Table 5 presents our replication results for FSS's full sample period 1928 to 1984. Our estimation of the GARCH-in-mean model of Eq. (8a) and Eq. (8b) shows a point estimate of $\beta = 0.073$ and a standard error of 0.024 for the standard deviation specification, and a point estimate of $\beta = 2.412$ and a standard error of 0.932 for the variance specification. Both of these estimates are positive, indicating a positive risk-return tradeoff, and statistically significant at the 1% level. Moreover, our point estimates of β are nearly identical to those obtained by FSS. Specifically, FSS report a point estimate of $\beta = 0.073$ and a standard error of 0.023 for Eq. (8a) and $\beta = 2.41$ and a standard error of 0.934 for Eq. (8b). Overall, our results in Table 5 confirm FSS's finding of a positive and significant relation between risk and returns when estimated using GARCH-in-mean models.

Using monthly realized volatility and a WLS regression approach, we find in Table 4 an insignificant relation between predicted market volatility and market risk premium. However, we find in Table 5 a significant positive relation between conditional market volatility and market excess returns using daily data and a GARCH-in-mean model. One possible source of this discrepancy is data frequency. To investigate this possibility, FSS estimate the GARCH-in-mean models using monthly returns in Tables 6(a) and 6(b). We also replicate these analyses and report the results in our Tables 6(a) and 6(b).

Table 6(a) continue to report a positive relation between conditional market volatility and market excess return, although this positive relation is statistically weaker than that in Table 5 for daily data. Specifically, Panel A shows a point estimate of $\beta = 0.218$ and a standard error of 0.132 for the standard deviation specification and a point estimate of $\beta = 1.630$ and a standard error of 0.871 for the variance specification. These estimates are statistically significant at the 10% level,

	$\alpha \times 10^3$	β	$a \times 10^5$	b	c_1	c_2	θ
Panel A: January 1928 to December 1984							
Std. Dev.	−0.177	0.073	0.060	0.917	0.121	−0.040	−0.146
Eq. (8a), Eq. (5e)	(0.171)	(0.024)	(0.006)	(0.003)	(0.007)	(0.007)	(0.009)
Variance	0.188	2.412	0.060	0.917	0.121	−0.040	−0.146
Eq. (8b), Eq. (5e)	(0.080)	(0.932)	(0.006)	(0.003)	(0.007)	(0.007)	(0.009)
Panel B: January 1928 to December 1952							
Std. Dev.	0.100	0.048	0.150	0.894	0.109	−0.008	−0.084
Eq. (8a), Eq. (5e)	(0.274)	(0.030)	(0.014)	(0.004)	(0.009)	(0.009)	(0.012)
Variance	0.375	1.495	0.150	0.894	0.109	−0.008	−0.084
Eq. (8b), Eq. (5e)	(0.139)	(0.992)	(0.014)	(0.004)	(0.009)	(0.009)	(0.012)
Panel C: January 1953 to December 1984							
Std. Dev.	−0.425	0.113	0.050	0.921	0.130	−0.058	−0.196
Eq. (8a), Eq. (5e)	(0.281)	(0.045)	(0.008)	(0.004)	(0.010)	(0.011)	(0.012)
Variance	−0.029	7.121	0.050	0.921	0.131	−0.058	−0.196
Eq. (8b), Eq. (5e)	(0.136)	(2.838)	(0.008)	(0.004)	(0.010)	(0.011)	(0.012)

Table 5: GARCH-in-Mean Models at Daily Frequency.

Description: Replication of Table 5 of French *et al.* (1987). This table reports the results of GARCH-in-mean models for daily market excess returns. $R_{mt} - R_{ft}$ is the daily excess return to S&P composite portfolio. In Panels A, B, and C, respectively, the sample periods are January 1928 to December 1984, January 1928 to December 1952 and January 1953 to December 1984. Standard errors are reported in parentheses.

$$R_{mt} - R_{ft} = \alpha + \beta \sigma_t + \varepsilon_t - \theta \varepsilon_{t-1}, \tag{8a}$$

$$R_{mt} - R_{ft} = \alpha + \beta \sigma_t^2 + \varepsilon_t - \theta \varepsilon_{t-1}, \tag{8a}$$

$$\sigma_t^2 = a + b\sigma_{t-1}^2 + c_1 \varepsilon_{t-1}^2 + c_2 \varepsilon_{t-2}^2. \tag{5e}$$

Interpretation: The results are very close to those in French *et al.* (1987). The coefficients and standard errors for β show that there is a positive and statistically significant risk-return tradeoff.

and are nearly identical to those obtained by FSS. Specifically, the corresponding numbers reported in FSS are $\beta = 0.224$ (SE $= 0.132$) and $\beta = 1.693$ (SE $= 0.873$), respectively.

In Table 6(b), FSS estimate regressions of market excess returns on conditional market volatility using a WLS regression approach, where the conditional volatility is obtained from the monthly GARCH-in-mean model. Here, we find little evidence of a significant relation between risk and return. And our results are once again qualitatively and quantitatively similar to those of FSS.

	α	β	$a \times 10^3$	b	c_1	c_2	θ
Panel A: January 1928 to December 1984							
Std. Dev.	−0.002	0.218	0.083	0.811	0.058	0.110	−0.070
Eq. (8a), Eq. (5e)	(0.006)	(0.132)	(0.032)	(0.028)	(0.045)	(0.055)	(0.038)
Variance	0.004	1.630	0.085	0.809	0.061	0.106	−0.069
Eq. (8b), Eq. (5e)	(0.002)	(0.871)	(0.032)	(0.028)	(0.045)	(0.054)	(0.037)
Panel B: January 1928 to December 1952							
Std. Dev.	0.011	0.004	0.065	0.842	0.136	0.018	−0.079
Eq. (8a), Eq. (5e)	(0.009)	(0.175)	(0.068)	(0.034)	(0.098)	(0.104)	(0.057)
Variance	0.010	0.548	0.067	0.840	0.140	0.014	−0.081
Eq. (8b), Eq. (5e)	(0.004)	(1.085)	(0.070)	(0.035)	(0.101)	(0.107)	(0.057)
Panel C: January 1953 to December 1984							
Std. Dev.	−0.020	0.658	0.173	0.745	−0.000	0.157	−0.050
Eq. (8a), Eq. (5e)	(0.013)	(0.354)	(0.093)	(0.078)	(0.000)	(0.054)	(0.053)
Variance	−0.006	7.351	0.168	0.748	0.000	0.156	−0.050
Eq. (8b), Eq. (5e)	(0.006)	(4.066)	(0.091)	(0.078)	(0.000)	(0.053)	(0.052)

Table 6(a): Comparison of ARIMA with GARCH Predictions of Stock Market Volatility and the Risk-Return Tradeoff.

Description: Replication of Table 6(a) of French *et al.* (1987). This table reports the results of GARCH-in-mean models for monthly market excess returns. $R_{mt} - R_{ft}$ is the monthly excess return to S&P composite portfolio. In Panel A, B, and C, respectively, the sample periods are January 1928 to December 1984, January 1928 to December 1952 and January 1953 to December 1984. Standard errors are reported in parentheses.

$$R_{mt} - R_{ft} = \alpha + \beta\sigma_t + \varepsilon_t - \theta\varepsilon_{t-1}, \tag{8a}$$
$$R_{mt} - R_{ft} = \alpha + \beta\sigma_t^2 + \varepsilon_t - \theta\varepsilon_{t-1}, \tag{8b}$$

$$\sigma_t^2 = a + b\sigma_{t-1}^2 + c_1\varepsilon_{t-1}^2 + c_2\varepsilon_{t-2}^2. \tag{5e}$$

Interpretation: The results are nearly identical to those reported in French *et al.* (1987). The coefficient and standard error for β show that there is a positive and statistically significant risk-return tradeoff.

Overall, we successfully replicate the entire set of tables and results of FSS. In some cases, our replication results are nearly identical to those of FSS. In other cases, our results are quantitatively similar and qualitatively identical to those of FSS. The main results of our replications are that (1) using the realized volatility approach, we find an insignificant relation between market risk premium and predicted market volatility; (2) however, we find strong evidence of a negative relation between market risk premium and unexpected market volatility; this negative relation represents indirect evidence of a positive relation between market

	α	β
Panel A: January 1928 to December 1984		
Monthly GARCH Std. Dev.	0.0036	0.0458
	(0.0057)	(0.1318)
	[0.0057]	[0.1309]
Monthly GARCH Variance	0.0048	0.3356
	(0.0024)	(0.9595)
	[0.0024]	[0.9420]
Panel B: January 1928 to December 1952		
Monthly GARCH Std. Dev.	0.0213	−0.2383
	(0.0089)	(0.1773)
	[0.0089]	[0.1768]
Monthly GARCH Variance	0.0122	−0.8580
	(0.0042)	(1.1154)
	[0.0041]	[1.0734]
Panel C: January 1953 to December 1984		
Monthly GARCH Std. Dev.	−0.0105	0.3574
	(0.0092)	(0.2337)
	[0.0086]	[0.2140]
Monthly GARCH Variance	−0.0033	4.2654
	(0.0045)	(2.6010)
	[0.0042]	[2.3225]

Table 6(b): Comparison of ARIMA with GARCH Predictions of Stock Market Volatility and the Risk-Return Tradeoff.

Description: Replication of Table 6(b) of French *et al.* (1987). This table presents results from weighted least squares regressions of monthly market excess returns on predicted standard deviations or variance of market returns from the monthly GARCH-in-mean model. $R_{mt} - R_{ft}$ is the monthly excess return to value-weighted portfolio of all NYSE stocks. The predicted variance from the GARCH model is used as the weight to standardize each observation. In Panel A, B, and C, respectively, the sample periods are January 1928 to December 1984, January 1928 to December 1952 and January 1953 to December 1984. Standard errors are reported in parentheses. The numbers in brackets are standard errors based on White's (1980) consistent heteroskedasticity correction.

$$R_{mt} - R_{ft} = \alpha + \beta\sigma_t + \varepsilon_t, \tag{10a}$$

$$R_{mt} - R_{ft} = \alpha + \beta\sigma_t^2 + \varepsilon_t. \tag{10b}$$

Interpretation: The results are very close to those in French *et al.* (1987). There is little evidence of a significant relation between predicted volatility (variance) and market excess returns.

return and *ex ante* market volatility; (3) using a GARCH-in-mean model, we find a positive relation between conditional market volatility and market excess return at the daily frequency.

3.2 Out-of-Sample Results

Having replicated the results of FSS in sample, we perform an out of sample analysis to investigate whether the main results of FSS hold in 1985 to 2018. In addition to the out-of-sample period 1985 to 2018, we also report results for the combined FSS sample period and the out-of-sample period, i.e., 1928 to 2018. For brevity, we focus on the main analyses in FSS's Tables 4 and 5 in this out-of-sample study.

In Table 7, we repeat the analysis in FSS's Table 4 and estimate a WLS regressions of monthly market excess returns on predictable ($\hat{\sigma}^{\mathrm{p}}_{mt}$) and unpredictable components ($\hat{\sigma}^{\mathrm{pu}}_{mt}$) of standard deviations or variance of market returns for the out-of-sample periods. Specifically, we estimate regression Eqs. (6) and (7) for the sample period 1985 to 2018 in Panel A and for the sample period 1928 to 2018 in Panel B.

Similar to the results for 1928 to 1984, we find a statistically insignificant relation between monthly market excess returns and the predictable components of standard deviations ($\hat{\sigma}_{mt}$) or variance ($\hat{\sigma}^2_{mt}$) for the univariate regression Eq. (6). This insignificant relation persists in the bivariate regression Eq. (7), where we include both the predictable components of standard deviations ($\hat{\sigma}^2_{mt}$) or variance ($\hat{\sigma}_{mt}$) and the unexpected standard deviations ($\hat{\sigma}^{\mathrm{u}}_{mt}$) or variance ($\hat{\sigma}^{2\mathrm{u}}_{mt}$). However, we find a negative and significant relation between monthly market excess returns and the unpredictable components of standard deviations or variance. Specifically, the coefficient estimate on γ is −0.849 (SE = 0.103) for the standard deviation specification and is −4.524 (SE = 0.764) in the variance specification. These results are similar to what we find in Table 4 for FSS's original sample period. Recall the corresponding coefficient estimates for γ are −0.999 and −4.274 in Table 4. The results in Panel B for the 1928 to 2018 period are also qualitatively similar. Overall, our results for the out-of-sample analysis are consistent with the in-sample results reported in Table 4. That is, we find an insignificant relation between market excess returns and the predictable components of market volatility and a negative and highly significant relation between market excess returns and the unexpected market volatility.

We also perform an out-of-sample analysis of the results reported in Table 5 of FSS for the GARCH-in-mean model. We report our findings in Table 8. Panel A reports results for the sample period from January 1985 to December 2018 and Panel B reports results for the full sample period from January 1928 to December 2018. Table 8 presents strong evidence of volatility persistence. More importantly, we find a positive and statistically significant relation between conditional market

	Eq. (6)		Eq. (7)		
	α	β	α	β	γ
Panel A: January 1985 to December 2018					
σ_{mt}	0.0103	−0.1004	0.0152	−0.2284	−0.8492
	(0.0050)	(0.1404)	(0.0046)	(0.1276)	(0.0889)
	[0.0052]	[0.1436]	[0.0048]	[0.1311]	[0.1028]
σ^2_{mt}	0.0080	−0.7290	0.0089	−1.0767	−4.5235
	(0.0027)	(1.3928)	(0.0024)	(1.2652)	(0.4844)
	[0.0027]	[1.3410]	[0.0025]	[1.2215]	[0.7643]
Panel B: January 1928 to December 2018					
σ_{mt}	0.0069	−0.0240	0.0107	−0.1158	−0.9222
	(0.0033)	(0.0898)	(0.0030)	(0.0828)	(0.0651)
	[0.0033]	[0.0881]	[0.0031]	[0.0846]	[0.0761]
σ^2_{mt}	0.0060	0.0390	0.0068	−0.2438	−4.3670
	(0.0016)	(0.7539)	(0.0015)	(0.7061)	(0.3509)
	[0.0017]	[0.7435]	[0.0016]	[0.7637]	[0.5766]

Table 7: Weighted Least Squares Regressions at Monthly Frequency—Out-of-Sample Results.

Description: This table presents results from weighted least square regressions of monthly market excess returns on predictable ($\hat{\sigma}^{\mathrm{p}}_{mt}$) and unpredictable components ($\hat{\sigma}^{\mathrm{pu}}_{mt}$) of standard deviations or variance of market returns. $R_{mt} - R_{ft}$ is the monthly excess return to S&P composite portfolio. The predicted variance of S&P composite portfolio, $\hat{\sigma}^2_{mt}$, is used as the weight to standardize each observation. In Panels A and B, respectively, the sample periods are January 1985 to December 2018 and January 1928 to December 2018. Standard errors are reported in parentheses. The numbers in brackets are standard errors based on White's (1980) consistent heteroskedasticity correction.

$$R_{mt} - R_{ft} = \alpha + \beta \hat{\sigma}^{\mathrm{p}}_{mt} + \varepsilon_t, \tag{6}$$

$$R_{mt} - R_{ft} = \alpha + \beta \hat{\sigma}^{\mathrm{p}}_{mt} + \Upsilon \hat{\sigma}^{\mathrm{pu}}_{mt} + \varepsilon_t. \tag{7}$$

Interpretation: Out-of-sample evidence confirms the in-sample result. There is an insignificant relation between predicted volatility (or variance) and market excess returns, but a strong negative relation between the unexpected market volatility and market excess returns. The negative relation between unexpected market volatility and market excess returns implies an indirect evidence for a positive relation between expected market volatility and expected risk premiums.

volatility and market excess returns. Specifically, the coefficient estimate on the GARCH-in-mean term is 0.097 (SE = 0.033) in the volatility specification and is 3.392 (SE = 1.327) in the variance specification. These results are qualitatively identical and quantitatively similar to those for the 1928 to 1984 sample period. Recall that in Table 5, the corresponding coefficient estimates are 0.074 (SE = 0.024) and 2.412 (SE = 0.932). The results for the 1928 to 2018 sample period

	$\alpha \times 10^3$	β	$a \times 10^5$	b	c_1	c_2	θ
Panel A: January 1985 to December 2018							
Std. Dev.	−0.229	0.097	0.176	0.888	0.092	0.006	0.009
Eq. (8a), Eq. (5e)	(0.270)	(0.033)	(0.015)	(0.006)	(0.005)	(0.008)	(0.012)
Variance	0.294	3.392	0.174	0.889	0.093	0.004	0.009
Eq. (8b), Eq. (5e)	(0.123)	(1.327)	(0.015)	(0.006)	(0.005)	(0.008)	(0.012)
Panel B: January 1928 to December 2018							
Std. Dev.	−0.183	0.080	0.082	0.913	0.112	−0.029	−0.091
Eq. (8a), Eq. (5e)	(0.143)	(0.019)	(0.005)	(0.002)	(0.004)	(0.005)	(0.007)
Variance	0.223	2.735	0.082	0.913	0.112	−0.030	−0.091
Eq. (8b), Eq. (5e)	(0.066)	(0.755)	(0.005)	(0.002)	(0.004)	(0.005)	(0.007)

Table 8: GARCH-in-Mean Models at Daily Frequency—Out-of-Sample Results.

Description: This table reports the results of GARCH-in-mean models for daily market excess. $R_{mt} - R_{ft}$ is the daily excess return to S&P composite portfolio. In Panels A and B, respectively, the sample periods are January 1985 to December 2018 and January 1928 to December 2018. Standard errors are reported in parentheses.

$$R_{mt} - R_{ft} = \alpha + \beta\sigma_t + \varepsilon_t - \theta\varepsilon_{t-1}, \tag{8a}$$

$$R_{mt} - R_{ft} = \alpha + \beta\sigma_t^2 + \varepsilon_t - \theta\varepsilon_{t-1}, \tag{8b}$$

$$\sigma_t^2 = a + b\sigma_{t-1}^2 + c_1\varepsilon_{t-1}^2 + c_2\varepsilon_{t-2}^2. \tag{5e}$$

Interpretation: Out-of-sample evidence confirms the in-sample result. There is a positive and statistically significant relation between conditional market volatility and market excess returns.

(reported in Panel B) are similar. Overall, we confirm FSS's original findings for the GARCH-in-mean model in our out-of-sample analyses.[7]

3.3 Robustness Tests

In this section, we perform a number of robustness tests. For brevity, we report the results of all robustness tests for the 1928 to 2018 sample period. The results for the 1928 to 1984 and 1985 to 2018 sub-periods are qualitatively similar. French *et al.* (1987) use daily S&P 500 index returns (ex-dividend) to estimate market volatility and NYSE portfolio returns as their estimate of market expected returns because the CRSP daily index returns were not available prior to 1963 when FSS carried out their study. In our first robustness test, we use CRSP value-weighted index

[7] We estimate the realized volatility model (regression Eq. (7)) as well as the GARCH-in-mean model [Eq. (8b)] using an expanding window approach. The start year of the expanding window is 1928, while the end year changes from 1985 to 2018. We then plot the coefficient estimates for β and γ over time in Figures 1 and 2. We find both β and γ are quite stable during 1985 to 2018.

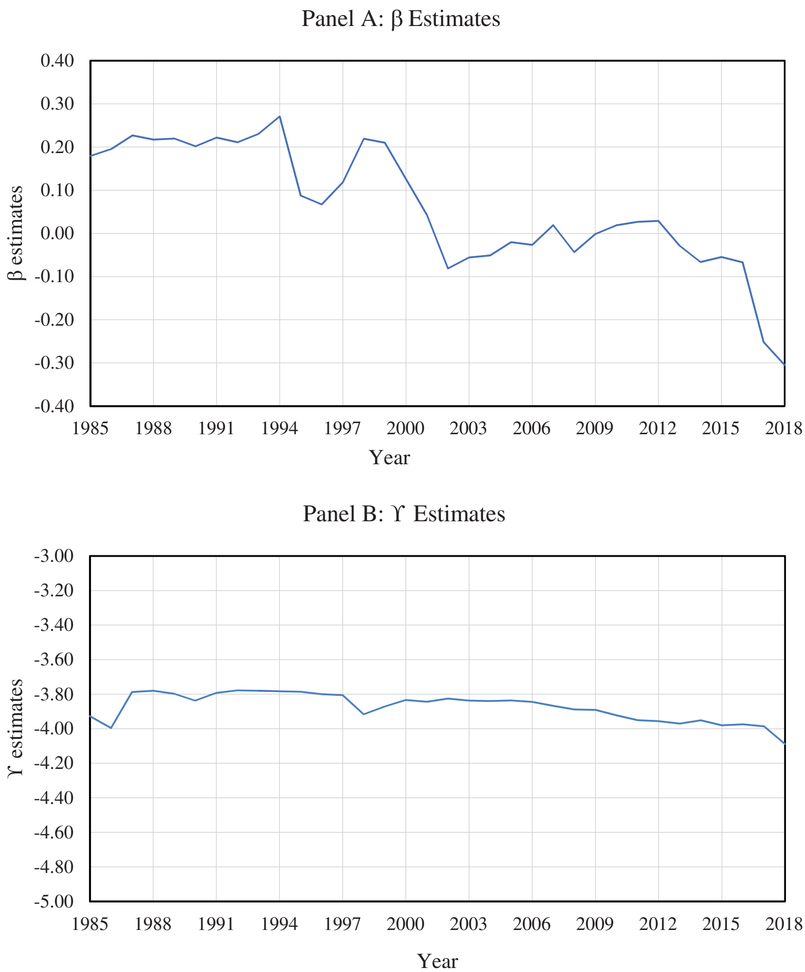

Figure 1: Expanding Window Analysis using Realized Volatility Model.

Description: Expanding window analysis using realized volatility model, $R_{mt} - R_{ft} = \alpha + \beta\hat{\sigma}^2_{mt} + \Upsilon\hat{\sigma}^{2u}_{mt} + \varepsilon_t$, where $R_{mt} - R_{ft}$ is the monthly excess return to S&P composite portfolio, and $\hat{\sigma}^2_{mt}$ and $\hat{\sigma}^{2u}_{mt}$ are predictable and unpredictable components of variance of market returns. The start year of the expanding window is 1928, while the end year changes from 1985 to 2018. Panel A plots the coefficient estimates for β, and Panel B plots the coefficient estimates for γ over time.

Interpretation: Both the coefficient estimates for β and γ are fairly stable over the past 30 years.

returns for both the estimation of market volatility and market expected returns. In Panel A of Table 9, we find that the results are qualitatively identical to those reported in Tables 4 (in-sample) and 7 (out-of-sample). That is, we continue to find an insignificant relation between the predicted component of market volatility

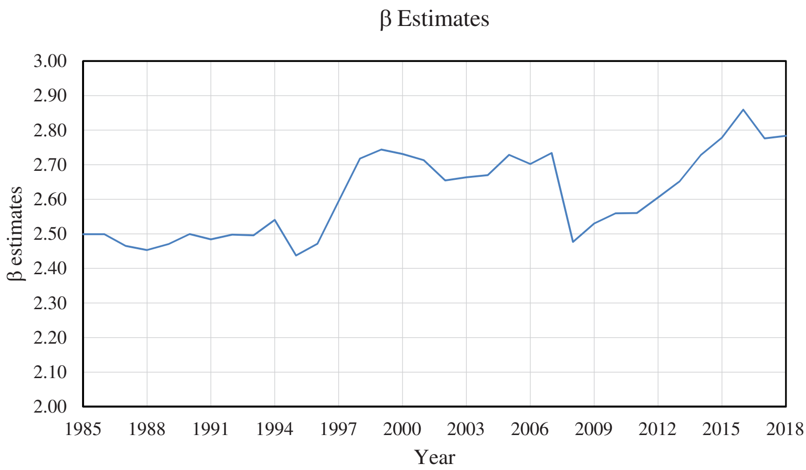

Figure 2: Expanding Window Analysis using GARCH-in-Mean Model.

Description: Expanding window analysis using GARCH-in-Mean model, $R_{mt} - R_{ft} = \alpha + \beta\sigma_t^2 + \varepsilon_t - \theta\varepsilon_{t-1}$, where $R_{mt} - R_{ft}$ is the monthly excess return to S&P composite portfolio, and $\sigma_t^2 = a + b\sigma_{t-1}^2 + c_1\varepsilon_{t-1}^2 + c_2\varepsilon_{t-2}^2$. The start year of the expanding window is 1928, while the end year changes from 1985 to 2018. The coefficient estimates for β are plotted over time.

Interpretation: The coefficient estimates for β are fairly stable over the past 30 years.

and market excess returns, and a negative and statistically significant relation between the unexpected market volatility and market returns.

FSS use a WLS approach to estimate the relation between market excess return and the predicted or unexpected market volatility, where the weight for each observation is the predicted market volatility. We examine whether their results are robust to the alternative OLS procedure while conducting inferences based on Newey and West (1987) standard errors. Panel B of Table 9 reports the results. We find the OLS results are qualitatively similar to the WLS results. Specifically, we continue to find an insignificant relation between market excess return and the predicted market volatility and strong evidence of a significantly negative relation between market excess return and the unpredicted market volatility.

Campbell (1987) and Scruggs (1998) point out that the difficulty in measuring a positive risk-return relation could stem from misspecification of Eq. (1). Following Merton (1973), they argue that if changes in the investment opportunity set are captured by state variables in addition to the conditional variance itself, then those variables must be included in the equation of expected returns. We therefore conduct a robustness test by including the T-bill rate, dividend yield, term spread, and default spread in the return-volatility equation (Fama and French, 1989) and present the results in Panel C of Table 9. We find that FSS's findings

	Eq. (6)		**Eq. (7)**		
	α	**β**	**α**	**β**	**γ**
Panel A: Daily and Monthly Returns on CRSP Value-Weighted Index					
Std. Dev.	0.0069	−0.0221	0.0109	−0.1184	−1.0082
	[0.0035]	[0.0960]	[0.0032]	[0.0902]	[0.0793]
Variance	0.0062	0.0161	0.0070	−0.2375	−4.5959
	[0.0018]	[0.8387]	[0.0017]	[0.8522]	[0.7186]
Panel B: OLS Regressions with Newey–West Standard Errors					
Std. Dev.	0.0005	0.1257	0.0026	0.0757	−0.6572
	(0.0067)	(0.1607)	(0.0069)	(0.1652)	(0.1253)
Variance	0.0028	1.1507	0.0039	0.8039	−2.5063
	(0.0027)	(0.9693)	(0.0028)	(1.0152)	(0.6722)

Panel C: OLS Regressions with Newey–West Standard Errors with Additional Controls

	α	**β**	**γ**	**T-bill**	**D/Y**	**Term**	**Default**
Eq. (6)	0.0259	0.0632		−0.0604	0.0066	0.1522	−0.0511
Std. Dev.	(0.0150)	(0.1859)		(0.0620)	(0.0036)	(0.1665)	(0.8704)
Eq. (6)	0.0293	1.4233		−0.0327	0.0069	0.1938	−0.5504
Variance	(0.0146)	(1.3321)		(0.0657)	(0.0036)	(0.1686)	(0.8673)
Eq. (7)	0.0263	−0.0875	−0.6685	−0.0783	0.0057	0.0344	0.4493
Std. Dev.	(0.0142)	(0.1768)	(0.1157)	(0.0580)	(0.0033)	(0.1521)	(0.8364)
Eq. (7)	0.0258	0.5275	−2.5130	−0.0534	0.0061	0.1083	−0.0572
Variance	(0.0139)	(1.3029)	(0.6279)	(0.0603)	(0.0034)	(0.1525)	(0.8221)

Table 9: Weighted Least Squares Regressions at Monthly Frequency—Robustness Tests.

Description: This table presents results from regressions of monthly market excess returns on predictable ($\hat{\sigma}^{\text{p}}_{mt}$) and unpredictable components ($\hat{\sigma}^{\text{pu}}_{mt}$) of standard deviations or variance of market returns. $R_{mt} - R_{ft}$ is the monthly market excess return. In Panel A, we use the CRSP value-weighted index returns and weighted least squares regressions. The predicted variance of the market portfolio, $\hat{\sigma}^2_{mt}$, is used to standardize each observation. Results in Panel B and C are based on ordinary least squares regressions. The sample period is January 1928 to December 2018. The numbers in brackets are standard errors based on White's (1980) consistent heteroskedasticity correction. The numbers in parentheses are calculated using Newey and West (1987) standard errors with six lags.

$$R_{mt} - R_{ft} = \alpha + \beta \hat{\sigma}^{\text{p}}_{mt} + \varepsilon_t, \tag{6}$$

$$R_{mt} - R_{ft} = \alpha + \beta \hat{\sigma}^{\text{p}}_{mt} + \Upsilon \hat{\sigma}^{\text{pu}}_{mt} + \varepsilon_t. \tag{7}$$

Interpretation: There is an insignificant relation between predicted volatility (or variance) and market excess returns, but a strong negative relation between the unexpected market volatility and market excess returns. The negative relation between unexpected market volatility and market excess returns implies an indirect evidence for a positive relation between expected market volatility and expected risk premiums.

regarding the relations between market excess returns and expected or unexpected market volatility are unaffected after controlling for these standard market return predictors.

In Table 10, we first re-estimate the GARCH-in-mean model by using the daily CRSP value-weighted index returns and find that the positive relation between market returns and conditional market volatility remain intact (Panel A). In addition, in Panel B of Table 10 we examine whether the positive market return-predicted volatility relation uncovered in a GARCH-in-mean model holds in the following EGARCH model:

$$R_{mt} - R_{ft} = \alpha + \beta \sigma_t + \varepsilon_t - \theta \varepsilon_{t-1}, \tag{8a}$$

$$R_{mt} - R_{ft} = \alpha + \beta \sigma_t^2 + \varepsilon_t - \theta \varepsilon_{t-1}, \tag{8b}$$

$$\log \sigma_t^2 = a + b \log \sigma_{t-1}^2 + c_1 g(z_{t-1}) + c_2 g(z_{t-2}), \tag{5f}$$

and

$$g(z_t) = \mathrm{d} z_t + (|z_t| - E|z_t|). \tag{5f}$$

Here, the mean equations are identical to those in the GARCH-in-mean model and our main hypothesis is to test whether $\beta > 0$. A positive β would indicate that the expected market excess return is positively related to conditional volatility or conditional variance. The main difference between the GARCH-in-mean model and the EGARCH-in-mean model lies in the variance equation. Specifically, the dependent variable in the variance equation is σ_t^2 in the GARCH model, and is $\ln \sigma_t^2$ in the EGARCH model. By modeling the logarithm of conditional variance, the EGARCH model guarantees that the conditional variance is positive. Also the $g(z_t)$ term allows for the possibility that volatility differs according to the sign of the return, i.e., volatility asymmetry. Results reported in Panel B continue to indicate a positive risk-return relation and the result is statistically significant when using conditional variance as a proxy for risk. Overall, results in Tables 9 and 10 indicate that the main findings of FSS are robust to alternative data and model specifications.

3.4 International Evidence

Next, we investigate whether the main findings of FSS extend to international stock markets. As described earlier, we obtain daily and monthly stock market returns of 23 developed countries from AQR. The sample period for most of these markets is 1985 to 2018. We examine the risk-return relation using both the realized volatility approach and the GARCH-in-mean approach and present the results in Tables 11 and 12, respectively.

Similar to the results for the U.S., we find in Table 11 that the relation between market return and the predicted market volatility is largely statistically

Panel A: CRSP Value-Weighted Index Returns							
	$\alpha \times 10^3$	β	$a \times 10^5$	b	$c1$	$c2$	θ
Std. Dev.	−0.194	0.109	0.115	0.892	0.116	−0.018	−0.133
Eqs. (8a), (5e)	(0.136)	(0.020)	(0.005)	(0.003)	(0.004)	(0.005)	(0.007)
Variance	0.342	3.702	0.115	0.892	0.116	−0.018	−0.132
Eqs. (8b), (5e)	(0.065)	(0.835)	(0.005)	(0.003)	(0.004)	(0.005)	(0.007)

Panel B: EGARCH								
	$\alpha \times 10^3$	β	a	b	$c1$	$c2$	θ	d
Std. Dev.	0.178	0.014	−0.110	0.988	0.213	−0.072	−0.094	−0.520
Eqs. (8a), (5f)	(0.128)	(0.017)	(0.007)	(0.001)	(0.006)	(0.006)	(0.007)	(0.017)
Variance	0.137	2.464	−0.121	0.987	0.212	−0.071	−0.096	−0.522
Eqs. (8b), (5f)	(0.065)	(0.724)	(0.007)	(0.001)	(0.006)	(0.006)	(0.007)	(0.017)

Table 10: GARCH-in-Mean Models of the Risk-Return Tradeoff at Daily Frequency—Robustness Tests Results.

Description: This table reports results from robustness tests using different GARCH-in-mean models. In Panel A, we use the CRSP value-weighted index returns. Results in Panel B are based on EGARCH-in-mean model. The sample period is January 1928 to December 2018. The numbers in parentheses are standard errors.

$$R_{mt} - R_{ft} = \alpha + \beta\sigma_t + \varepsilon_t - \theta\varepsilon_{t-1}, \quad (8a)$$

$$R_{mt} - R_{ft} = \alpha + \beta\sigma_t^2 + \varepsilon_t - \theta\varepsilon_{t-1}, \quad (8b)$$

$$\sigma_t^2 = a + b\sigma_{t-1}^2 + c_1\varepsilon_{t-1}^2 + c_2\varepsilon_{t-2}^2, \quad (5e)$$

$$\log\sigma_t^2 = a + b\log\sigma_{t-1}^2 + c_1 g(z_{t-1}) + c_2 g(z_{t-2}), \text{ and } g(z_t) = dz_t + (|z_t| - E|z_t|). \quad (5f)$$

Interpretation: There is a positive and statistically significant relation between market returns and conditional market volatility.

insignificant, whether we estimate the univariate regression Eq. (6) or the bivariate regression Eq. (7). For example, in regression Eq. (6) the coefficient estimates of β is positive and significant in Canada, negative and significant in Hong Kong and Portugal, and statistically insignificant in the other 20 markets. Similar to the U.S., we find strong evidence of a negative relation between the market excess return and the unexpected market volatility. Specifically, 21 of the 23 countries exhibit a statistically significant negative relation. Only Greece and Ireland exhibit an insignificant, but still negative relation.

In Table 12, we present the results for the GARCH-in-mean model. For brevity, we only report the coefficient for the GARCH-in-mean term in this table. Here, consistent with the results for the U.S., we find an overall positive relation between market return and conditional market volatility. Specifically, the coefficient estimate is positive in 18 out of 23 markets and is negative only in U.K., Greece,

	Eq. (6)		Eq. (7)		
Country	α	β	α	β	γ
Australia	0.0040	0.8851	0.0079	−0.5644	−4.9302
	[0.0046]	[1.1192]	[0.0045]	[1.0705]	[0.4394]
Austria	0.0070	−0.3042	0.0079	−0.5849	−2.7694
	[0.0045]	[1.1579]	[0.0044]	[1.0578]	[1.0760]
Belgium	0.0118	−1.4269	0.0137	−2.1901	−5.7226
	[0.0039]	[1.4367]	[0.0036]	[1.2260]	[0.5962]
Canada	−0.0193	6.9561	−0.0142	4.6538	−6.8637
	[0.0109]	[3.2278]	[0.0100]	[2.9670]	[1.1399]
Switzerland	0.0098	−1.2306	0.0113	−1.9246	−5.0089
	[0.0047]	[1.7683]	[0.0047]	[1.7327]	[0.9886]
Germany	0.0078	−0.7437	0.0110	−2.0891	−5.3621
	[0.0039]	[1.1710]	[0.0039]	[1.1784]	[0.9307]
Denmark	0.0170	−2.8041	0.0180	−3.2309	−5.0608
	[0.0039]	[1.4910]	[0.0037]	[1.3803]	[1.0353]
Spain	0.0071	0.1560	0.0088	−0.4402	−3.7460
	[0.0050]	[1.1545]	[0.0050]	[1.1376]	[0.7804]
Finland	0.0121	−0.4695	0.0149	−1.3238	−3.2960
	[0.0051]	[0.7622]	[0.0050]	[0.8032]	[0.9940]
France	0.0075	−0.2939	0.0094	−1.0134	−5.1437
	[0.0044]	[1.3175]	[0.0044]	[1.2468]	[0.8103]
UK	0.0087	−1.1520	0.0096	−1.5317	−4.3288
	[0.0036]	[1.3325]	[0.0035]	[1.2151]	[0.8106]
Greece	0.0166	−1.1388	0.0173	−1.2520	−0.5328
	[0.0073]	[0.7573]	[0.0073]	[0.8712]	[1.3494]
Hong Kong	0.0234	−2.5231	0.0268	−3.2147	−4.3296
	[0.0052]	[1.0914]	[0.0050]	[1.0610]	[0.4618]
Ireland	0.0106	−0.5380	0.0120	−0.8253	−1.7697
	[0.0046]	[1.0303]	[0.0047]	[1.1548]	[1.1955]
Israel	0.0003	0.5496	−0.0027	2.1389	−1.2812
	[0.0072]	[2.4214]	[0.0064]	[1.7882]	[0.1257]
Italy	0.0080	−0.6863	0.0129	−2.0549	−4.3370
	[0.0054]	[1.0129]	[0.0050]	[0.9920]	[0.5944]
Japan	−0.0011	0.9467	−0.0000	0.6185	−2.1463
	[0.0056]	[1.5775]	[0.0057]	[1.6412]	[0.8537]
Netherlands	0.0130	−1.8941	0.0145	−2.5555	−6.6008
	[0.0039]	[1.3118]	[0.0036]	[1.1333]	[0.7864]

Table 11: Weighted Least Squares Regressions at Monthly Frequency—International Evidence.

	Eq. (6)		Eq. (7)		
Country	**α**	**β**	**α**	**β**	**γ**
Norway	0.0132	−0.8326	0.0164	−1.6316	−5.4206
	[0.0053]	[0.9990]	[0.0050]	[0.9284]	[0.6037]
New Zealand	0.0102	−0.8267	0.0108	−1.0570	−3.5768
	[0.0058]	[1.8885]	[0.0057]	[1.8846]	[1.7627]
Portugal	0.0162	−3.3649	0.0175	−3.9073	−2.4870
	[0.0046]	[1.2171]	[0.0045]	[1.2229]	[1.1783]
Singapore	0.0023	1.4550	0.0033	1.1808	−2.8990
	[0.0037]	[1.2137]	[0.0036]	[1.2013]	[0.5987]
Sweden	0.0105	−0.3125	0.0138	−1.2279	−5.1638
	[0.0046]	[0.8781]	[0.0044]	[0.8346]	[0.8130]

Table 11: *Continued.*

Description: This table presents results from weighted least squares regressions of monthly market excess returns on predictable ($\hat{\sigma}^2_{mt}$) and unpredictable components ($\hat{\sigma}^{2u}_{mt}$) of variance of market returns for 23 developed countries. $R_{mt} - R_{ft}$ is the monthly excess return. The predicted variance of the market return, $\hat{\sigma}^2_{mt}$, is used as the weight to standardize each observation. The sample period is January 1928 to December 2018. The numbers in brackets are standard errors based on White's (1980) consistent heteroskedasticity correction.

$$R_{mt} - R_{ft} = \alpha + \beta \hat{\sigma}^2_{mt} + \varepsilon_t, \tag{6}$$

$$R_{mt} - R_{ft} = \alpha + \beta \hat{\sigma}^2_{mt} + \Upsilon \hat{\sigma}^{2u}_{mt} + \varepsilon_t. \tag{7}$$

Interpretation: There is an insignificant relation between predicted variance and market excess returns for 20 countries. There is a strong negative relation between the unexpected market variance and market excess returns for 21 countries.

Ireland, Israel, and New Zealand. Among the 18 positive coefficients, seven are statistically significant at the 5% level. Overall, we find the main findings of FSS for the U.S. extends to the international markets.

3.5 Business Cycle, Sentiment, and Risk-Return Tradeoff

Previous studies suggest that the risk-return relation may be time-varying (see e.g., Campbell, 1987; Harvey 1989, 2001; Lettau and Ludvigson, 2010). In this section, we first examine whether the risk-return relation vary with business cycles. We divide our sample period 1928 to 2018 into recessions and expansions by using NBER recession dates. We then repeat the main analyses of FSS separately for recession and expansion periods and report the results in Table 13. In Panel A, we report the results for the realized volatility approach. In Panel B, we report the results for the GARCH-in-mean approach.

In Panel A, we find a negative but insignificant relation between risk and return in univariate regressions during recession periods. In comparison, we find a

Country	β	S.E. (β)
Australia	1.9719	(1.2599)
Austria	2.392	(1.3637)
Belgium	0.2373	(1.5577)
Canada	5.2284	(1.0233)
Switzerland	2.6292	(1.6594)
Germany	2.5479	(1.196)
Denmark	0.0121	(1.6924)
Spain	2.9735	(1.1783)
Finland	2.0375	(0.9344)
France	3.3811	(1.2599)
UK	−0.0008	(1.3948)
Greece	−0.0115	(0.8808)
Hong Kong	1.4817	(1.0953)
Ireland	−0.0994	(0.972)
Israel	−0.2104	(0.1717)
Italy	2.2696	(1.1441)
Japan	4.5457	(1.4158)
Netherlands	1.5599	(1.2277)
Norway	0.0422	(1.1121)
New Zealand	−0.0363	(1.6612)
Portugal	0.132	(1.4144)
Singapore	2.023	(1.3044)
Sweden	1.8055	(1.0642)

Table 12: GARCH-in-Mean Models at Daily Frequency—International Evidence.

Description: This table reports the results of GARCH-in-mean models for daily market excess returns for 23 developed countries. $R_{mt} - R_{ft}$ is the daily market excess return for each country. We conduct GARCH-in-mean analysis according to Eqs. (8b) and (5e). We select the best model, i.e., combination of p and q, based on AIC, where and p and q can be 1, 2, and 3. Due to limited space, we only report the β coefficient and standard errors from Eq. (8b), which captures the risk-return relations. The sample period is January 1928 to December 2018. Standard errors are reported in parentheses.

$$R_{mt} - R_{ft} = \alpha + \beta \sigma_t^2 + \varepsilon_t - \theta \varepsilon_{t-1}, \tag{8b}$$

$$\sigma_t^2 = a + \sum_{k=1}^{P} b_k \sigma_{t-k}^2 + \sum_{k=1}^{q} c_k \varepsilon_{t-k}^2. \tag{5e}$$

Interpretation: There is an overall positive relation between market return and conditional market volatility. Specifically, the coefficient estimate is positive in 18 out of 23 markets and is negative only in U.K., Greece, Ireland, Israel, and New Zealand.

positive and marginally significant relation between risk and return in expansion periods. These findings suggest that the risk-return tradeoff is stronger during expansions than during recessions. This difference, however, largely disappears

Panel A: Weighted Least Squares Regressions of the Risk-Return Tradeoff					
	Eq. (6)		**Eq. (7)**		
	α	β	α	β	γ
Panel A.1: Recessions					
Std. Dev.	0.0134	-0.3456	0.0147	−0.2731	−0.8426
	(0.0125)	(0.2503)	(0.0115)	(0.2300)	(0.1412)
	[0.0124]	[0.2459]	[0.0118]	[0.2406]	[0.1133]
Variance	0.0007	-1.2076	0.0034	-0.9741	-2.9899
	(0.0063)	(1.5160)	(0.0059)	(1.4191)	(0.5733)
	[0.0063]	[1.4812]	[0.0060]	[1.5170]	[0.3368]
Panel A.2: Non-Recessions					
Std. Dev.	0.0031	0.1287	0.0089	-0.0409	-0.8205
	(0.0036)	(0.1032)	(0.0035)	(0.0991)	(0.0799)
	[0.0037]	[0.0996]	[0.0035]	[0.0988]	[0.1105]
Variance	0.0051	1.5777	0.0070	0.2177	-5.2372
	(0.0018)	(0.9718)	(0.0017)	(0.9322)	(0.5220)
	[0.0018]	[0.8907]	[0.0018]	[0.9794]	[1.1526]

Panel B: GARCH-in-Mean Models of the Risk-Return Tradeoff							
	$\alpha \times 10^3$	β	$a \times 10^5$	b	c_1	c_2	θ
Panel B.1: Recessions							
Std. Dev.	0.332	0.026	0.152	0.868	0.068	0.067	−0.103
Eqs. (8a), (5e)	(0.345)	(0.034)	(0.023)	(0.008)	(0.013)	(0.016)	(0.015)
Variance	0.483	0.812	0.153	0.868	0.069	0.066	−0.103
Eqs. (8b), (5e)	(0.197)	(1.019)	(0.023)	(0.008)	(0.014)	(0.016)	(0.015)
Panel B.2: Non-Recessions							
Std. Dev.	-0.260	0.113	0.078	0.920	0.124	-0.051	−0.088
Eqs. (8a), (5e)	(0.177)	(0.025)	(0.005)	(0.002)	(0.004)	(0.005)	(0.008)
Variance	0.255	4.994	0.078	0.921	0.124	-0.051	−0.088
Eqs. (8b), (5e)	(0.082)	(1.219)	(0.005)	(0.002)	(0.004)	(0.005)	(0.008)

Table 13: Business Cycles and the Risk-Return Tradeoff.

Table 13: *Continued.*

Description: This table reports the risk-return tradeoff during recessions and non-recessions. Panel A presents results from weighted least squares regressions of monthly S&P excess returns on predictable ($\hat{\sigma}^{\mathrm{p}}_{mt}$) and unpredictable components ($\hat{\sigma}^{\mathrm{pu}}_{mt}$) of standard deviations or variance of market returns. $R_{mt} - R_{ft}$ is the monthly S&P excess return. The predicted variance of the market portfolio, $\hat{\sigma}^2_{mt}$, is used to standardize each observation.

$$R_{mt} - R_{ft} = \alpha + \beta \hat{\sigma}^{\mathrm{p}}_{mt} + \varepsilon_t, \tag{6}$$

$$R_{mt} - R_{ft} = \alpha + \beta \hat{\sigma}^{\mathrm{p}}_{mt} + \Upsilon \hat{\sigma}^{\mathrm{pu}}_{mt} + \varepsilon_t. \tag{7}$$

Panel B reports results from GARCH-in-mean models using daily S&P excess returns. The sample period is July 1965 to December 2018. The numbers in parentheses are standard errors. The numbers in brackets are standard errors based on White's (1980) consistent heteroskedasticity correction.

$$R_{mt} - R_{ft} = \alpha + \beta \sigma_t + \varepsilon_t - \theta \varepsilon_{t-1}, \tag{8a}$$

$$R_{mt} - R_{ft} = \alpha + \beta \sigma^2_t + \varepsilon_t - \theta \varepsilon_{t-1}, \tag{8b}$$

$$\sigma^2_t = a + b\sigma^2_{t-1} + c_1 \varepsilon^2_{t-1} + c_2 \varepsilon^2_{t-2}. \tag{5e}$$

Interpretation: The risk-return tradeoff is stronger during expansions than during recessions.

in bivariate regressions when we also include the unexpected market volatility. In both recessions and expansions, we find a significant and negative relation between market excess returns and the unexpected market volatility, consistent with the full sample results.

In Panel B of Table 13, we repeat the GARCH-in-mean analysis separately for recessions and expansions. The results show a positive but insignificant relation between risk (both variance and standard deviation proxies) and return during recession periods but a positive and statistically significant relation between risk (both variance and standard deviation proxies) and return during expansion. This result is consistent with the previous finding for the univariate regression in Panel A that the risk-return tradeoff is stronger during expansions than during recessions. This finding is somewhat puzzling. One might argue that because risk aversion is higher during recessions, the risk-return tradeoff should be more favorable during recessions in order to induce investors to hold the market portfolio (Lettau and Ludvigson, 2010).

Next, we investigate whether the relation between risk and return differs between high- and low-sentiment periods. Given that high investor sentiment might induce overvaluation (i.e., low subsequent return) and high volatility (Baker and Wurgler, 2006; Stambaugh *et al.*, 2012), one might expect the risk-return tradeoff to be less favorable during high-sentiment periods than during low-sentiment periods. In a similar vein, Yu and Yuan (2011) argue that sentiment investors exert a greater influence on prices during high-sentiment periods. Because sentiment investors also tend to misestimate risk and return, the positive risk-return relation are likely to be weaker during high-sentiment periods. We test this prediction by dividing the sample period into low sentiment and high sentiment periods using the investor sentiment data of Baker and Wurgler (2006). We then repeat our realized volatility analysis and the GARCH-in-mean analysis separately for high- and low-sentiment periods.

We present the results for the realized volatility approach in Panel A of Table 14. We first discuss results for Eq. (6) that investigates the relation between risk and return in a univariate regression. In Panel A.1, we find a negative but not significant relation between risk and return in high sentiment periods and in Panel A.2, we also find a negative but insignificant relation between risk and return in low sentiment periods. The results for Eq. (7), which includes both the predicted and unpredicted market volatility in the regressions of market excess returns, continue to indicate a negative but insignificant risk-return relation during both high and low sentiment periods for both the standard deviation and variance specification. Consistent with the full-sample results, we find a negative and statistically significant strong relation between the unpredicted component of risk and return during both high and low sentiment periods, using either standard deviation or variance as a proxy for risk.

Panel A: Weighted Least Squares Regressions of the Risk-Return Tradeoff					
	Eq. (6)		**Eq. (7)**		
	α	β	α	β	γ
Panel A.1: High Sentiment Period					
Std. Dev.	0.0088	−0.0978	0.0133	−0.1981	−0.6245
	(0.0073)	(0.2011)	(0.0071)	(0.1949)	(0.1246)
	[0.0069]	[0.1900]	[0.0071]	[0.1939]	[0.1807]
Variance	0.0065	−0.7385	0.0073	−0.9002	−3.5621
	(0.0038)	(2.0734)	(0.0036)	(1.9843)	(0.6463)
	[0.0035]	[1.8321]	[0.0034]	[1.7437]	[0.5228]
Panel A.2: Low Sentiment Period					
Std. Dev.	0.0074	−0.0680	0.0115	−0.1923	−0.8654
	(0.0056)	(0.1585)	(0.0053)	(0.1491)	(0.1253)
	[0.0061]	[0.1645]	[0.0058]	[0.1567]	[0.1481]
Variance	0.0054	−0.1544	0.0062	−0.8343	−5.7715
	(0.0031)	(1.6509)	(0.0029)	(1.5570)	(0.8891)
	[0.0034]	[1.6480]	[0.0032]	[1.6376]	[1.2951]

Table 14: Investor Sentiment and the Risk-Return Tradeoff.

Panel B: GARCH-in-Mean Models of the Risk-Return Tradeoff							
	$\alpha \times 10^3$	β	$a \times 10^5$	b	c_1	c_2	θ
Panel B.1: High Sentiment Period							
Std. Dev.	−0.250	0.098	0.147	0.890	0.107	−0.009	−0.075
Eqs. (8a), (5e)	(0.346)	(0.044)	(0.016)	(0.006)	(0.006)	(0.009)	(0.014)
Variance	0.234	4.013	0.146	0.890	0.108	−0.010	−0.075
Eqs. (8b), (5e)	(0.163)	(2.060)	(0.016)	(0.006)	(0.006)	(0.009)	(0.014)
Panel B.2: Low Sentiment Period							
Std. Dev.	−0.130	0.074	0.140	0.894	0.065	0.027	−0.077
Eqs. (8a), (5e)	(0.307)	(0.041)	(0.015)	(0.006)	(0.009)	(0.010)	(0.013)
Variance	0.119	4.966	0.142	0.894	0.068	0.024	−0.077
Eqs. (8b), (5e)	(0.137)	(1.703)	(0.015)	(0.006)	(0.009)	(0.010)	(0.013)

Table 14: *Continued.*

Description: This table reports the risk-return tradeoff during high and low sentiment periods. Panel A presents results from weighted least squares regressions of monthly S&P excess returns on predictable ($\hat{\sigma}^{\mathrm{p}}_{mt}$) and unpredictable components ($\hat{\sigma}^{\mathrm{pu}}_{mt}$) of standard deviations or variance of market returns. $R_{mt} - R_{ft}$ is the monthly S&P excess return. The predicted variance of the market portfolio, $\hat{\sigma}^2_{mt}$, is used to standardize each observation.

$$R_{mt} - R_{ft} = \alpha + \beta \hat{\sigma}^{\mathrm{p}}_{mt} + \varepsilon_t, \tag{6}$$

$$R_{mt} - R_{ft} = \alpha + \beta \hat{\sigma}^{\mathrm{p}}_{mt} + \Upsilon \hat{\sigma}^{\mathrm{pu}}_{mt} + \varepsilon_t. \tag{7}$$

Panel B reports results from GARCH-in-mean models using daily S&P excess returns. The sample period is July 1965 to December 2018. The numbers in parentheses are standard errors. The numbers in brackets are standard errors based on White's (1980) consistent heteroskedasticity correction.

$$R_{mt} - R_{ft} = \alpha + \beta \sigma_t + \varepsilon_t - \theta \varepsilon_{t-1}, \tag{8a}$$

$$R_{mt} - R_{ft} = \alpha + \beta \sigma^2_t + \varepsilon_t - \theta \varepsilon_{t-1}, \tag{8b}$$

$$\sigma^2_t = a + b\sigma^2_{t-1} + c_1 \varepsilon^2_{t-1} + c_2 \varepsilon^2_{t-2}. \tag{5e}$$

Interpretation: There is little evidence that the risk-return tradeoff differs significantly between high- and low-sentiment periods.

We also redo the GARCH-in-mean analysis separately for high and low sentiment periods and present the results in Panel B of Table 14. The results show a positive and significant relation between risk (for both variance and standard deviation proxies) and return during both high and low sentiment periods and the point estimates are not significantly different across the two periods. Taken together, the results in Table 14 show little evidence that the risk-return tradeoff differs significantly between high- and low-sentiment periods. This finding is somewhat different from Yu and Yuan (2011), who find that the risk-return tradeoff is more pronounced during low-sentiment periods.[8]

4 Conclusions

We replicate the findings of French, Schwert, and Stambaugh (FSS, 1987) almost exactly. Consistent with FSS, we find modest evidence of a positive relation between the market excess return and the predicted market volatility and strong evidence of a negative relation between the market excess return and the unexpected market volatility during 1928 to 1984. These results persist during 1985 to 2018 and are robust to alternative data and model specifications. We extend the analysis to 23 developed countries and find qualitatively similar results. We also show that the positive market return-market volatility relation is stronger during expansion than during recession and does not vary significantly with investor sentiment.

The risk-return relation is a fundamental issue in finance and the lack of consensus on the exact nature of this relation suggests that it will continue to be an active area of research. In our view, the main challenge to identify the risk-return relation is to pin down the expected market return. Given the relatively short sample period and the time-varying nature of the market risk premium, estimating expected market returns using conditioning information is likely to be a fruitful approach. We do caution that researchers need to guard against data mining because theory offers little guidance as to what conditioning information to use to model risk and return. Finally, research into the performance of volatility-managed portfolios can also generate new insights into the risk-return tradeoff.

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[8]Yu and Yuan (2011) sample and methodology are different from ours. They use NYSE/AMEX index returns during 1963 to 2004, and we use the S&P 500 index returns during 1965 to 2018. Yu and Yuan use lagged realized variance as a proxy for conditional variance, whereas we use predicted realized variance from an ARIMA model following FSS. Yu and Yuan use annual sentiment index, whereas we use monthly sentiment index.

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