

**An Introductory Comparison of Matlab and R
Applications to Ordinary and Partial Differential Equations**

**W. E. Schiesser
Lehigh University
Bethlehem, USA¹**

¹Iacocca B312, Lehigh University, 111 Research Drive, Bethlehem, PA
18015 USA; wes1@lehigh.edu; 610-758-4264

Features

Matlab

- Convenient graphics
 - 2D, 3D (in-line)
 - Color
- Quality utilities
 - ODE integrators (e.g., ode15s)
 - Extensive linear algebra routines

Features

Matlab

- Convenient graphics
 - 2D, 3D (in-line)
 - Color
- Quality utilities
 - ODE integrators (e.g., ode15s)
 - Extensive linear algebra routines

R

- Convenient graphics
 - 2D, 3D (in-line)
 - Color
 - Many options
- Quality utilities
 - ODE integrators (e.g., ode, lsoda, lsodes)
 - Extensive linear algebra routines

Features (cont'd)

Matlab

- Complicated, expensive annual licenses
- Limits use

R

- Open source (no cost)
- Available to anyone with an Internet connection
- Easily downloaded

ODEs

Single ODE for tumor growth

$$\frac{dV}{dt} = \lambda e^{-\alpha t} V; \quad V(t_0) = V_0$$

V tumor volume

t time

$\lambda, \alpha, t_0, V_0$ constants

ODEs

Single ODE for tumor growth

$$\frac{dV}{dt} = \lambda e^{-\alpha t} V; \quad V(t_0) = V_0$$

V tumor volume

t time

$\lambda, \alpha, t_0, V_0$ constants

Analytical solution

$$V(t) = V_0 e^{(\lambda/\alpha)(1-e^{-\alpha t})}$$

Limiting solution

$$\lambda = \alpha > 0; \quad V_0 = 1$$

$$V(t \rightarrow \infty) = e$$

Braun, M. (1993), *Differential Equations and Their Applications*, 4th ed., Springer-Verlag, New York, 52-53

Two interpretations:

$$\frac{dV}{dt} = (\lambda e^{-\alpha t}) V; \quad \frac{dV}{dt} = \lambda (e^{-\alpha t}) V;$$

ODE Routines

Matlab

```
function yt=tumor_1(t,y)
%
% Function tumor_1 computes the derivative for the
% tumor growth ODE
%
global alpha lambda icalse ncall
%
% Calls to tumor_1
ncall=ncall+1;
%
% ODE
yt=lambda(icalse)*exp(-alpha*t)*y;
```

R

```
tumor_1=function(t,y,parms){
#
# Function tumor_1 computes the derivative for the
# tumor growth ODE
#
# Calls to tumor_1
ncall<-ncall+1;
#
# ODE
yt=lambda[icalse]*exp(-alpha*t)*y;
#
# Return derivative
return(list(c(yt)));
}
```

ODE Main Program - Matlab

```
%
% Clear previous files
clear all
clc
%
% Parameters, variable shared by other routines
global alpha lambda icase ncall
%
% Initial condition
for(icas=1:3)
    if(icas==1)lambda(1)=1    ;end
    if(icas==2)lambda(2)=0.75;end
    if(icas==3)lambda(3)=1.25;end
    V0=1;alpha=1;ncall=0;
%
% t scale
tout=[0:0.5:10]';nout=21;
%
% ODE integration
[t,V]=ode15s(@tumor_1,tout,V0);
%
% Display heading
fprintf('\n icase = %2d',icas);
%
% Display calls to tumor_1
fprintf('\n\n Calls to tumor_1 = %3d\n',ncall);
fprintf('\n      t      V(t)(num)  V(t)(anal)    diff\n');
%
% Display numerical output
for(it=1:nout)
    Va(it)=V0*exp(lambda(icas)/alpha*(1-exp(-alpha*tout(it))));
    Vplot(it,icas)=V(it,1);
    Vaplot(it,icas)= Va(it);
```



```

        diff(it,icase)=(Vplot(it,icase)-Vaplot(it,icase))...
                        /Vplot(it,icase);
        fprintf('%10.2f%12.4f%12.4f%12.6f\n',...
            tout(it),Vplot(it,icase),Vaplot(it,icase),diff(it,icase));
    end
end
%
% Plot numerical, analytical solutions
figure(1)
plot(tout,Vplot,'-',tout,Vaplot,'o');
xlabel('t');ylabel('V(t)');
title('V(t), solid - num, points - anal');
%
% Plot difference of solutions
figure(2)
plot(tout,diff,'-');
xlabel('t');ylabel('diff');
title('diff = num - anal (rel), V(t)');

```

ODE Main Program - R

```
#
# Delete previous workspaces
rm(list=ls(all=TRUE))
#
# Access ODE integrator
library("deSolve");
#
# Access ODE file
setwd("c:/R/odepde_intro/ode");
source("tumor_1.R");
#
# Matrices for plotted output
nout=21;
Vplot=matrix(0,nrow=nout,ncol=3);
Vaplot=matrix(0,nrow=nout,ncol=3);
diff=matrix(0,nrow=nout,ncol=3);
#
# Initial condition
lambda=rep(0,3);
for(icase in 1:3){
  if(icase==1){lambda[1]=1 ;}
  if(icase==2){lambda[2]=0.75;}
  if(icase==3){lambda[3]=1.25;}
  V0=1;alpha=1;ncall=0;
#
# t scale
tout=seq(from=0,to=10,by=0.5);
#
# ODE integration
out=ode(func=tumor_1,times=tout,y=V0,parms);
#
# Display heading
V=rep(0,nout);Va=rep(0,nout);
```

```

V=out[,-1];
cat(sprintf("\n\n  icase = %2d\n",icase));
#
# Display calls to ODE routine
cat(sprintf("\n  ncall = %3d\n",ncall));
#
# Display numerical output
cat(sprintf("\n          t          V(t)(num)    V(t)(anal)    diff\n"));
for(it in 1:nout){
    Va[it]=V0*exp(lambda[icase]/alpha*(1-exp(-alpha*tout[it])));
    Vplot[it,icase]= V[it];
    Vaplot[it,icase]=Va[it];
    diff[it,icase]=(Vplot[it,icase]-Vaplot[it,icase])/
                    Vplot[it,icase];
    cat(sprintf("%10.2f%12.4f%12.4f%12.6f\n",
        tout[it],Vplot[it,icase],Vaplot[it,icase],diff[it,icase]));
}
}
#
# Plot numerical, analytical solutions
par(mfrow=c(1,1))
plot(tout,Vplot[,1],xlab="t",ylab="V(t)",
     xlim=c(0,10),ylim=c(1,5),
     main="V(t), solid - num, points - anal",type="l",lwd=2);
points(tout,Vaplot[,1],pch="1");
lines(tout, Vplot[,2],type="l",lwd=2);
points(tout,Vaplot[,2],pch="2");
lines(tout, Vplot[,3],type="l",lwd=2);
points(tout,Vaplot[,3],pch="3");
#
# Plot difference of solutions
par(mfrow=c(1,1))
matplot(tout,diff,type="l",xlab="t",ylab="diff",
        col="black",lwd=2,lty=1,main="diff = num - anal (rel)");

```

Numerical Solution - Matlab

icase = 1

Calls to tumor_1 = 53

t	V(t)(num)	V(t)(anal)	diff
0.00	1.0000	1.0000	0.000000
0.50	1.4770	1.4821	-0.003475
1.00	1.8742	1.8816	-0.003954
1.50	2.1674	2.1747	-0.003365
2.00	2.3680	2.3742	-0.002617
2.50	2.4980	2.5041	-0.002440
3.00	2.5799	2.5863	-0.002473
3.50	2.6297	2.6374	-0.002923
4.00	2.6598	2.6689	-0.003449
4.50	2.6779	2.6883	-0.003849
5.00	2.6890	2.7000	-0.004104
5.50	2.6955	2.7072	-0.004327
6.00	2.6996	2.7116	-0.004422
6.50	2.7020	2.7142	-0.004522
7.00	2.7036	2.7158	-0.004513
7.50	2.7046	2.7168	-0.004486
8.00	2.7053	2.7174	-0.004475
8.50	2.7057	2.7177	-0.004462
9.00	2.7059	2.7179	-0.004450
9.50	2.7061	2.7181	-0.004437
10.00	2.7061	2.7182	-0.004441

icase = 2

Calls to tumor_1 = 51

t	V(t)(num)	V(t)(anal)	diff
---	-----------	------------	------

0.00	1.0000	1.0000	0.000000
0.50	1.3401	1.3433	-0.002388
1.00	1.6027	1.6066	-0.002410

.
.
.

Solution for t = 1.50 to 8.50 removed

.
.
.

9.00	2.1192	2.1168	0.001129
9.50	2.1193	2.1169	0.001157
10.00	2.1194	2.1169	0.001172

icase = 3

Calls to tumor_1 = 59

t	V(t)(num)	V(t)(anal)	diff
0.00	1.0000	1.0000	0.000000
0.50	1.6328	1.6353	-0.001522
1.00	2.1922	2.2037	-0.005250

.
.
.

Solution for t = 1.50 to 8.50 removed

.
.
.

9.00	3.4788	3.4898	-0.003166
9.50	3.4790	3.4900	-0.003173
10.00	3.4791	3.4901	-0.003168

Numerical Solution - R

icase = 1

ncall = 100

t	V(t)(num)	V(t)(anal)	diff
0.00	1.0000	1.0000	0.000000
0.50	1.4821	1.4821	0.000001
1.00	1.8816	1.8816	-0.000001
1.50	2.1747	2.1747	-0.000000
2.00	2.3742	2.3742	0.000002
2.50	2.5041	2.5041	0.000001
3.00	2.5863	2.5863	0.000001
3.50	2.6374	2.6374	0.000001
4.00	2.6690	2.6689	0.000001
4.50	2.6883	2.6883	0.000000
5.00	2.7000	2.7000	-0.000001
5.50	2.7072	2.7072	-0.000001
6.00	2.7115	2.7116	-0.000002
6.50	2.7142	2.7142	-0.000002
7.00	2.7158	2.7158	-0.000002
7.50	2.7168	2.7168	-0.000002
8.00	2.7174	2.7174	-0.000003
8.50	2.7177	2.7177	-0.000003
9.00	2.7179	2.7179	-0.000003
9.50	2.7181	2.7181	-0.000003
10.00	2.7182	2.7182	-0.000003

icase = 2

ncall = 88

t	V(t)(num)	V(t)(anal)	diff
---	-----------	------------	------

0.00	1.0000	1.0000	0.000000
0.50	1.3433	1.3433	0.000000
1.00	1.6065	1.6066	-0.000002

.
.
.

Solution for t = 1.50 to 8.50 removed

.
.
.

9.00	2.1168	2.1168	-0.000004
9.50	2.1169	2.1169	-0.000004
10.00	2.1169	2.1169	-0.000004

icase = 3

ncall = 102

t	V(t)(num)	V(t)(anal)	diff
0.00	1.0000	1.0000	0.000000
0.50	1.6353	1.6353	0.000002
1.00	2.2037	2.2037	0.000000

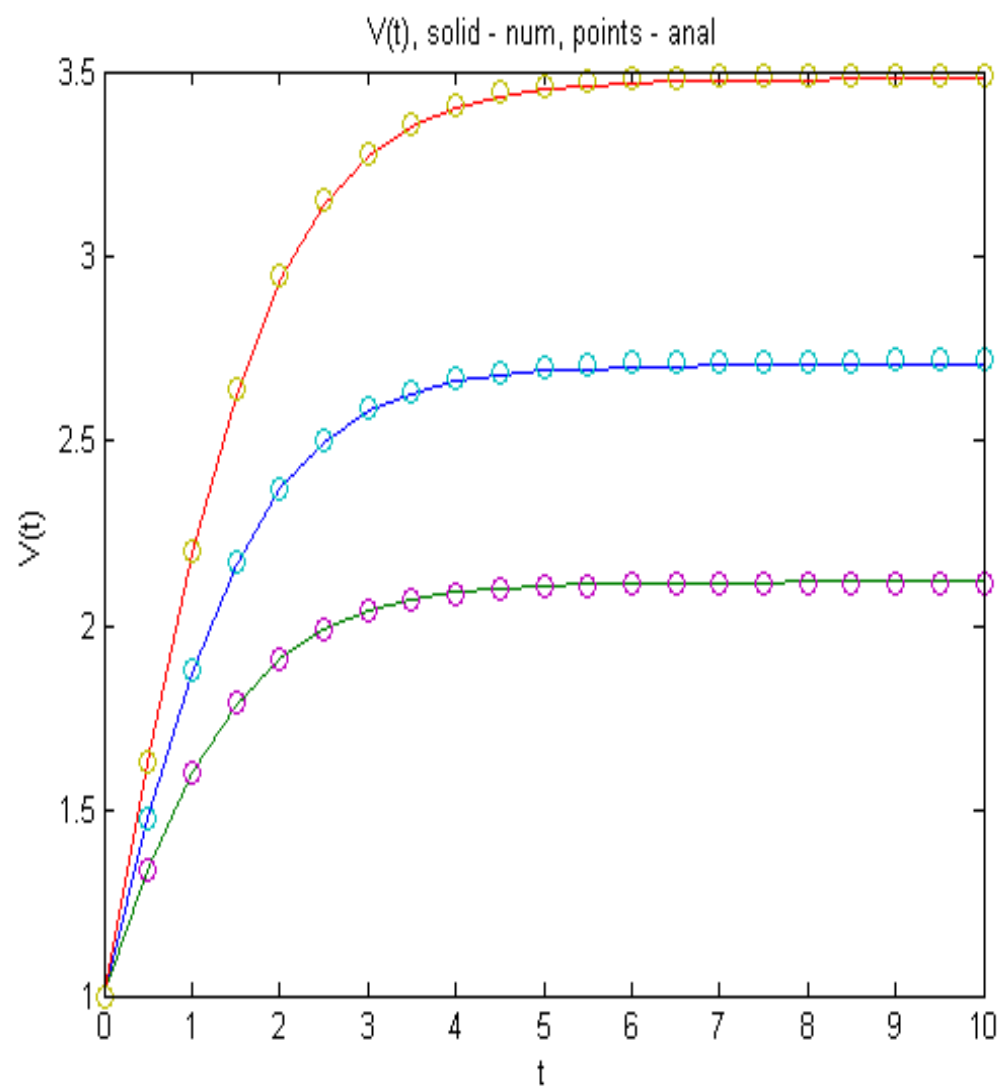
.
.
.

Solution for t = 1.50 to 8.50 removed

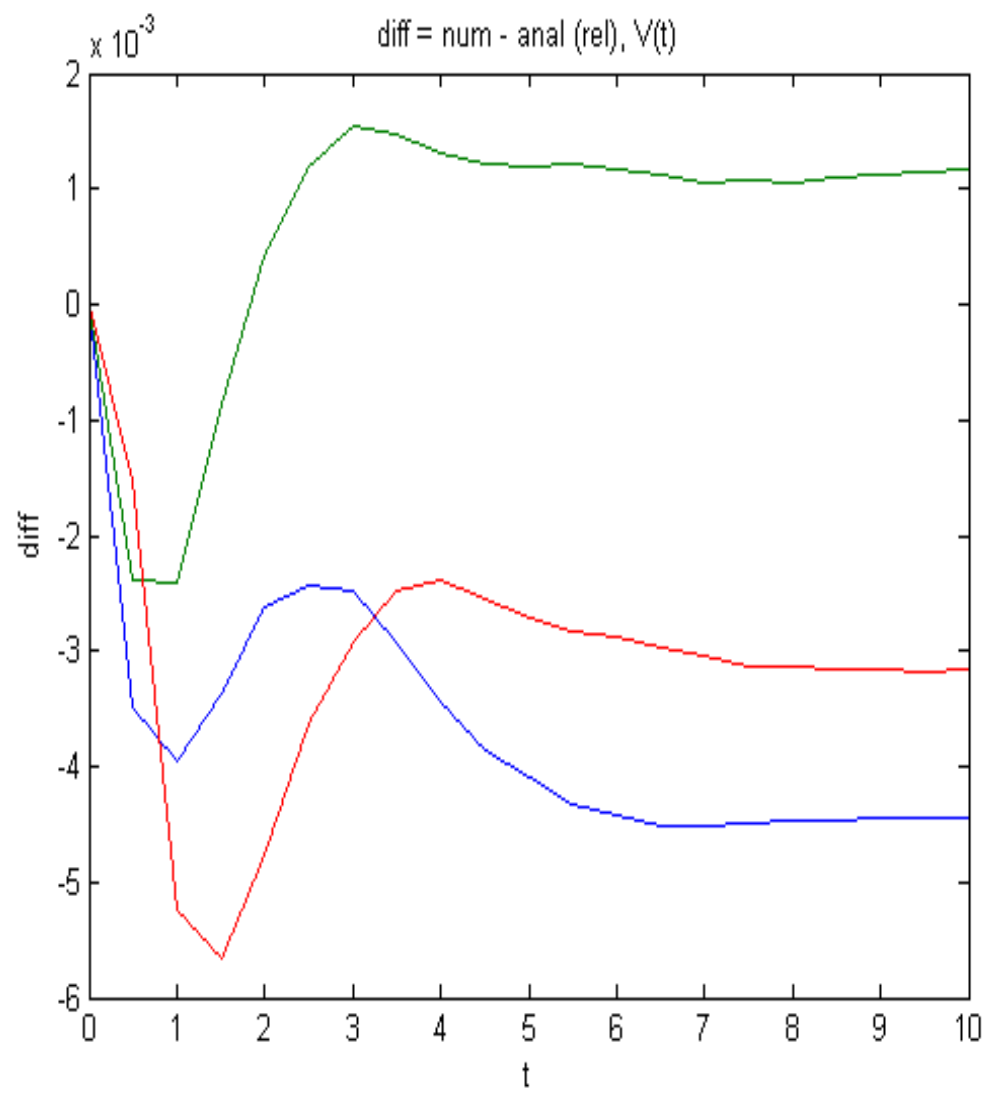
.
.
.

9.00	3.4898	3.4898	-0.000006
9.50	3.4900	3.4900	-0.000006
10.00	3.4901	3.4901	-0.000006

Matlab output

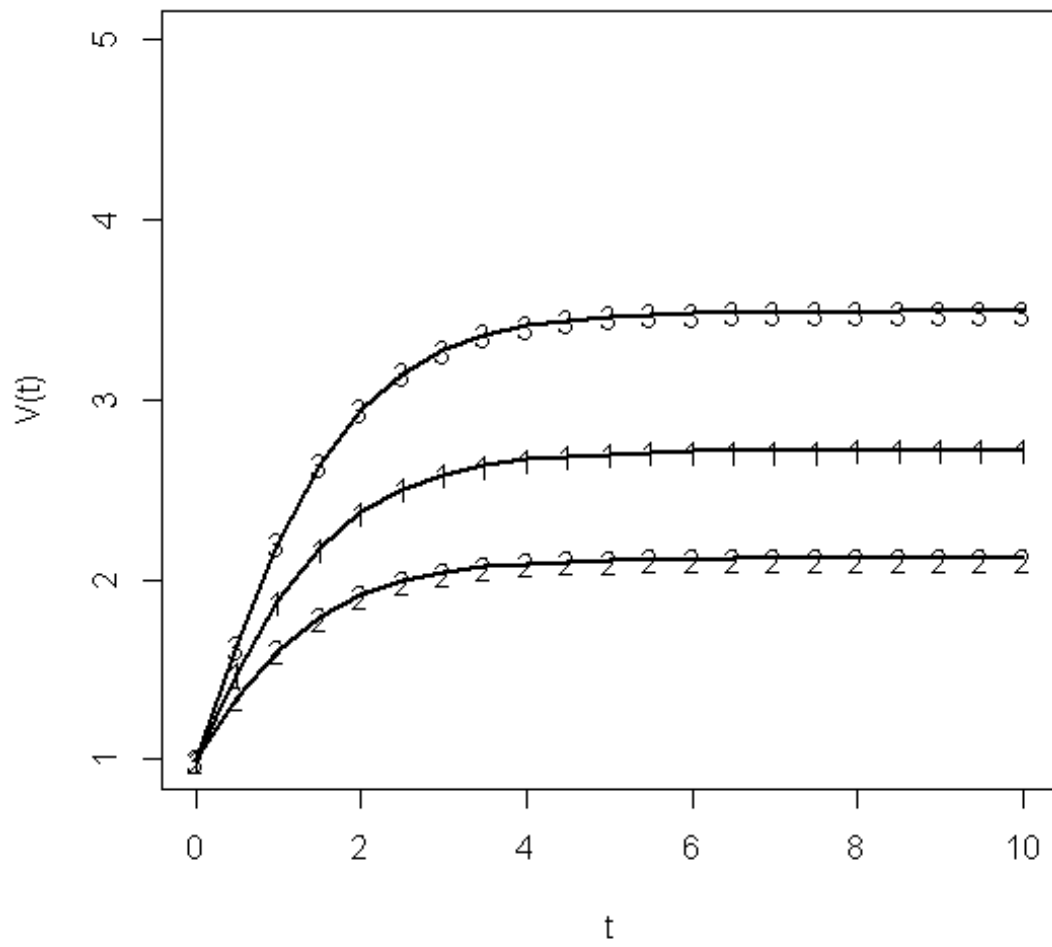


Matlab errors



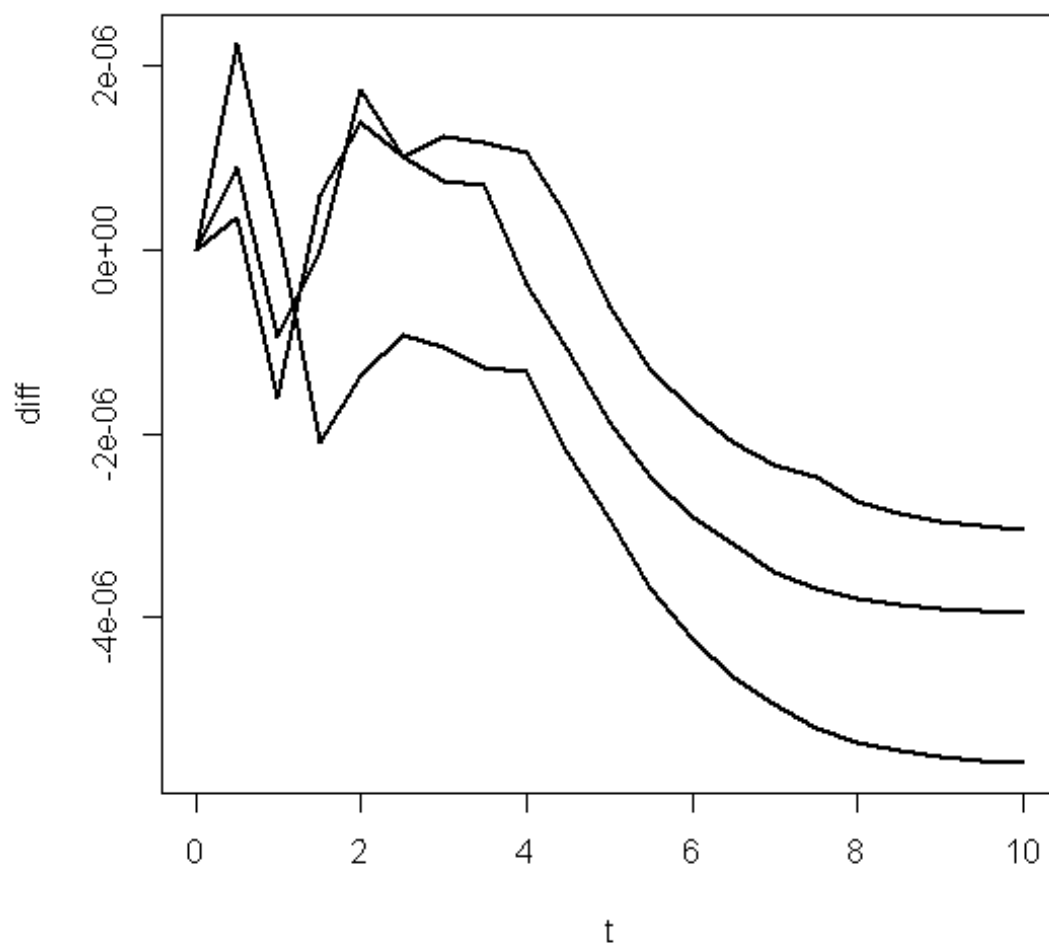
R output

V(t), solid - num, points - anal



R errors

diff = num - anal (rel)



PDEs

Fick's second law

$$\frac{\partial u}{\partial t} = \frac{\partial \left[D \frac{\partial u}{\partial x} \right]}{\partial x}$$

Variables parameters	Interpretation
-------------------------	----------------

u	dependent variable
x	boundary value independent variable
t	initial value independent variable
D	diffusivity (a parameter to be specified)

(1) $D = \text{constant}$ linear

(2) $D = u$ nonlinear

(3) $D = e^u$ nonlinear

PDEs

Fick's second law

$$\frac{\partial u}{\partial t} = \frac{\partial \left[D \frac{\partial u}{\partial x} \right]}{\partial x}$$

$$u(x, t = 0) = 0$$

$$u(x = 0, t) = 1; \quad \frac{\partial u(x = L, t)}{\partial x} = 0$$

BC 1: Dirichlet; BC 2: Neumann

Analytical solution, D=1

$$u(x, t) = 1 - 2 \sum_{i=1}^{\infty} \frac{1}{\lambda_i} e^{-(\lambda_i^2/L^2)t} \sin(\lambda_i x/L)$$

with the eigenvalues

$$\lambda_i = (i - 1/2)\pi, \quad i = 1, 2, \dots$$

ODE Routine - Matlab

```
function ut=pde_1(t,u)
%
% Parameters
global xl xu nx ncall ndss ncase
%
% BC at x = 0 (Dirichlet)
u(1)=1;
%
% ux
if (ndss== 2) ux=dss002(xl,xu,nx,u); % second order
elseif(ndss== 4) ux=dss004(xl,xu,nx,u); % fourth order
elseif(ndss== 6) ux=dss006(xl,xu,nx,u); % sixth order
elseif(ndss== 8) ux=dss008(xl,xu,nx,u); % eighth order
elseif(ndss==10) ux=dss010(xl,xu,nx,u); % tenth order
end
%
% BC at x = 1 (Neumann)
ux(nx)=0;
%
% Diffusivity
for i=1:nx
    if(ncase==1)D(i)=1          ;end
    if(ncase==2)D(i)=u(i)       ;end
    if(ncase==3)D(i)=exp(u(i));end
    ux(i)=D(i)*ux(i);
end
%
% uxx
if (ndss== 2) uxx=dss002(xl,xu,nx,ux); % second order
elseif(ndss== 4) uxx=dss004(xl,xu,nx,ux); % fourth order
elseif(ndss== 6) uxx=dss006(xl,xu,nx,ux); % sixth order
elseif(ndss== 8) uxx=dss008(xl,xu,nx,ux); % eighth order
```

```

elseif(ndss==10) uxx=dss010(xl,xu,nx,ux); % tenth order
end
%
% PDE
ut=uxx';
ut(1)=0;
%
% Increment calls to pde_1
ncall=ncall+1;

```

ODE Routine - R

```
pde_1=function(t,u,parms){
#
# BC at x = 0 (Dirichlet)
  u[1]=1;
#
# ux
  if      (ndss== 2){ux=dss002(xl,xu,nx,u); # second order
}else if(ndss== 4){ux=dss004(xl,xu,nx,u); # fourth order
}else if(ndss== 6){ux=dss006(xl,xu,nx,u); # sixth order
}else if(ndss== 8){ux=dss008(xl,xu,nx,u); # eighth order
}else if(ndss==10){ux=dss010(xl,xu,nx,u); # tenth order
}
#
# BC at x = 1 (Neumann)
  ux[nx]=0;
#
# Diffusivity
  D=rep(0,nx);
  for(i in 1:nx){
    if(ncase==1){D[i]=1      ;}
    if(ncase==2){D[i]=u[i]   ;}
    if(ncase==3){D[i]=exp(u[i]);}
    ux[i]=D[i]*ux[i];
  }
#
# uxx
  if      (ndss== 2){uxx=dss002(xl,xu,nx,ux); # second order
}else if(ndss== 4){uxx=dss004(xl,xu,nx,ux); # fourth order
}else if(ndss== 6){uxx=dss006(xl,xu,nx,ux); # sixth order
}else if(ndss== 8){uxx=dss008(xl,xu,nx,ux); # eighth order
}else if(ndss==10){uxx=dss010(xl,xu,nx,ux); # tenth order
}
```



```

#
# PDE
  ut=rep(0,nx);
  ut=uxx;
  ut[1]=0;
#
# Increment calls to pde_1
  ncall <- ncall+1;
#
# Return derivative vector
  return(list(c(ut)));
}

```

Main Program - Matlab

```
%
% Clear previous files
clear all
clc
%
% Parameters shared with the ODE routine
global xl xu nx ncall ndss ncase
%
% Grid in x, initial condition
xl=0;xu=1;nx=26;
for ix=1:nx
    xg(ix)=(xu-xl)*(ix-1)/(nx-1);
    u0(ix)=0;
end
%
% Independent variable for ODEs
t0=0.0;tf=1.5;nout=11;
tout=linspace(t0,tf,nout);
ncall=0;
%
% ODE integration
ncase=1;ndss=4;
[t,u]=ode15s(@pde_1,tout,u0);
%
% Display selected output
for it=1:nout
    if(it==1);u(it,1)=0;end
    if(it> 1);u(it,1)=1;end
    fprintf('\n      t      x      u(x,t)\n');
    for ix=1:5:nx
        fprintf('%7.2f%8.2f%12.5f\n',t(it),xg(ix),u(it,ix));
    end
end
end
```

```
fprintf('\n ncall = %4d\n',ncall);  
%  
% Plot numerical solution  
figure(1);  
plot(xg,u);  
xlabel('x');ylabel('u(x,t)');  
title('u(x,t) vs x, t=0,0.15,...,1.5');
```

Main Program - R

```
#
# Remove previous workspaces
rm(list=ls(all= TRUE));
#
# Access ODE integrator
library("deSolve");
#
# Access functions
setwd("c:/R/pde_intro");
source("pde_1.R");
source("dss004.R");
#
# Grid in x, initial condition
xl=0;xu=1;nx=26;
xg=rep(0,nx);u0=rep(0,nx);
for(ix in 1:nx){
  xg[ix]=(xu-xl)*(ix-1)/(nx-1);
  u0[ix]=0;
}
#
# Independent variable for ODEs
t0=0;tf=1.5;nout=11;
tout=seq(from=0,to=tf,by=(tf-t0)/(nout-1));
ncall=0;
#
# ODE integration
ncase=1;ndss=4;
out=lsodes(y=u0,times=tout,func=pde_1,parms=NULL)
#
# Display selected output
for(it in 1:nout){
  if(it==1){out[it,2]=0;}
  if(it> 1){out[it,2]=1;}
```

```

cat(sprintf("\n      t      x      u(x,t)\n"));
for(ix in 1:nx){
  if((ix-1)*(ix- 6)*(ix-11)*(ix-16)
      *(ix-21)*(ix-26)==0){
    cat(sprintf("%7.2f%8.2f%12.5f\n",
      out[it,1],xg[ix],out[it,ix+1]));
  }
}
}
cat(sprintf("\n ncall = %4d\n",ncall));
#
# Plot numerical solution
par(mfrow=c(1,1));
matplot(x=xg,y=t(out[,-1]),type="l",xlab="x",
  ylab="u(x,t) vs x, t=0,0.15,...,1.5",xlim=c(xl,xu),
  main="u(x,t) vs x, t=0,0.15,...,1.5;",lty=1,lwd=2);

```

Matlab output, $D = 1$ (ncase = 1)

t	x	u(x,t)
0.00	0.00	0.000000
0.00	0.20	0.000000
0.00	0.40	0.000000
0.00	0.60	0.000000
0.00	0.80	0.000000
0.00	1.00	0.000000

(output for t = 0.15
to 0.30 removed)

t	x	u(x,t)
0.45	0.00	1.000000
0.45	0.20	0.87040
0.45	0.40	0.75347
0.45	0.60	0.66065
0.45	0.80	0.60105
0.45	1.00	0.58052

(output for t = 0.60
to 1.35 removed)

t	x	u(x,t)
1.50	0.00	1.000000
1.50	0.20	0.99033
1.50	0.40	0.98160
1.50	0.60	0.97467
1.50	0.80	0.97022
1.50	1.00	0.96869

ncall = 154

R output, D = 1 (ncase = 1)

t	x	u(x,t)
0.00	0.00	0.000000
0.00	0.20	0.000000
0.00	0.40	0.000000
0.00	0.60	0.000000
0.00	0.80	0.000000
0.00	1.00	0.000000

.

(output for t = 0.15
to 0.30 removed)

.

t	x	u(x,t)
0.45	0.00	1.000000
0.45	0.20	0.87036
0.45	0.40	0.75342
0.45	0.60	0.66064
0.45	0.80	0.60107
0.45	1.00	0.58055

.

(output for t = 0.60
to 1.35 removed)

.

t	x	u(x,t)
1.50	0.00	1.000000
1.50	0.20	0.99028
1.50	0.40	0.98152
1.50	0.60	0.97456
1.50	0.80	0.97009
1.50	1.00	0.96855

ncall = 230

Comparison of Matlab, R, analytical solutions $D = 1$ (ncase = 1)

Matlab

t	x	u(x,t)
0.45	0.00	1.00000
0.45	0.20	0.87040
0.45	0.40	0.75347
0.45	0.60	0.66065
0.45	0.80	0.60105
0.45	1.00	0.58052

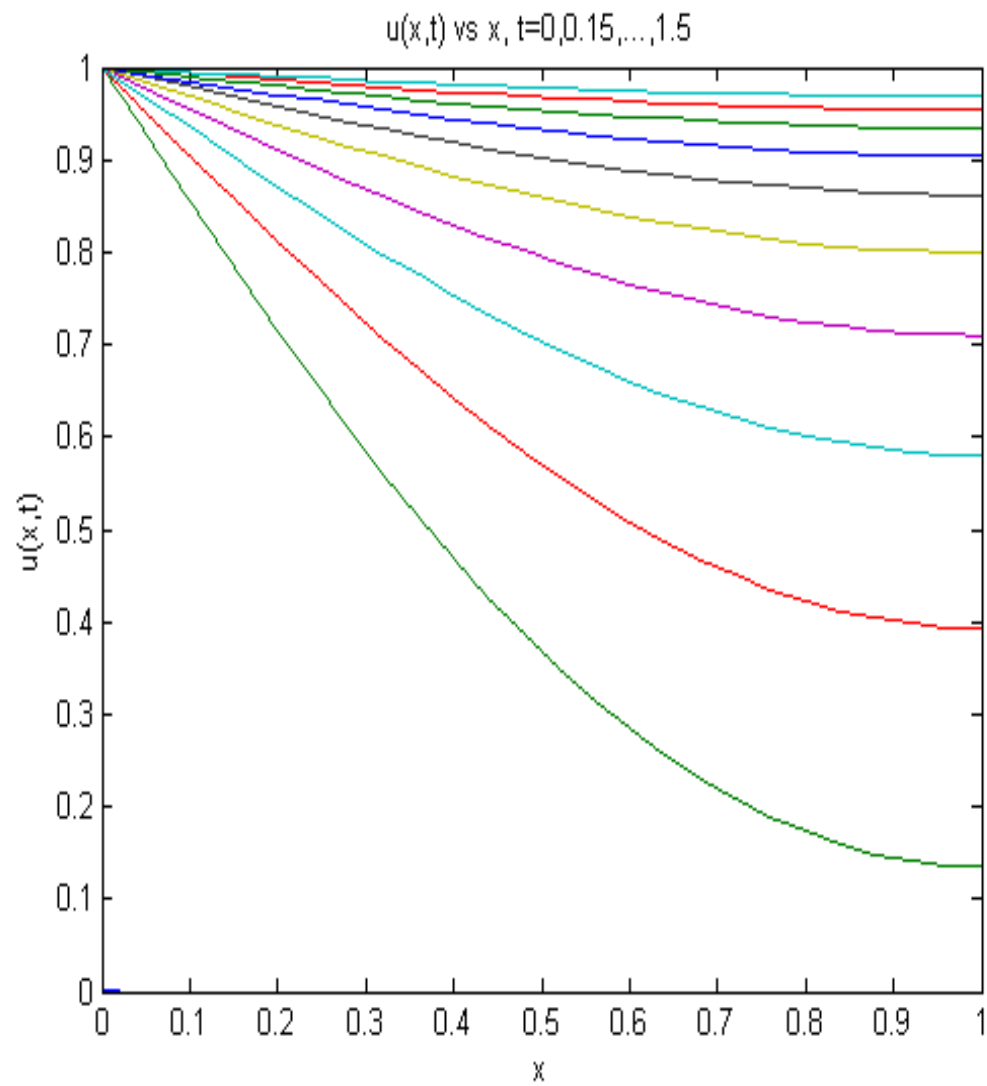
R

t	x	u(x,t)
0.45	0.00	1.00000
0.45	0.20	0.87036
0.45	0.40	0.75342
0.45	0.60	0.66064
0.45	0.80	0.60107
0.45	1.00	0.58055

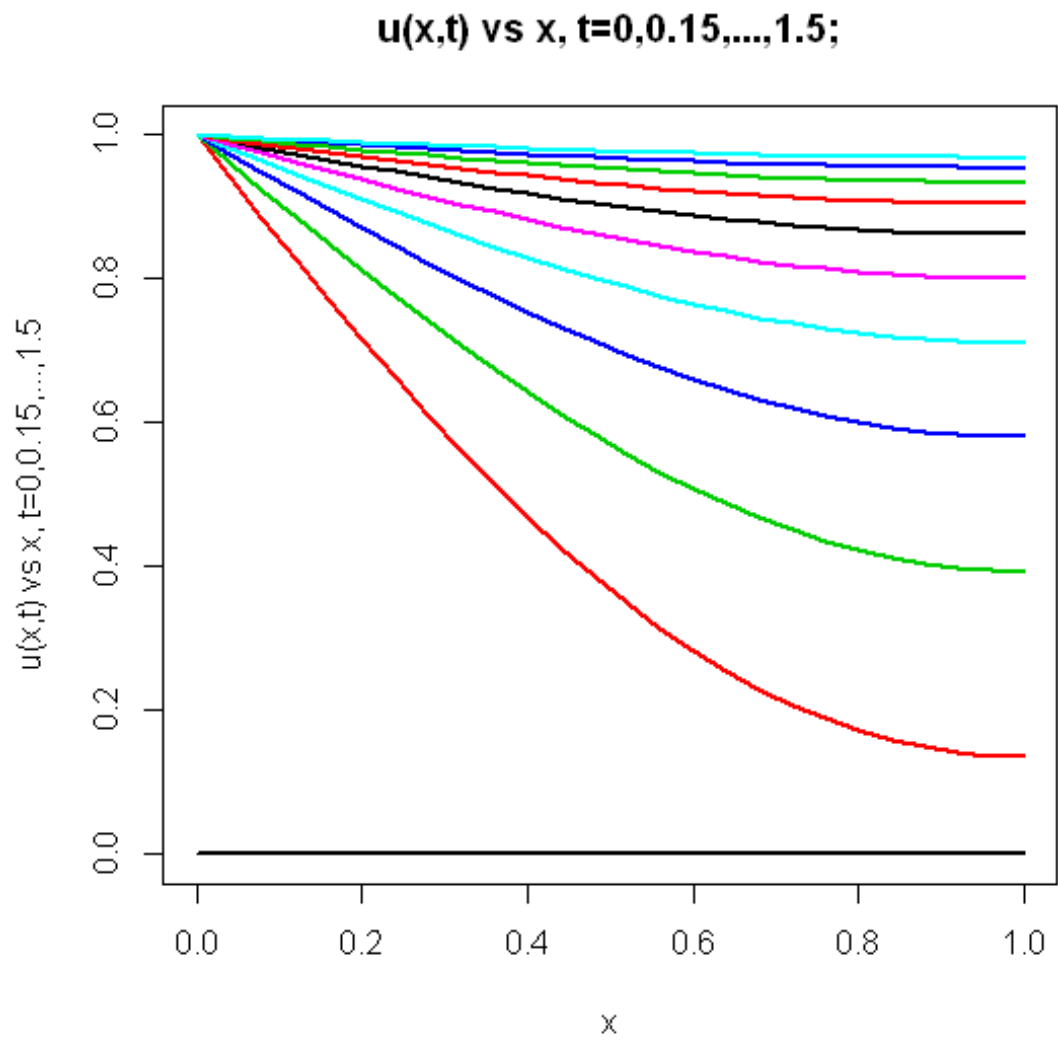
Analytical

t	x	u(x,t)
0.45	0.00	1.00000
0.45	0.20	0.87036
0.45	0.40	0.75342
0.45	0.60	0.66064
0.45	0.80	0.60107
0.45	1.00	0.58055

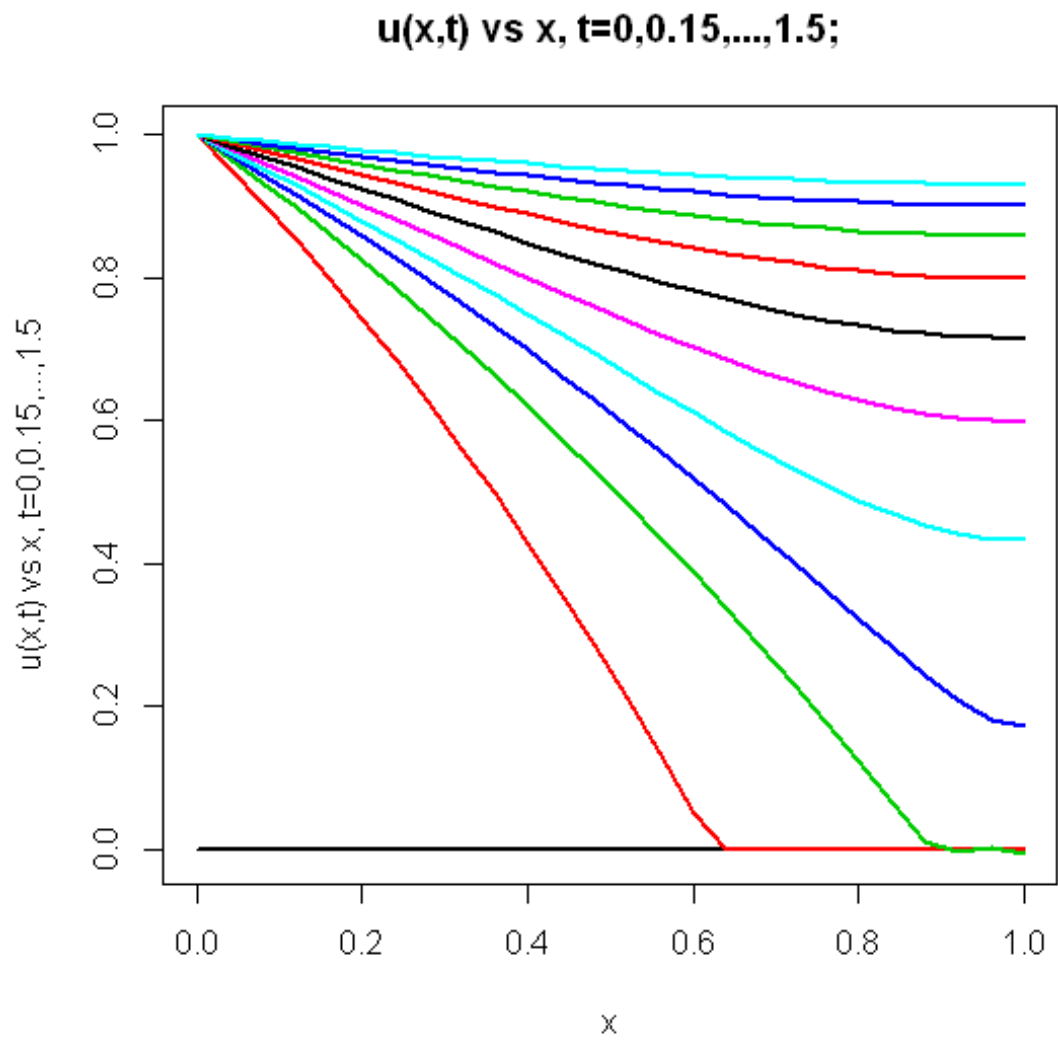
$u(x, t)$ vs x with t as a parameter (Matlab) for ncase=1



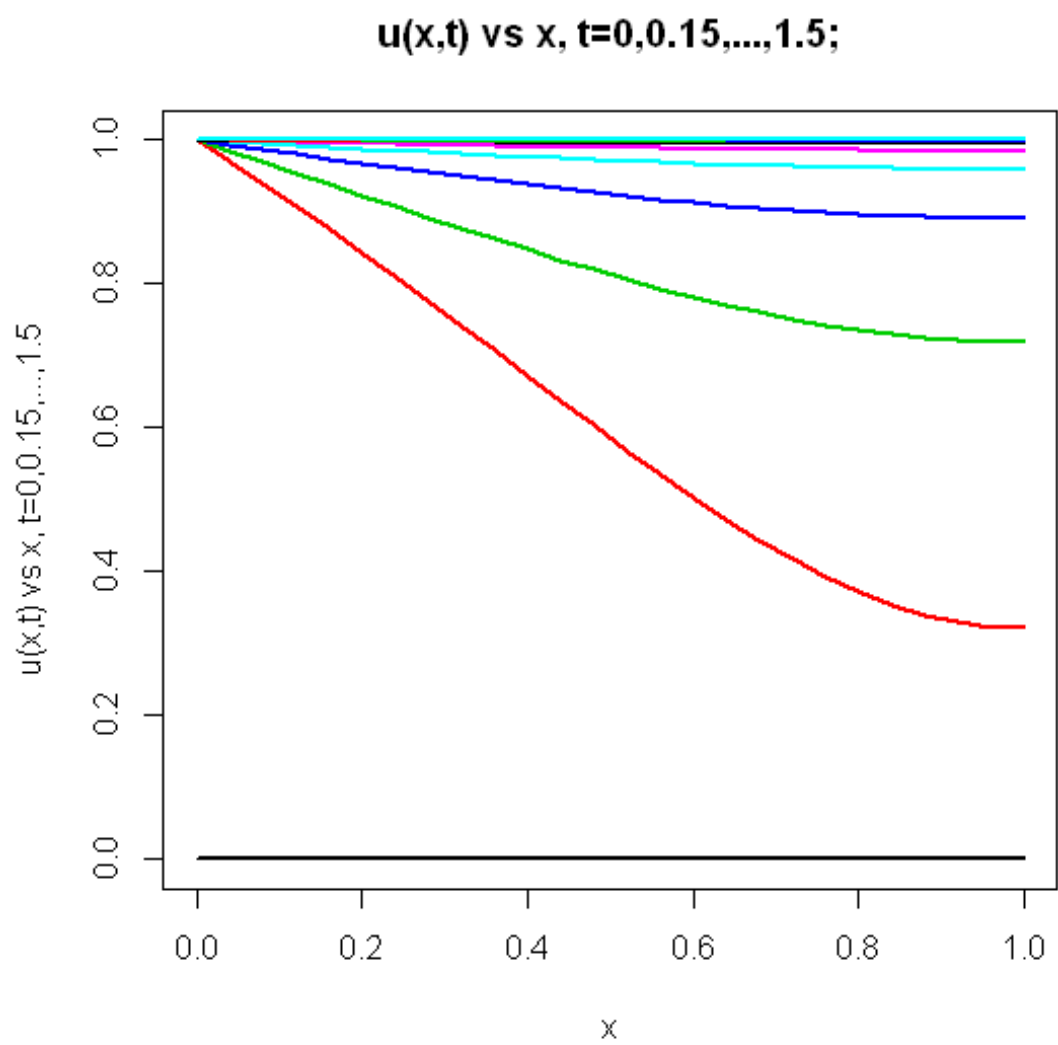
$u(x, t)$ vs x with t as a parameter (R) for ncase=1



$u(x, t)$ vs x with t as a parameter (R) for ncase=2

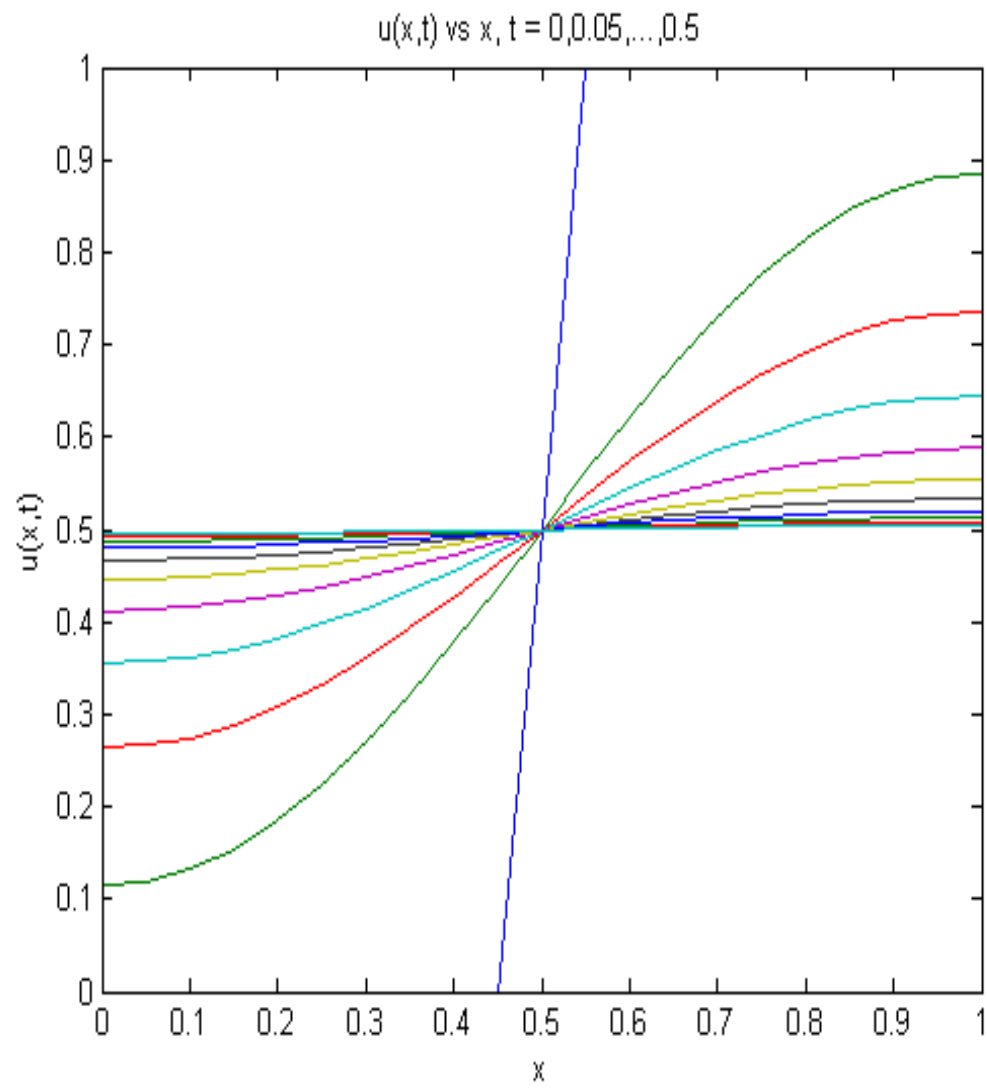


$u(x, t)$ vs x with t as a parameter (R) for ncase=3



$u(x, t)$ vs x with t as a parameter (Matlab) for ncase=1

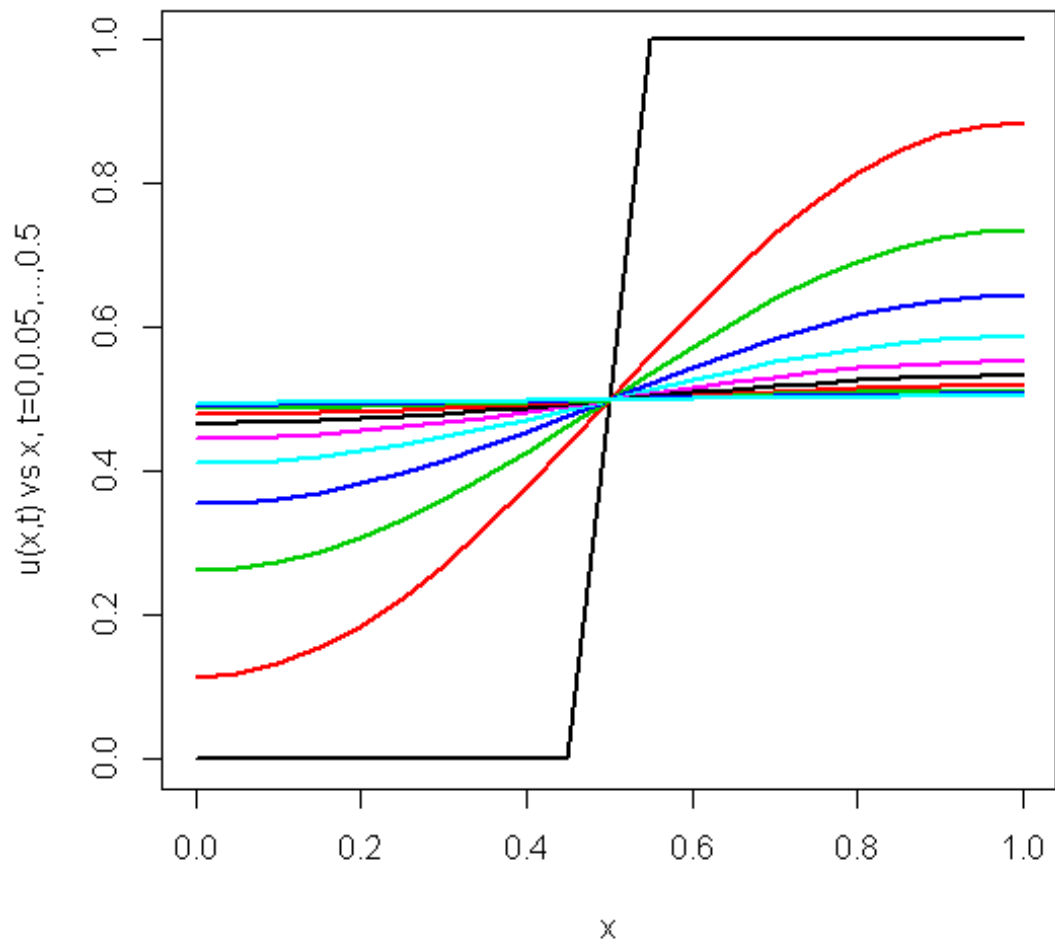
Discontinuous IC



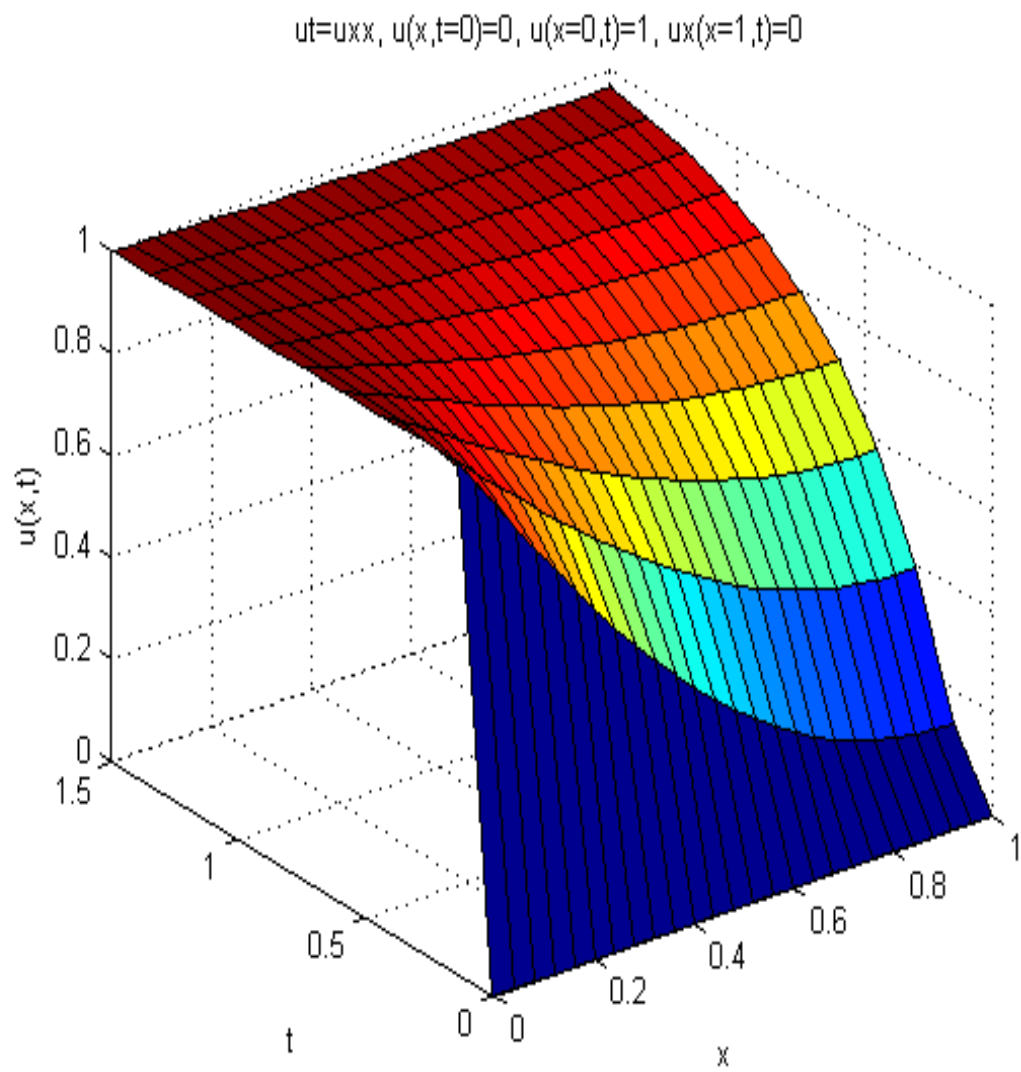
$u(x, t)$ vs x with t as a parameter (R) for ncase=1

Discontinuous IC

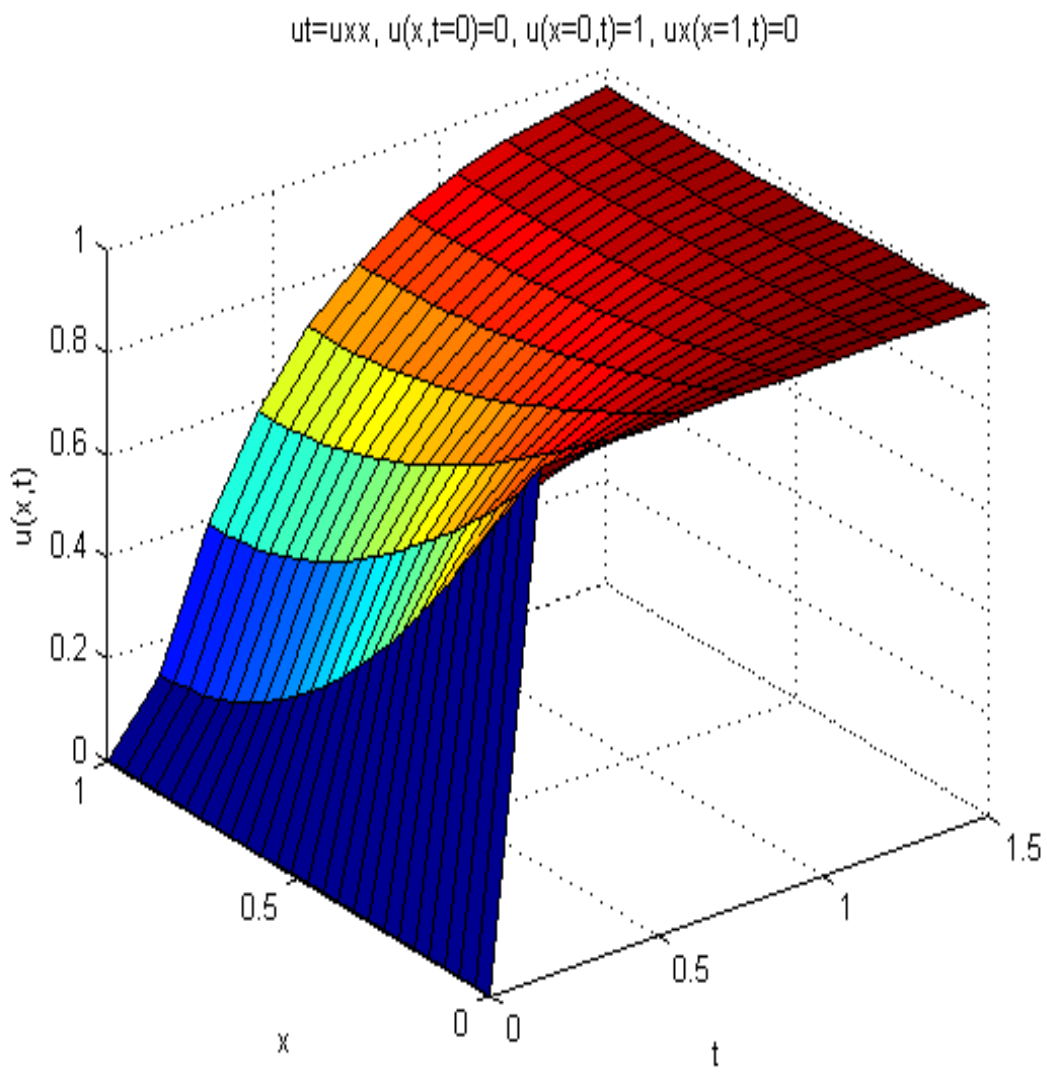
$u(x, t)$ vs x , $t=0, 0.05, \dots, 0.5$;



3D perspective of $u(x, t)$ vs x and t from surf (Matlab)

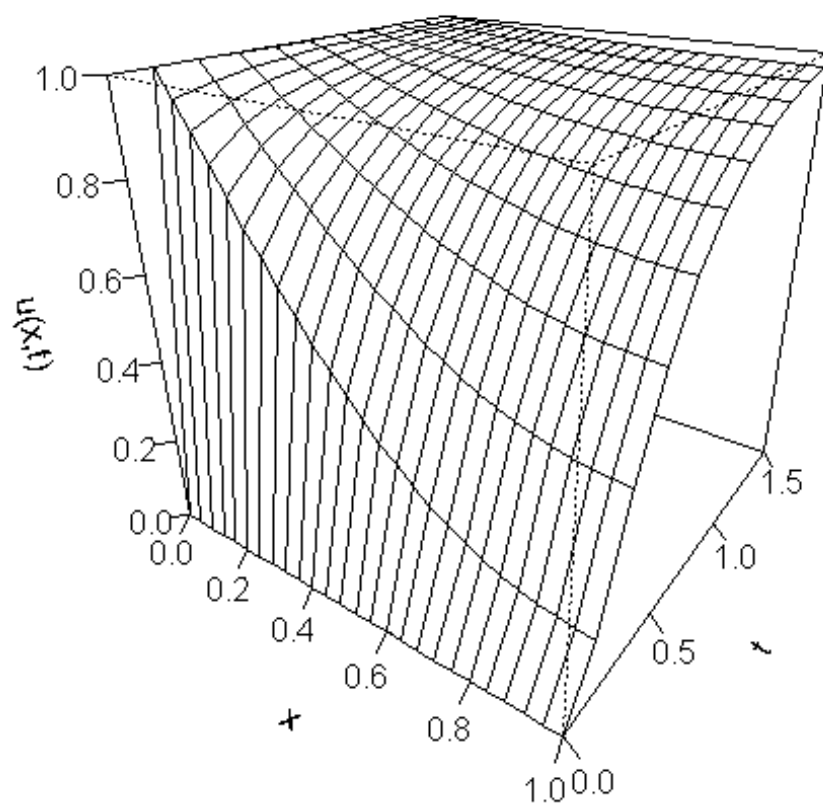


3D perspective of $u(x, t)$ vs x and t from surf (Matlab)



3D perspective of $u(x, t)$ vs x and t from persp (R), $\theta = 35$

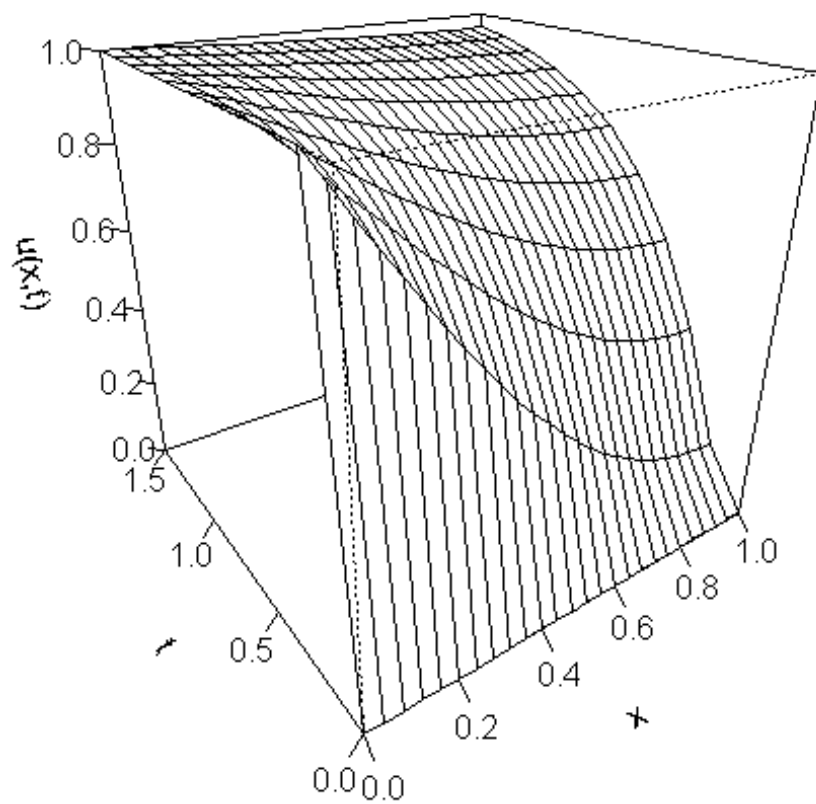
3D PDE plotting



$$u_t = u_{xx}, u(x, t=0) = 0, u(x=0, t) = 1, u_x(x=1, t) = 0$$

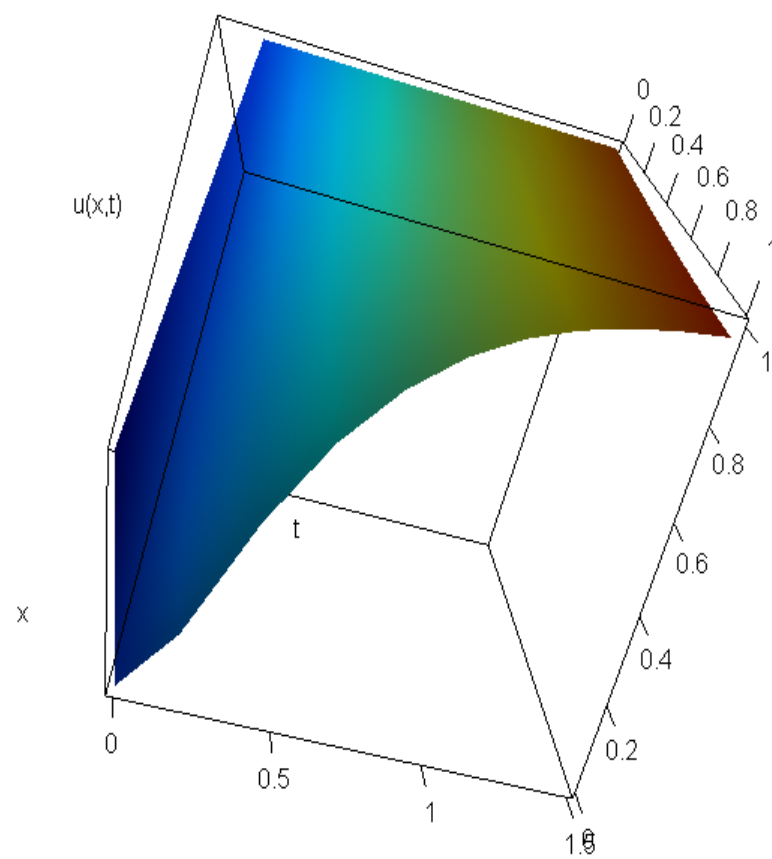
3D perspective of $u(x, t)$ vs x and t from persp (R), $\theta = -35$

3D PDE plotting



$$u_t = u_{xx}, u(x, t=0) = 0, u(x=0, t) = 1, u_x(x=1, t) = 0$$

3D perspective of $u(x, t)$ vs x and t from persp3d (R)



In summary, R is:

- A quality scientific computing system
 - An extensive library of utilities and applications (e.g., deSolve, persp3d, Rstudio)
 - A fully functional alternative to Matlab

In summary, R is:

- A quality scientific computing system
 - An extensive library of utilities and applications (e.g., deSolve, persp3d, Rstudio)
 - A fully functional alternative to Matlab
- Open source (no cost)

In summary, R is:

- A quality scientific computing system
 - An extensive library of utilities and applications (e.g., deSolve, persp3d, Rstudio)
 - A fully functional alternative to Matlab
- Open source (no cost)
- Available to anyone with an Internet connection

In summary, R is:

- A quality scientific computing system
 - An extensive library of utilities and applications (e.g., deSolve, persp3d, Rstudio)
 - A fully functional alternative to Matlab
- Open source (no cost)
- Available to anyone with an Internet connection
- Easily downloaded and used (e.g., by faculty and students); WES glad to help

R is available from the Internet in Linux, Mac, Windows formats: <http://cran.fhcrc.org/>. Also, the ODE/DAE library deSolve[1] can be downloaded for use with R.

The editor Rstudio is recommended for use in working with R and for facilitating graphical output: <http://rstudio.org/>. deSolve can be conveniently downloaded via Rstudio.

References

- [1] Soetaert, K., J. Cash and F. Mazzia (2012), *Solving Differential Equations in R*, Springer-Verlag, Heidelberg, Germany

Thank you.

Questions?