Distributed Optimization with Pairwise Constraints and its Application to Multi-robot Path Planning

Subhrajit Bhattacharya  Vijay Kumar  Maxim Likhachev

Problem definition:
(Goal directed navigation of $N$ heterogeneous robots)

$\{\pi_1^*, \ldots, \pi_N^*\} = \arg\min_{\pi_1 \ldots \pi_N} \sum_{j=1}^{N} c(\pi_j)$

s.t. $\Omega_{ij}(\pi_i, \pi_j) = 0$ (e.g., time-parametrized distance constraint)

Subproblem: $\pi_{i}^{\text{iter}} = \arg\min_{\pi_i} c(\pi_i) + \sum_{j=1}^{N} \sum_{j \neq i} w_{i,j}^{\text{iter}} \cdot \Omega(\pi_i, \pi_j)$

Six robots planning iteratively to satisfy rendezvous constraints in an empty environment:

Challenges — a very large constrained optimization problem

- Size of joint state space increases exponentially with $N$
- Coupling due to constraints
- Need of fast optimal plan as well as guarantees
- Cluttered, non-trivial environment

Approach — iterative planning in individual state-space, with guarantees

- Distributed planning
  a. Model constraints as penalty added to the objective — soft constraints.
  b. Iterate over the robots, gradually increasing the penalty weights
- Fast, Efficient; Provably Complete, Optimal.
- Discretization of environment into a graph
Results

An exact implementation

(a) Symmetric case: $\alpha = 0, \beta = 1$
(b) Symmetric case: $\alpha = 1, \beta = 0.2$
(c) An asymmetric case: $\alpha = 1, \beta = 0$

Planning with tasks and constraints

Heterogeneous agents performing complex tasks in 3D:

- Mobile ground patrol
- Messenger UAV
- UAV exploring Building-1
- UAV exploring Building-2