



Moving Boundary PDE Analysis

Biomedical Applications in R

Mathematical models stated as systems of partial differential equations (PDEs) are broadly used in biology, chemistry, physics and medicine (physiology). These models describe the spatial and temporal variations of the problem system dependent variables, such as temperature, chemical and biochemical concentrations and cell densities, as a function of space and time (spatiotemporal distributions).

For a complete PDE model, initial conditions (ICs) specifying how the problem system starts and boundary conditions (BCs) specifying how the system is defined at its spatial boundaries, must also be included for a well-posed PDE model. In this book, PDE models are considered for which the physical boundaries move with time. For example, as a tumor grows, its boundary moves outward. In atherosclerosis, the plaque formation on the arterial wall moves inward, thereby restricting blood flow with serious consequences such as stroke and myocardial infarction (heart attack).

These two examples are considered as applications of the reported moving boundary PDE (MBPDE) numerical method (algorithm). The method is programmed in a set of documented routines coded in R, a quality, open-source scientific programming system. The routines are provided as a download so that the readers/analysts/researchers can use MBPDE models without having to first study numerical methods and computer programming.

Principal features of the book include:

- Discusses the numerical solution of partial differential equations (PDEs) with moving boundary conditions (MBPDEs)
- Provides a numerical method (algorithm) for the solution of MBPDEs
- Details a computer implementation of the numerical method through a series of documented routines in R
- Illustrates the use of the numerical algorithm and R routines with two important applications in biomedicine
- Discusses a generic methodology for the computer analysis of MBPDEs

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Biomedical Applications in R

William E. Schiesser

$$\begin{aligned} \frac{\partial u_1}{\partial t} &= \frac{\partial^2 u_1}{\partial r^2} + \frac{2}{r} \frac{\partial u_1}{\partial r} \\ -\chi_1 \left(u_1 \frac{\partial^2 u_2}{\partial r^2} + \frac{\partial u_1}{\partial r} \frac{\partial u_2}{\partial r} + \frac{2}{r} u_1 \frac{\partial u_2}{\partial r} \right) \\ -\chi_2 \left(u_1 \frac{\partial^2 u_3}{\partial r^2} + \frac{\partial u_1}{\partial r} \frac{\partial u_3}{\partial r} + \frac{2}{r} u_1 \frac{\partial u_3}{\partial r} \right) \\ &+ \mu u_1 (r_1 - u_1 - u_3) \\ \frac{\partial u_1(r=0, t)}{\partial r} &= 0 \\ \frac{\partial u_1(r=r_u, t)}{\partial r} &= 0 \\ u_1(r, t=0) &= h_1(r) \\ \frac{dr_u}{dt} &= k_{ru} u_1(r=r_u, t) \end{aligned}$$

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