Data-Driven Topological Methods for Reasoning about Motion

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Why Topology?

Reasoning about Motion

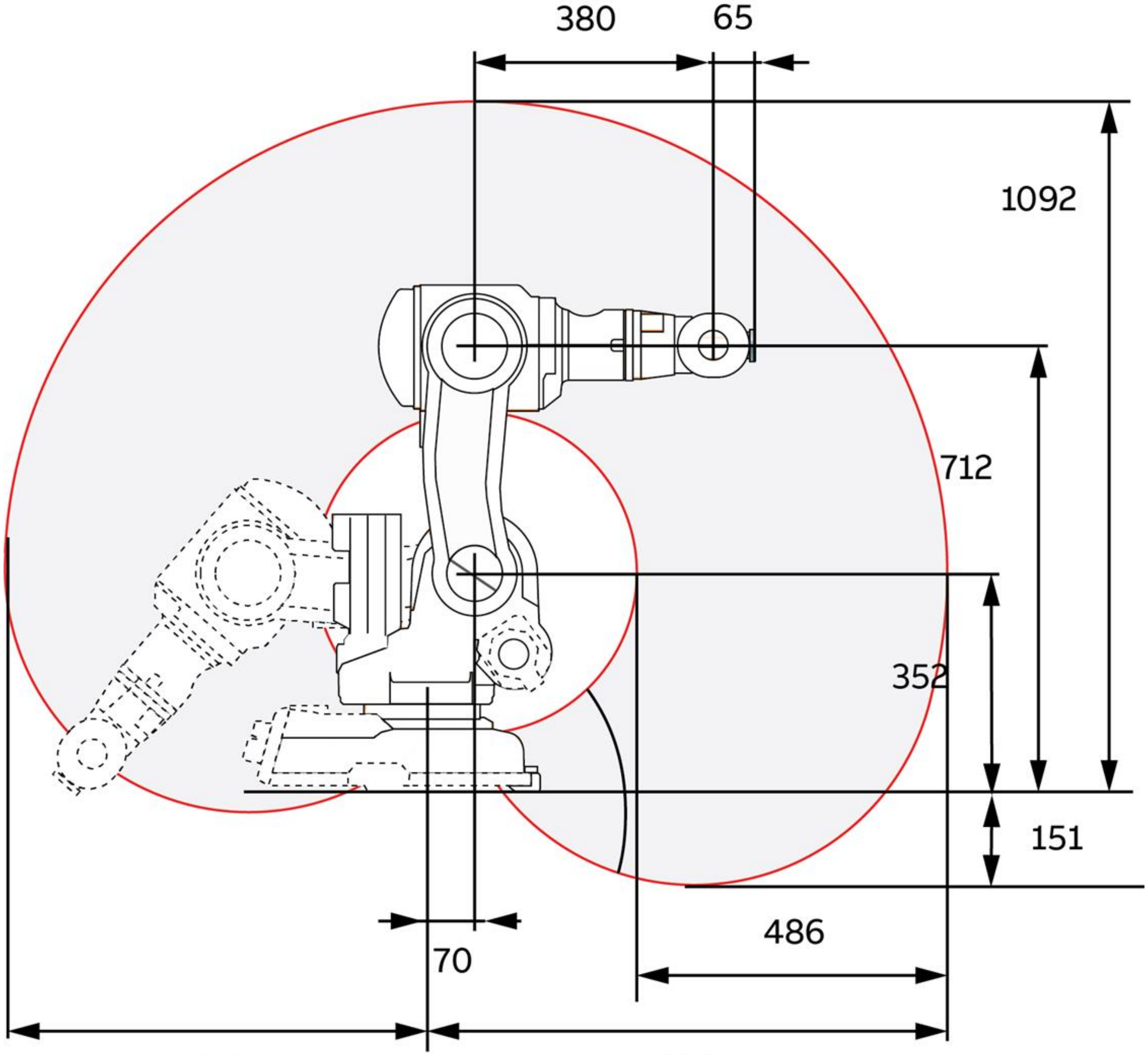
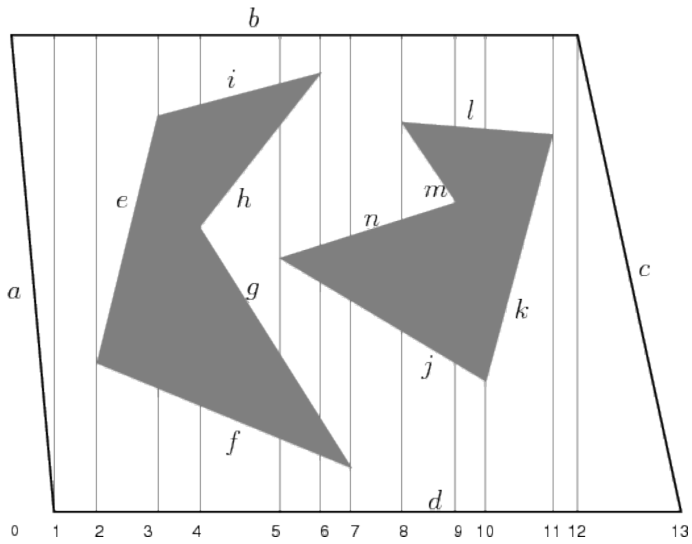


Figure: ABB

Robot Motion Planning

Algebraic Approach

- Schwarz & Sharir 1983
- Canny 1988
- Latombe 1991



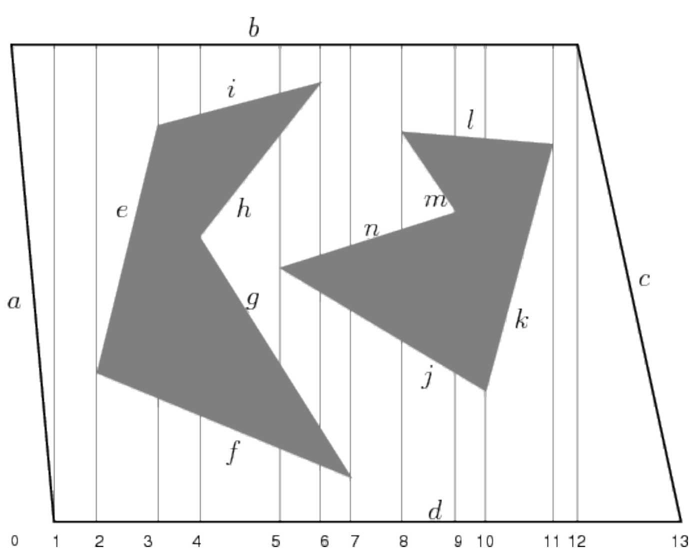
Robot Motion Planning

Algebraic Approach

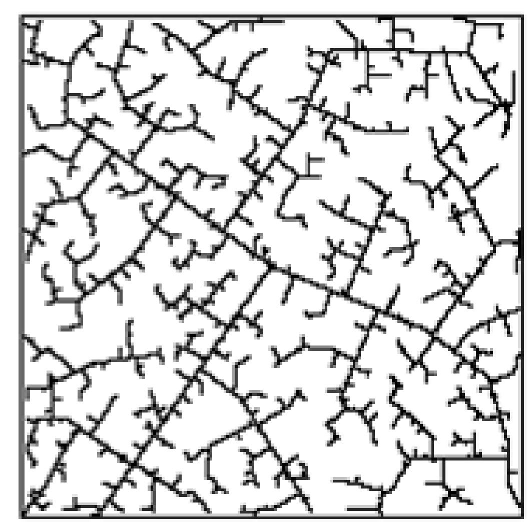
- Schwarz & Sharir 1983
- Canny 1988
- Latombe 1991

Sampling-based Revolution

- LaValle 2001
- Kavraki 1996
- Hsu 1997

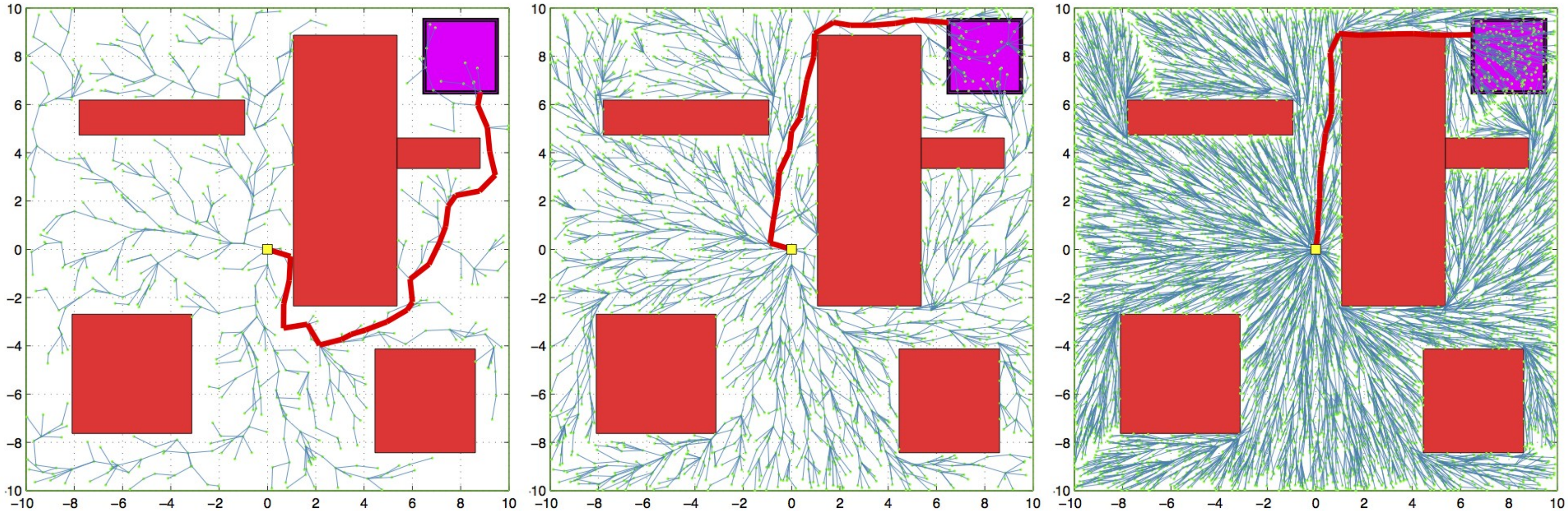


45 iterations



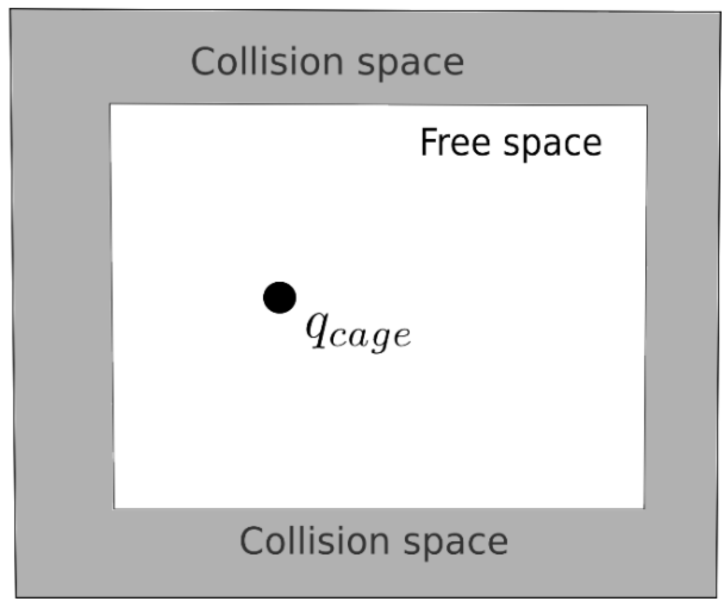
390 iterations

Karaman & Frazzoli 2011 - RRT*

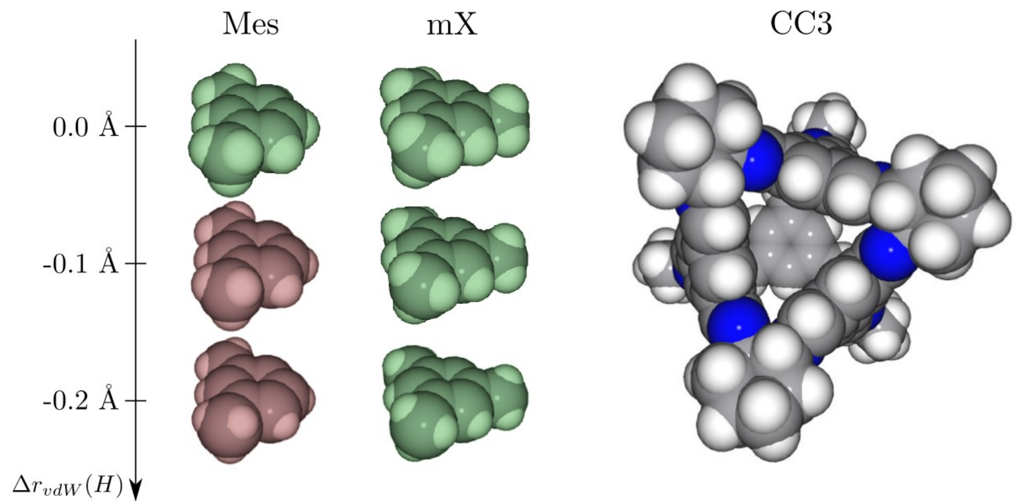


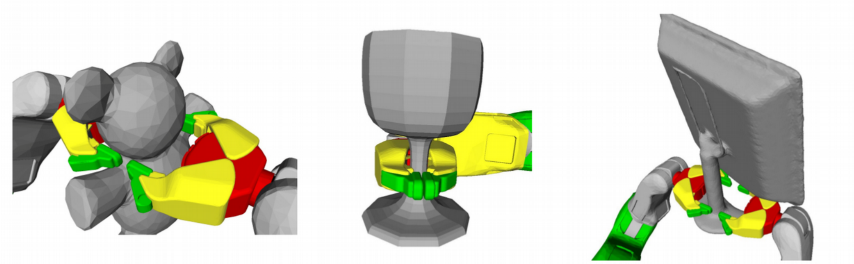
(d) RRT* in iteration 1,000 (e) RRT* in iteration 3,000 (f) RRT* in iteration 10,000

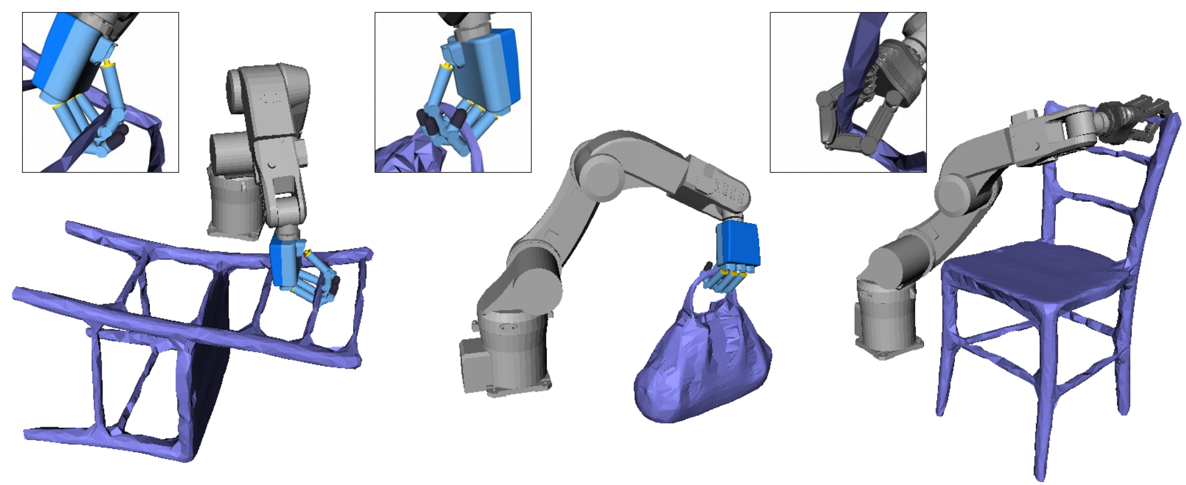
Path Non-Existence



Caging

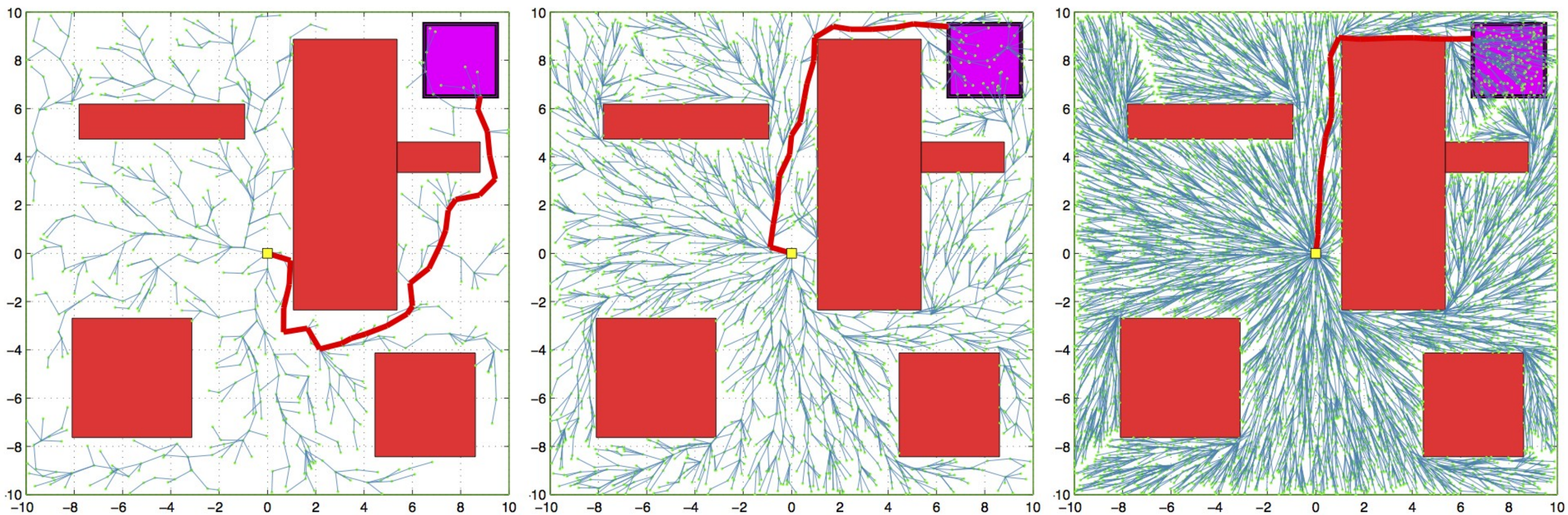






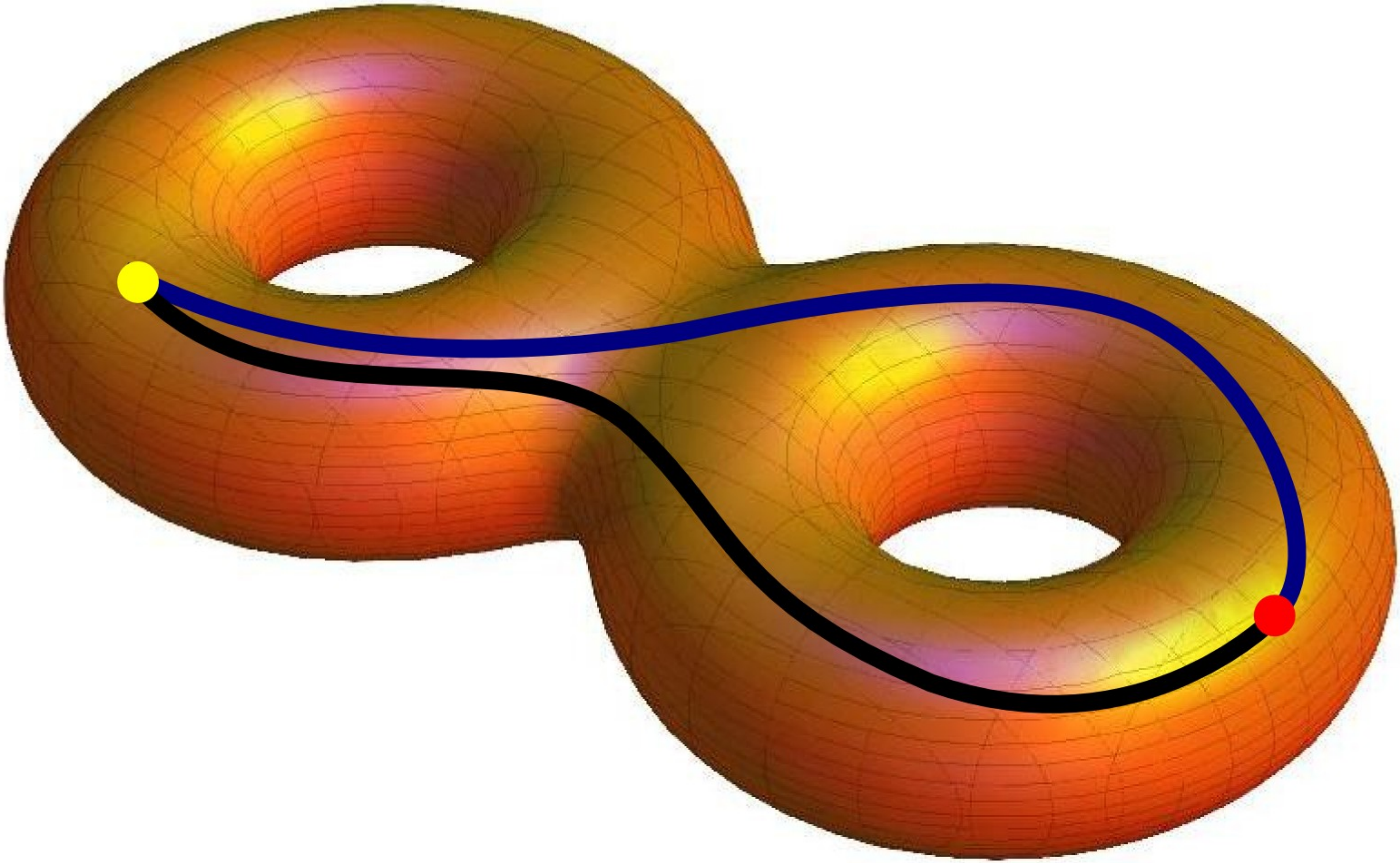
Top left, bottom left: A. Varava, D. Kragic, F. T. Pokorny, IJRR 2014
Top right: A. Varava, F. T. Pokorny, D. Devaurs, L. Kavraki, D. Kragic, Unpublished
Bottom right: J. A. Stork, F. T. Pokorny, D. Kragic, IROS 2013

Karaman & Frazzoli 2011 - RRT*

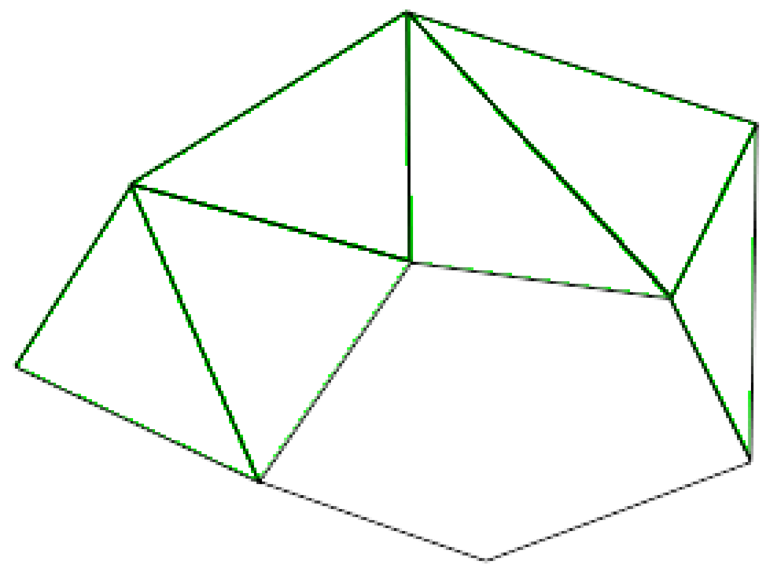


(d) RRT* in iteration 1,000 (e) RRT* in iteration 3,000 (f) RRT* in iteration 10,000

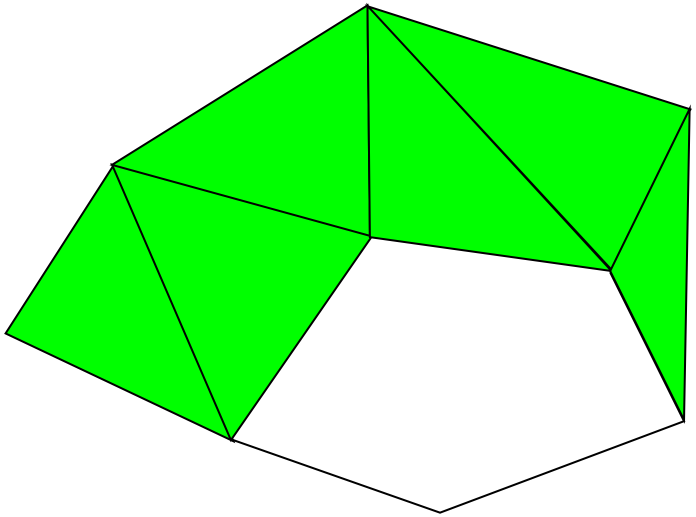
Homotopy Equivalence



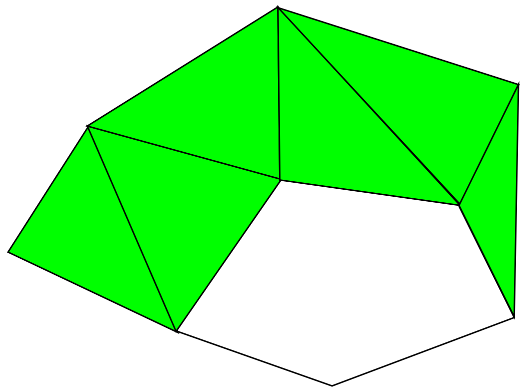
Graphs



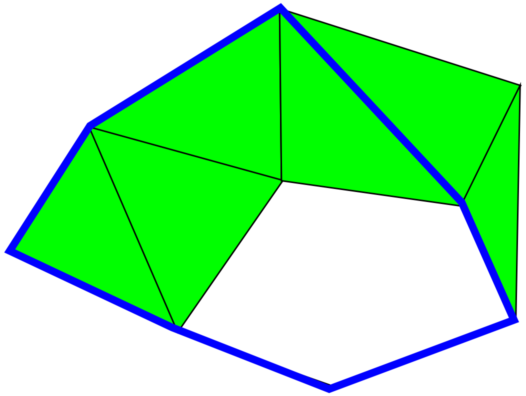
Simplicial Complexes



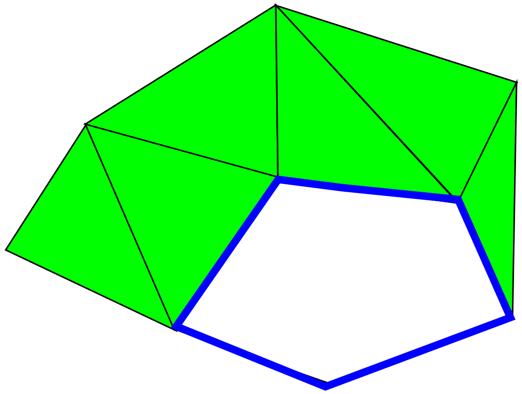
Simplicial Complexes



complex

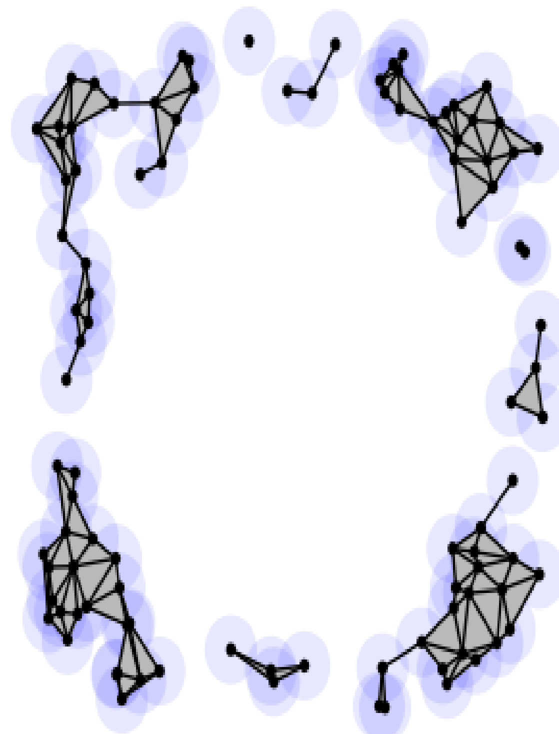


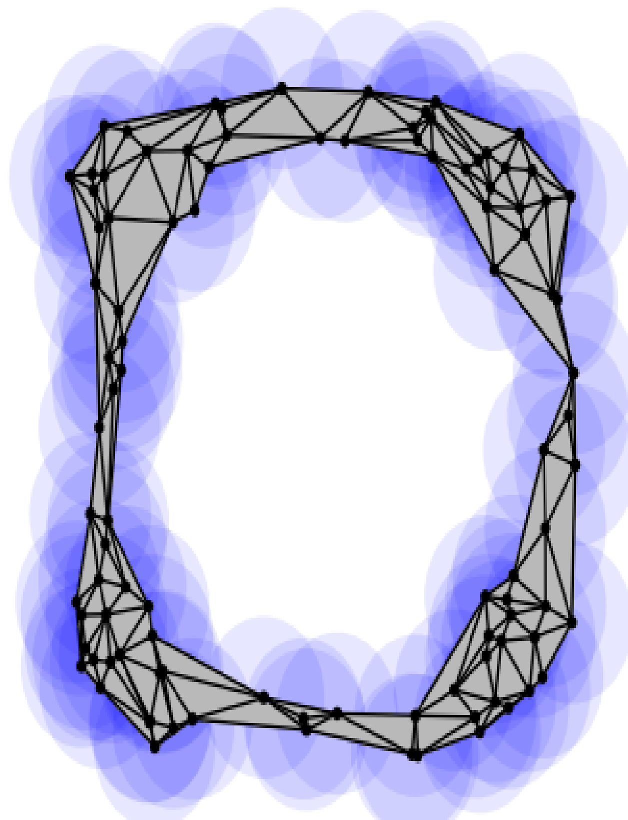
a 1-cycle

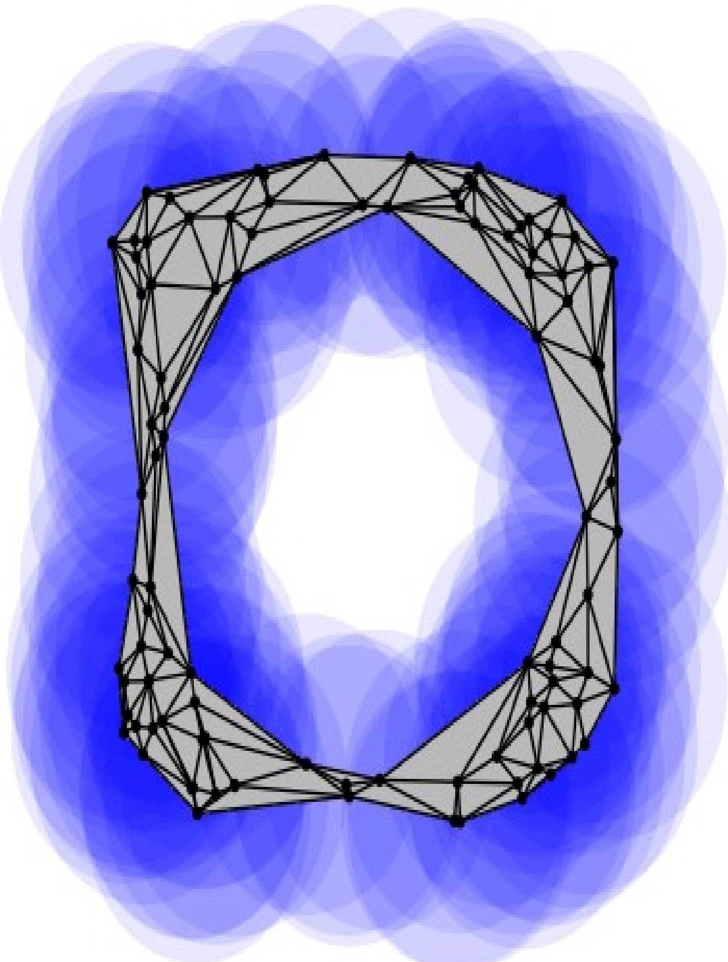


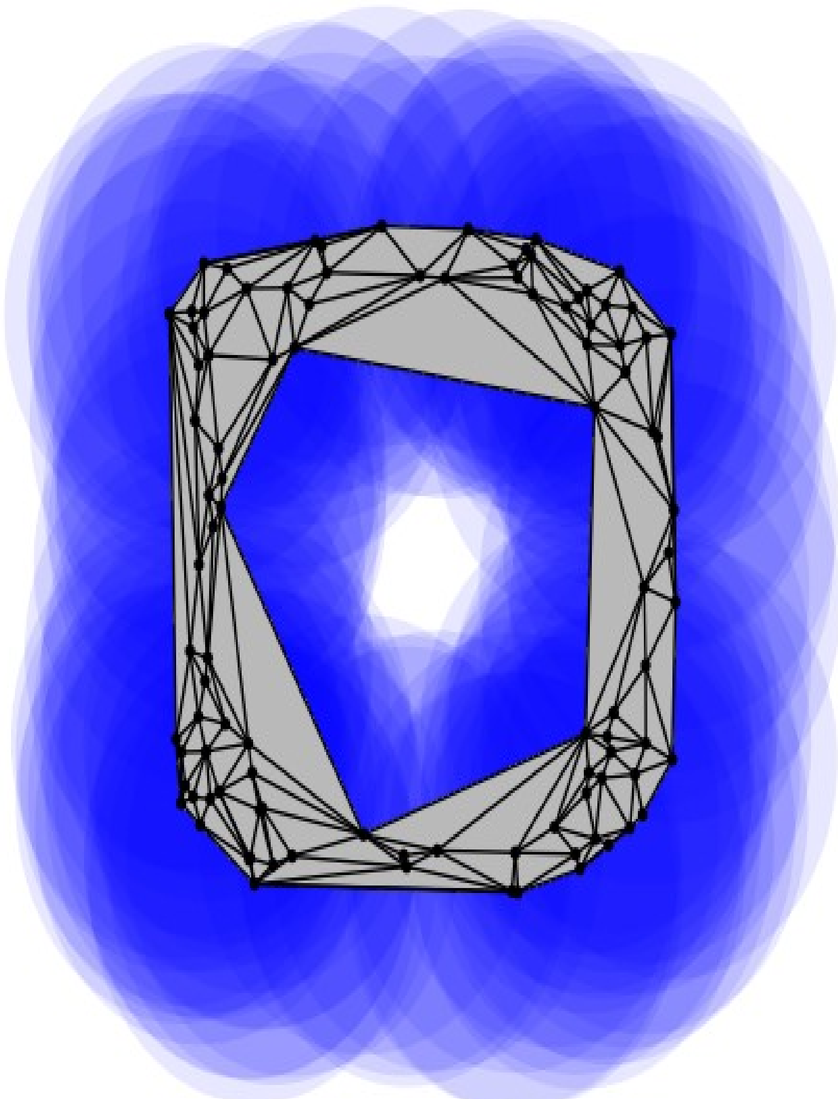
another 1-cycle

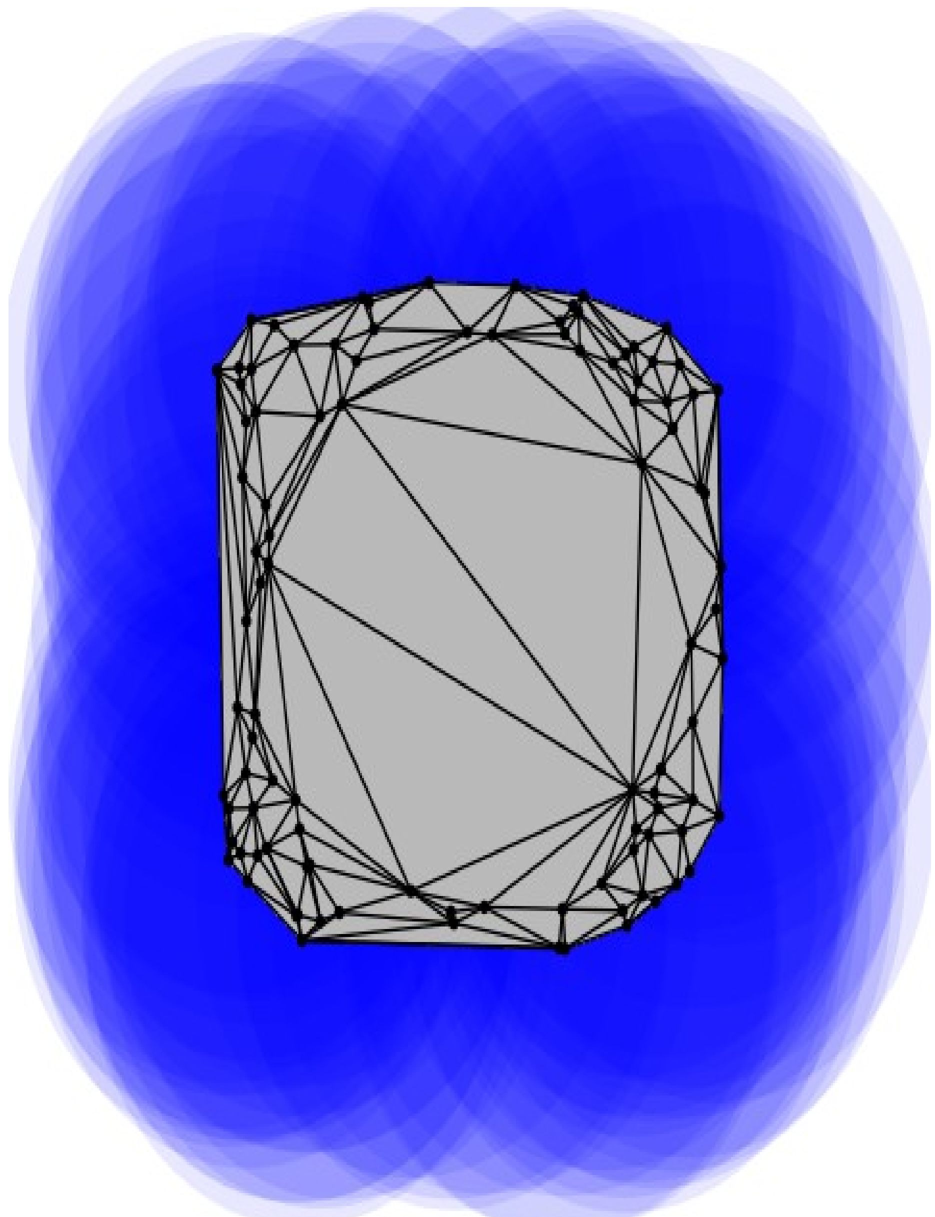
$$X_r = \bigcup_{x \in X} \mathbb{B}_r(x)$$



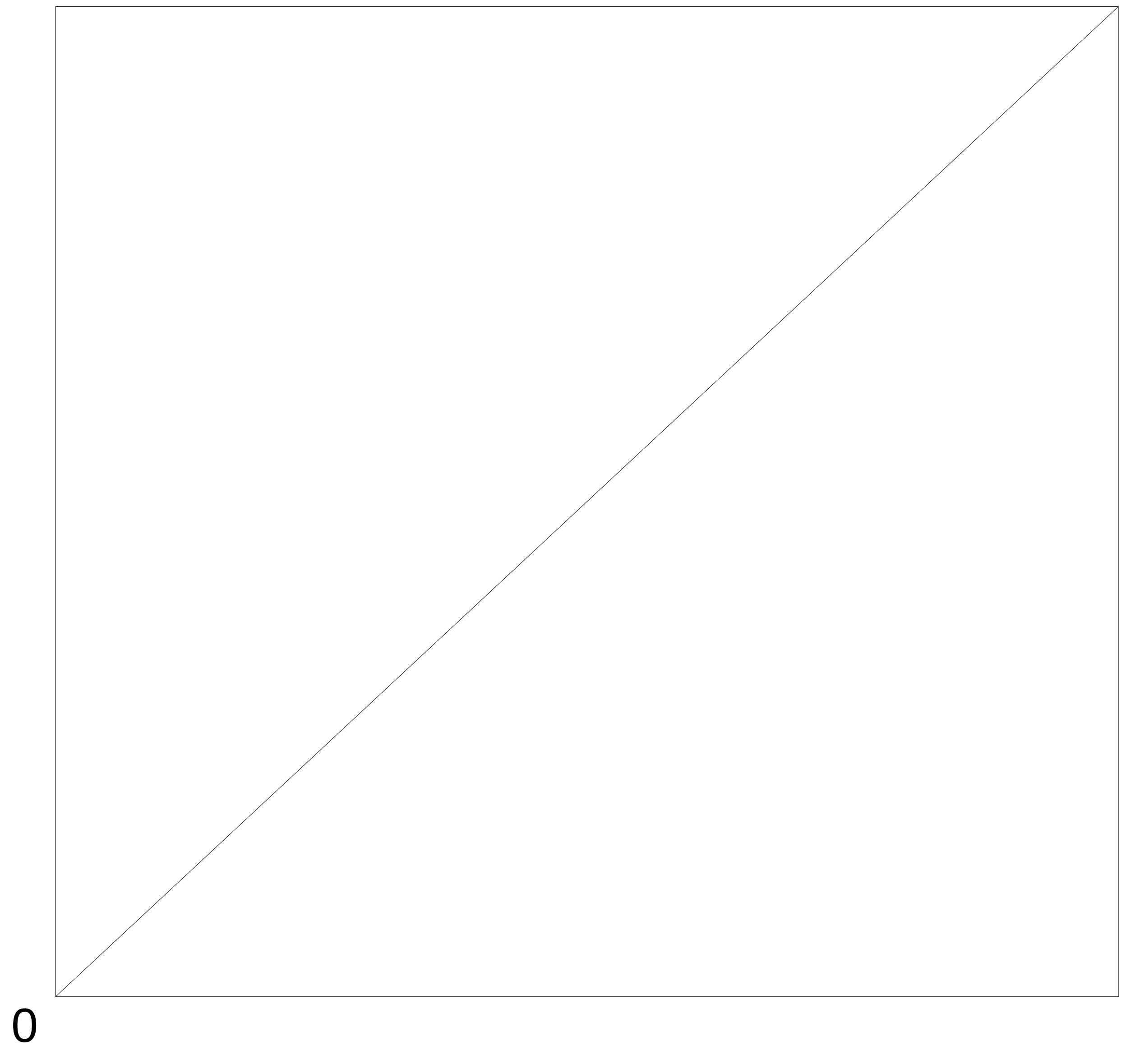




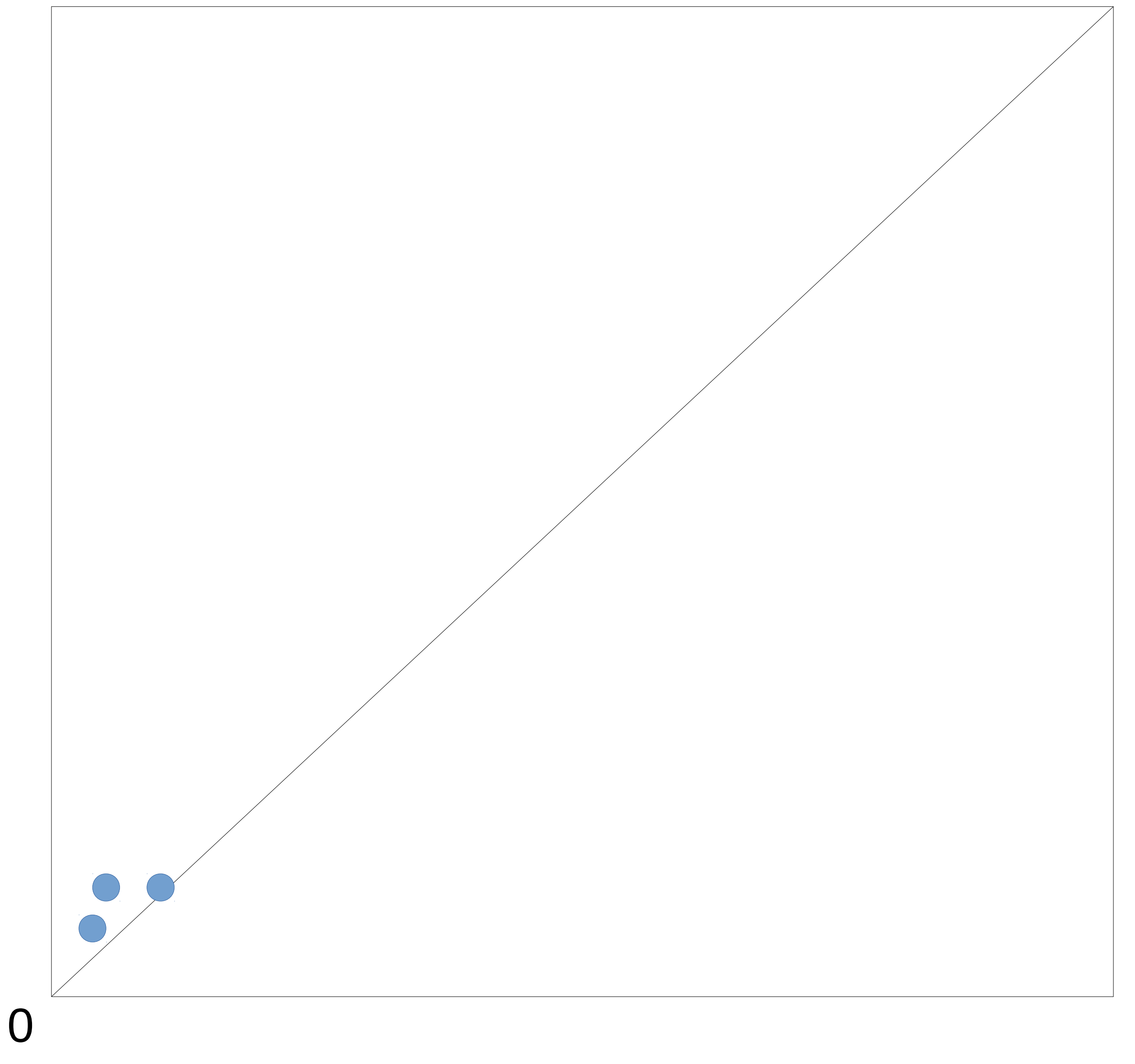




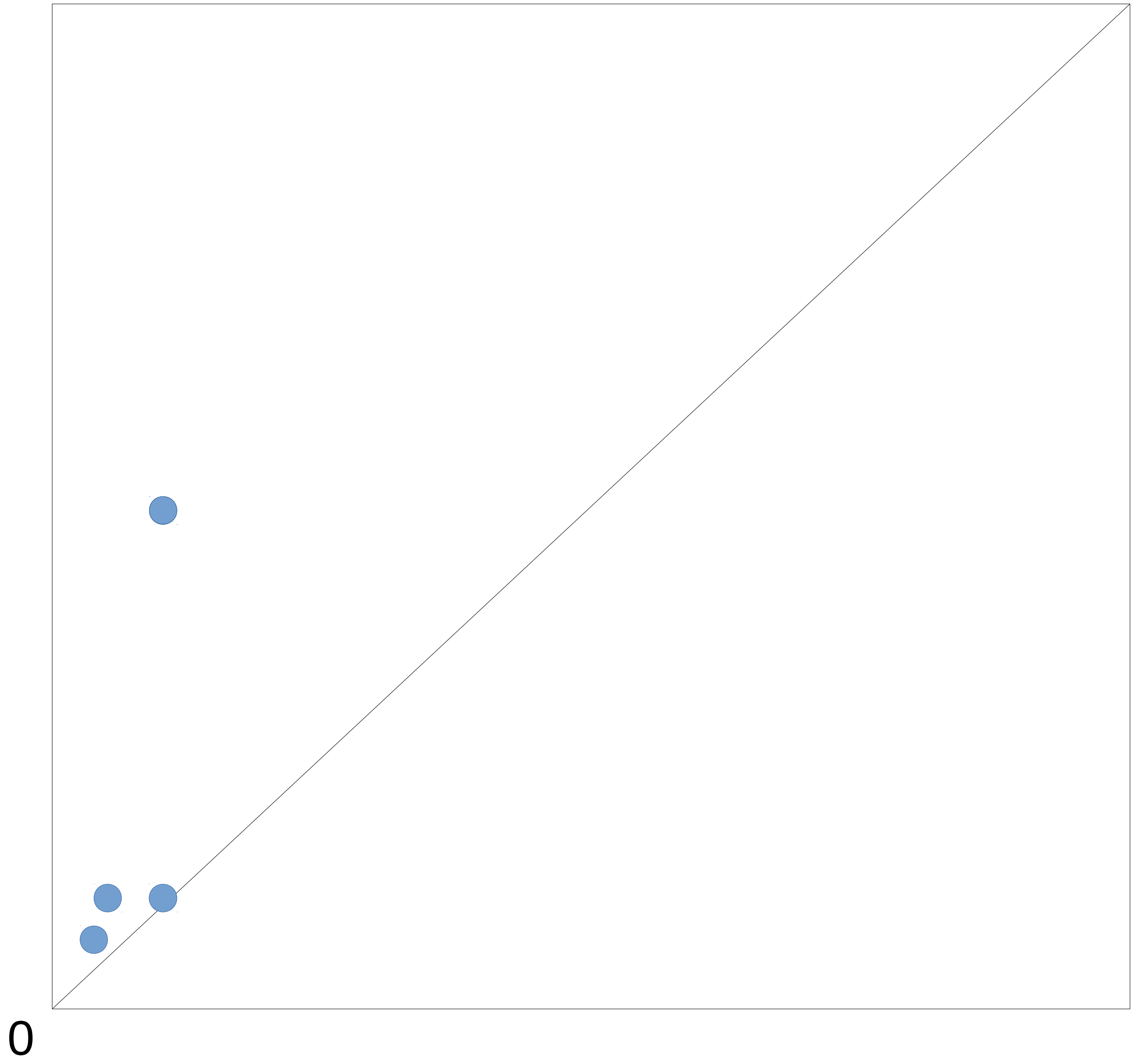
Persistence Diagram



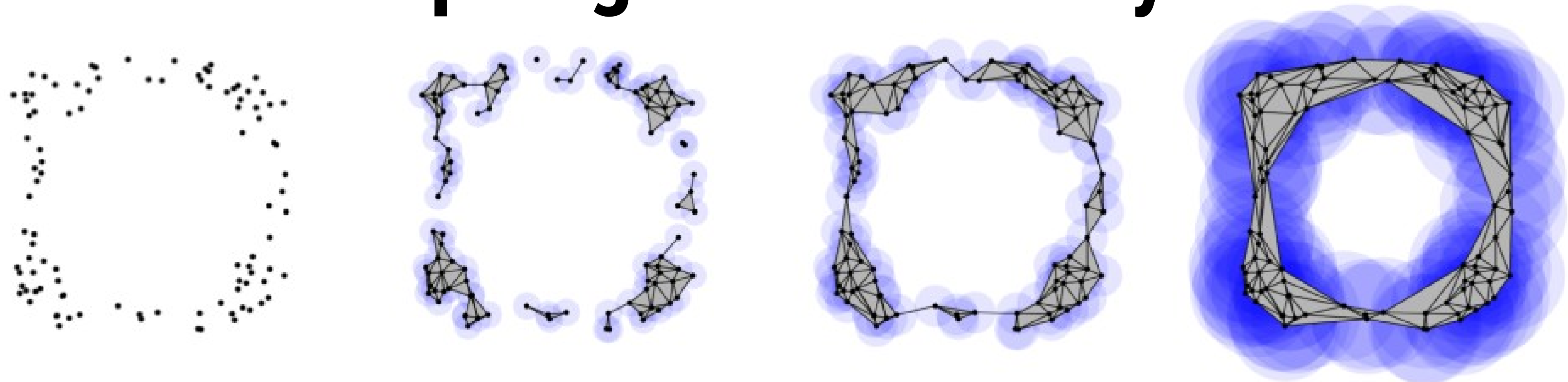
Persistence Diagram

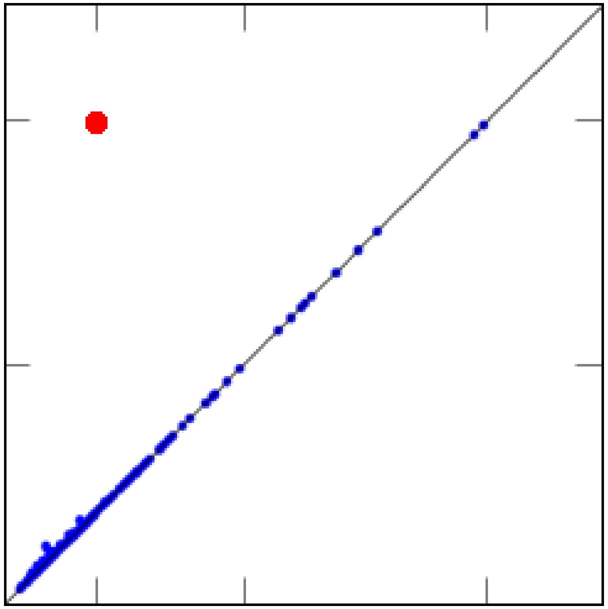


Persistence Diagram



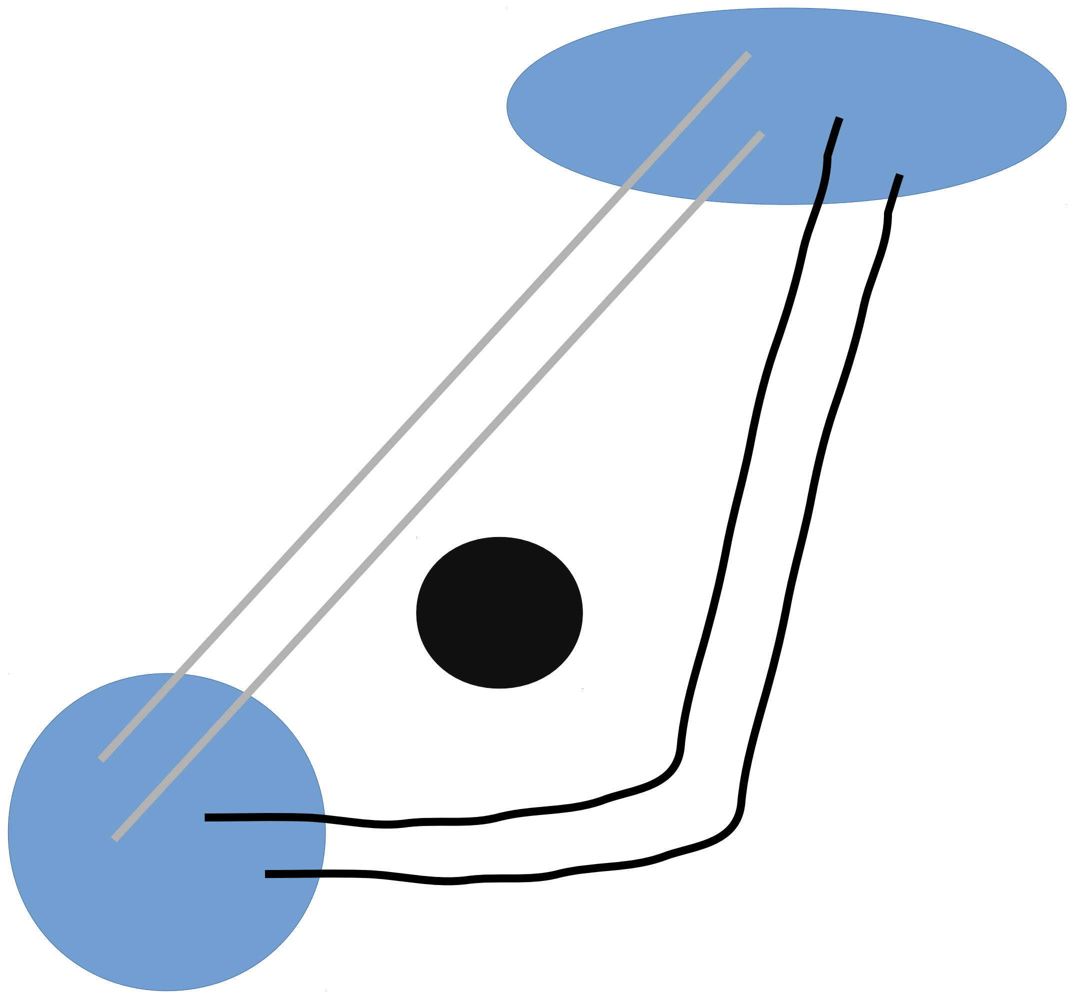
Topological Data Analysis

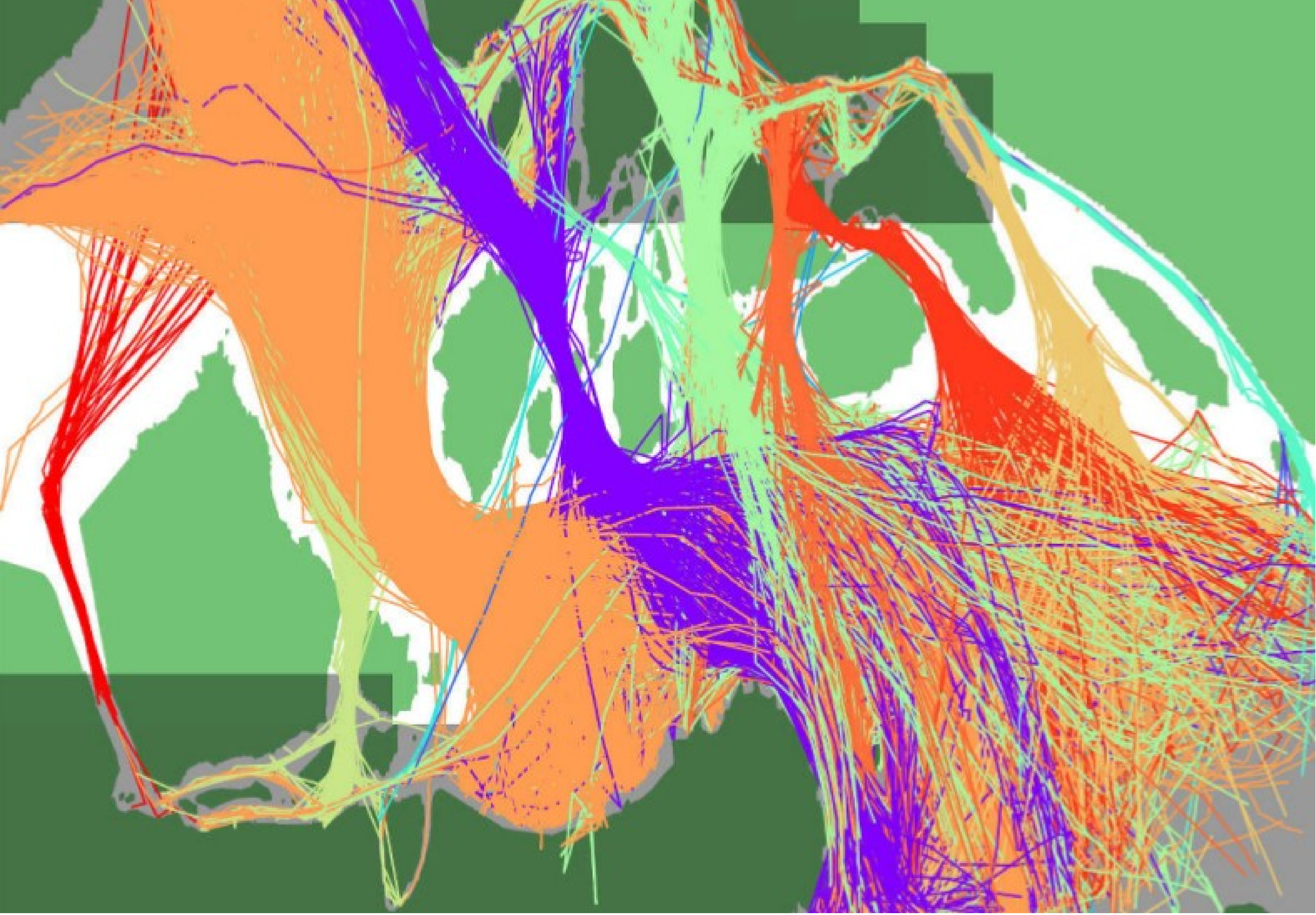




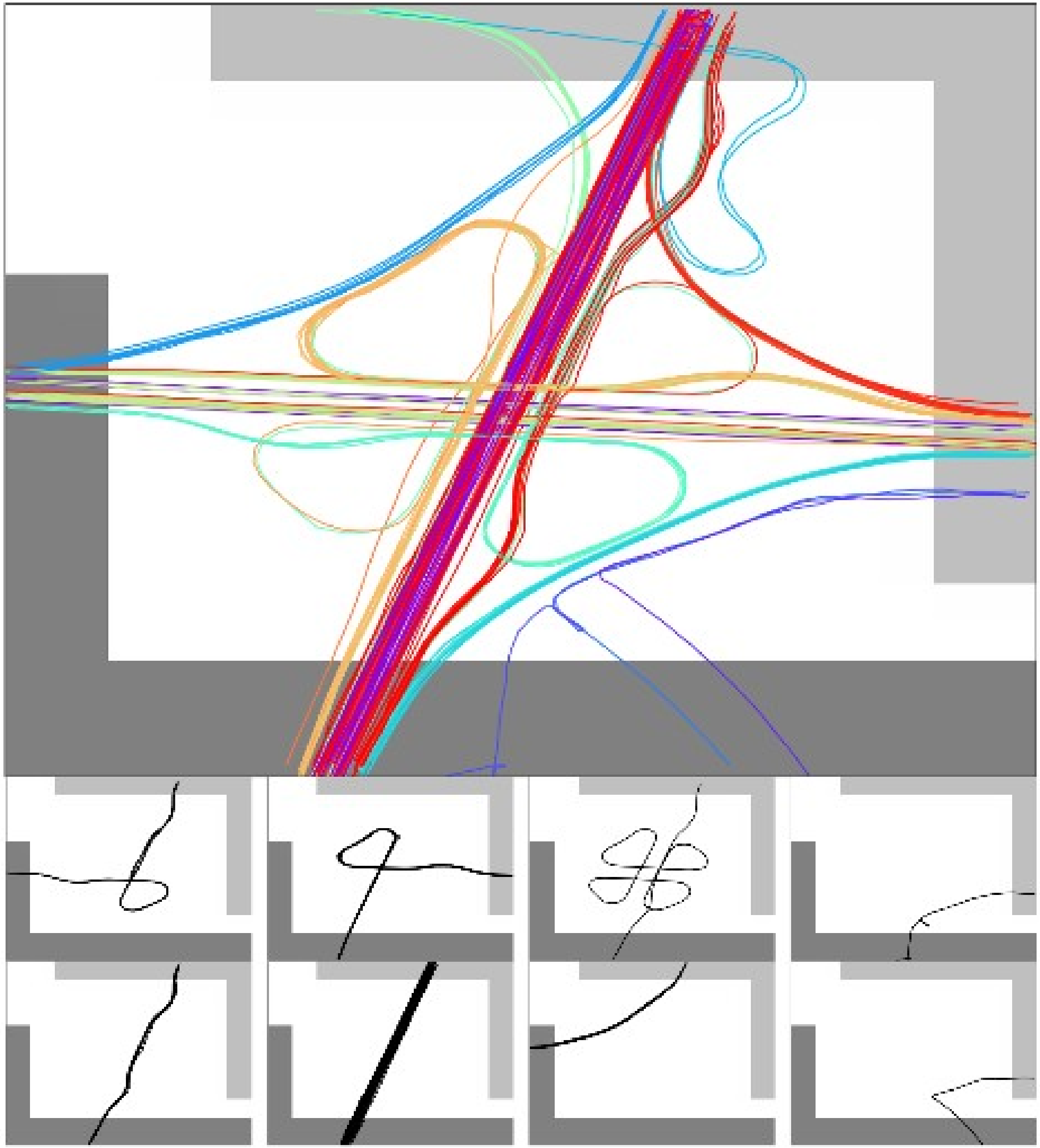
H. Edelsbrunner, J. Harer: Intro to Computational Topology
G. Calsson: “Topology and Data”, AMS Bulletin 2009
A. Zomorodian, P. Niyogi, P. Bubenik, F. Chazal, W. Chacholski, U. Bauer,...

Relative Homology & Homotopy



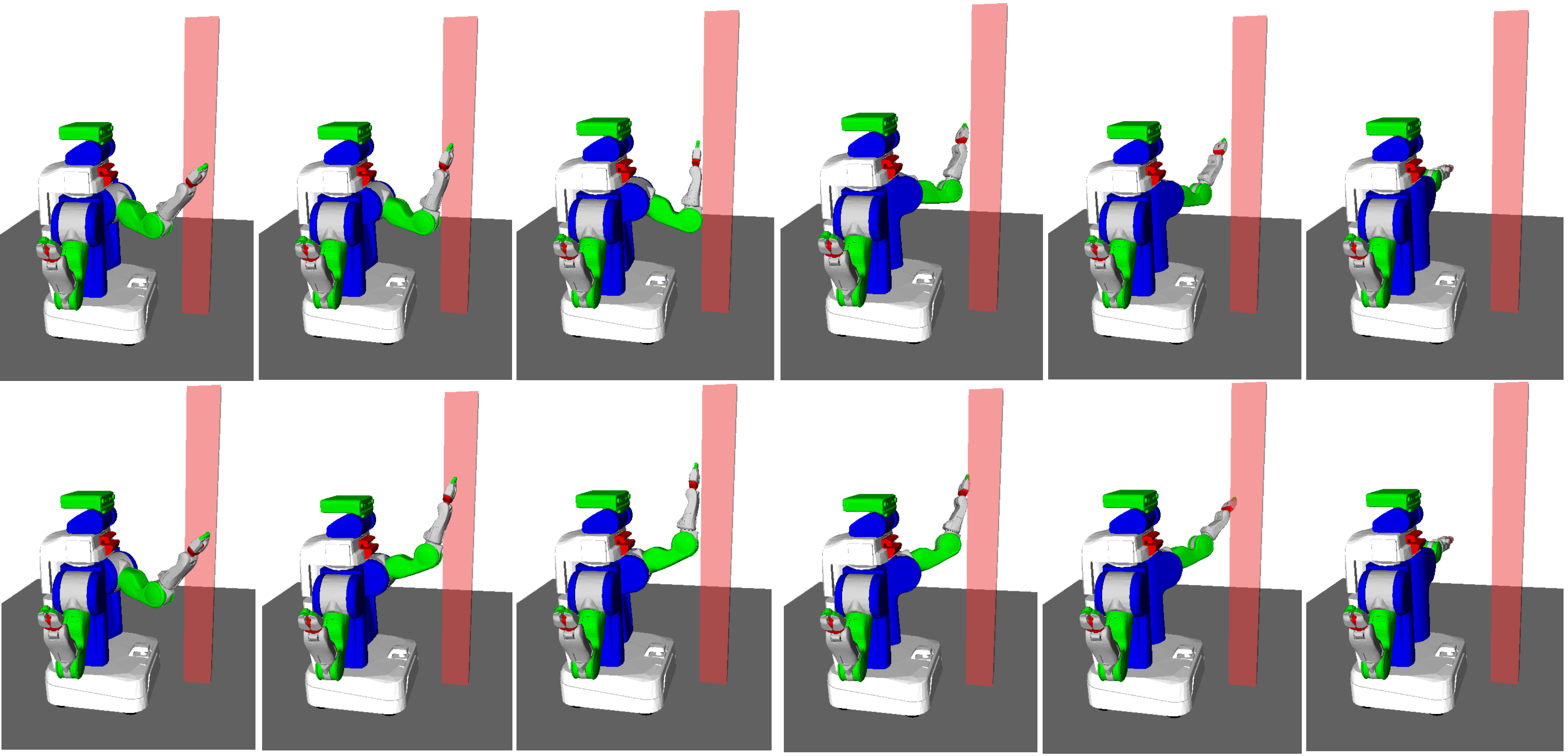


Topological Trajectory Clustering with Relative Persistent Homology
F. T. Pokorny, K. Goldberg, D Kragic, ICRA 2016



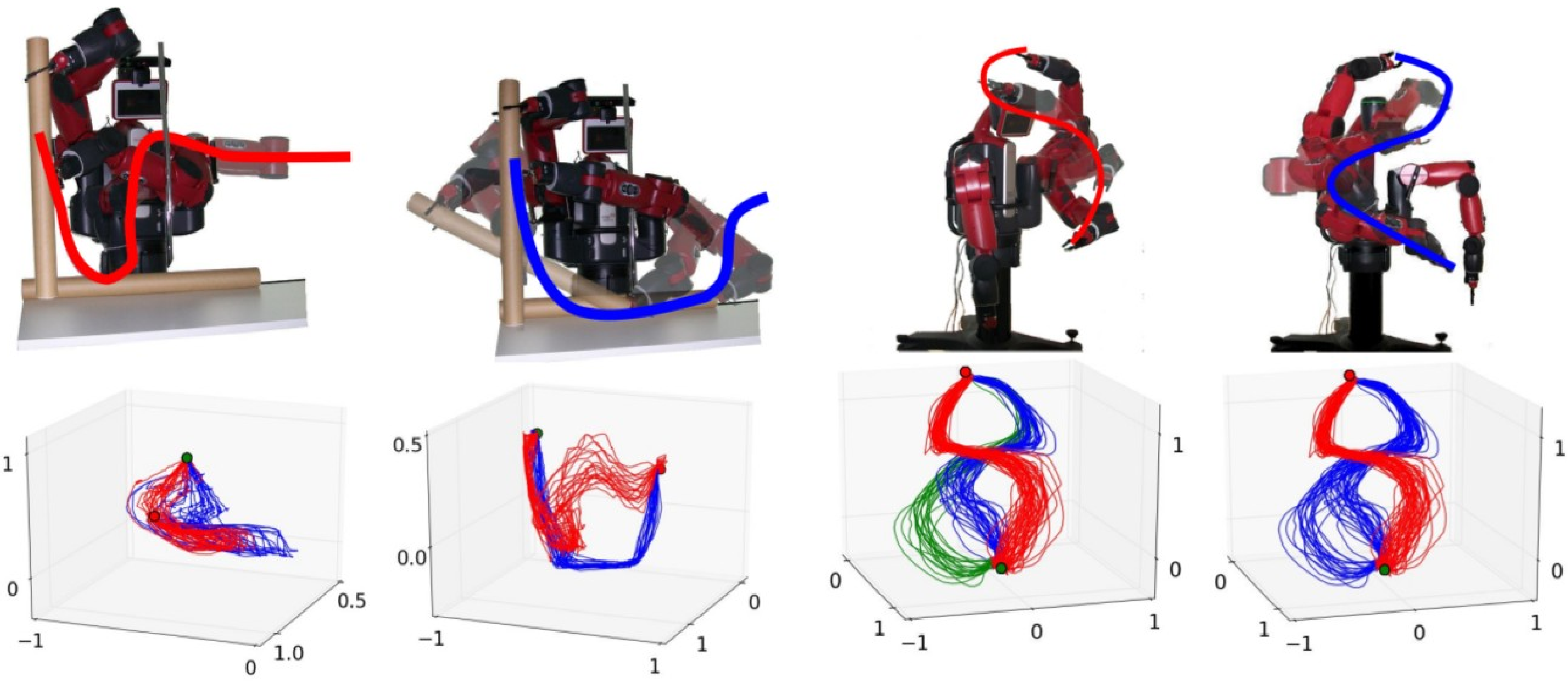
Topological Trajectory Clustering with Relative Persistent Homology
F. T. Pokorny, K. Goldberg, D Kragic, ICRA 2016

200.000 samples of 4DOF left arm, 3.6 million edges, 12.7 million triangles in 4D
Approx 80s total computation time



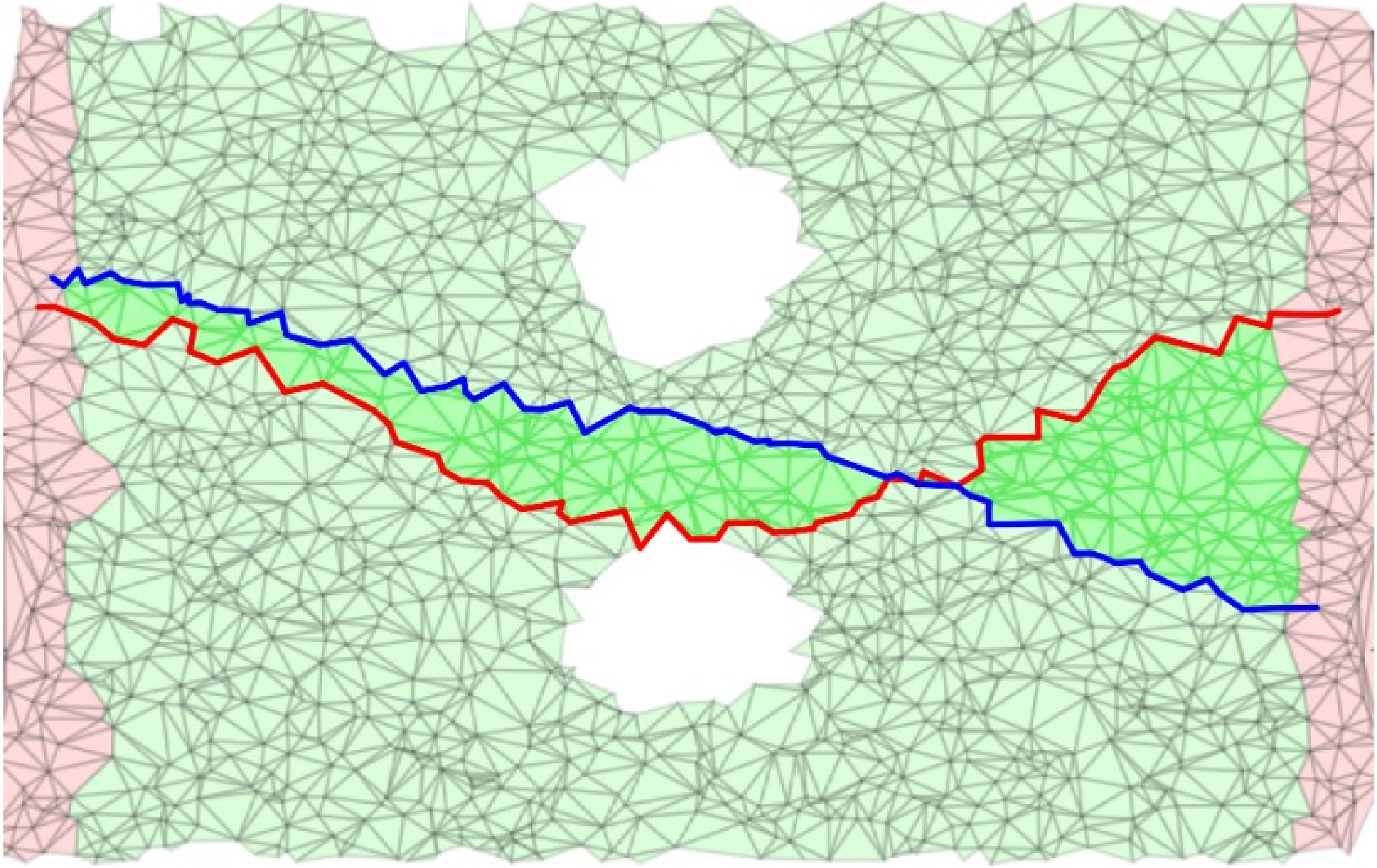
Data Driven Topological Motion Planning with Persistent Cohomology
F. T. Pokorny, D Kragic, RSS 2015

Topological Motion Clustering



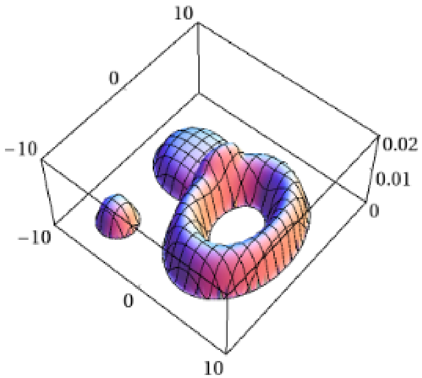
Multiscale Topological Trajectory Classification with Persistent Homology
F. T. Pokorny, M. Hawasly, S. Ramamoorthy, RSS 2014

Challenge: Topology vs Geometry

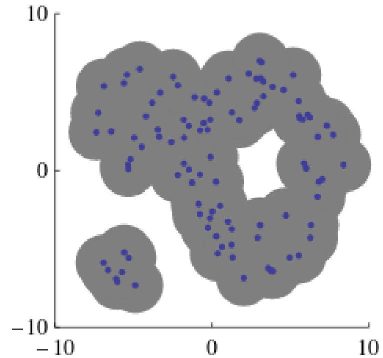


Path Clustering with Homology Area
J. F. Carvalho, M. Vejdemo-Johansson, D. Kragic, F. T. Pokorny
IEEE ICRA 2018

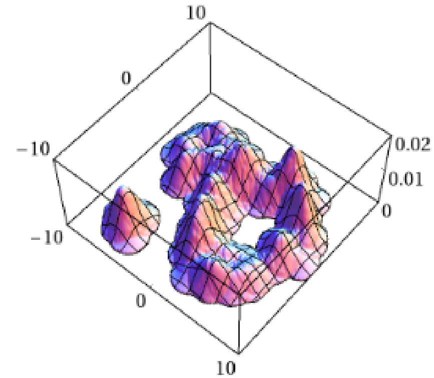
Challenge: ML integration



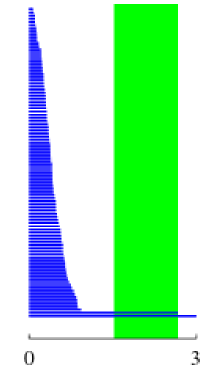
(a) density f



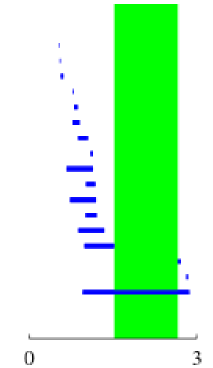
(b) 100 samples and $\Omega_{\varepsilon_{top}}$ in grey.



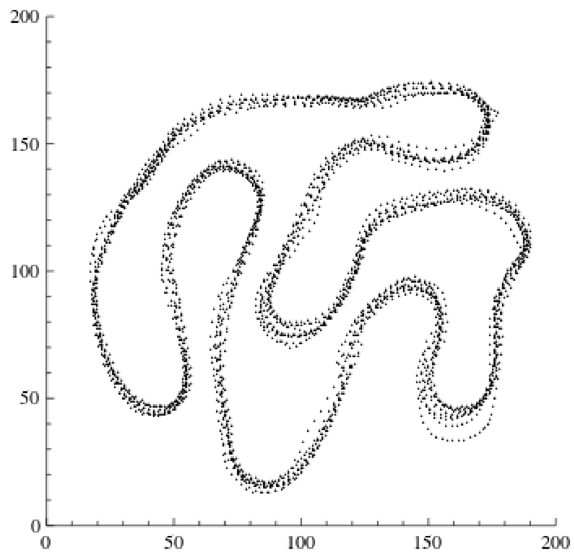
(c) $\hat{f}_{\varepsilon_{top},100}$ using just 100 samples as in 5(b)



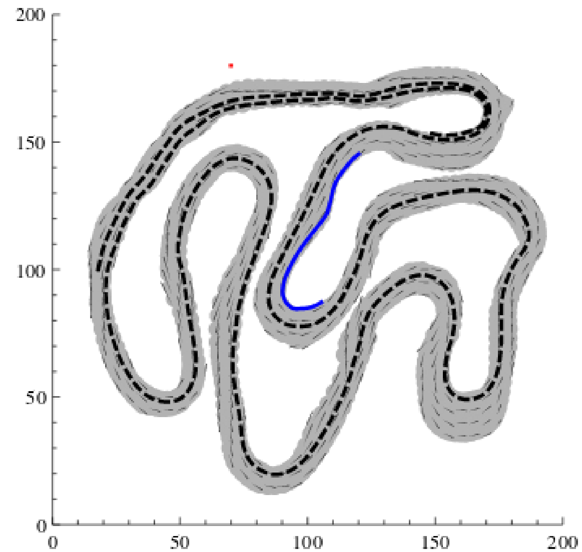
(d) barcode for b_0



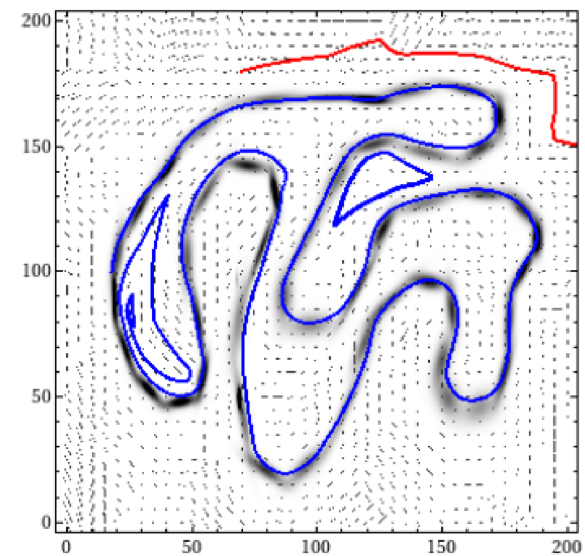
(e) barcode for b_1



(a) Position component of our racetrack data



(b) Projection of inferred support region, generated vector field and sample trajectories



(c) Inferred vector field, position likelihood and sample trajectories using GMR.

F. T. Pokorny, C.H. Ek, H.Kjellstrom, D. Kragic, **NIPS 2012**

Challenge: Complexity

Theorem (McMullen '70):

The number of d-simplices in the Delaunay triangulation of n points in d dimensions is at most

$$\binom{n - \lfloor \frac{d+1}{2} \rfloor}{n-d} + \binom{n - \lfloor \frac{d+2}{2} \rfloor}{n-d} = O(n^{\lfloor \frac{d+1}{2} \rfloor})$$

Challenge: Complexity

Theorem (Dwyer '91):

The number of d-simplices of n points drawn i.i.d from the unit ball in d dimensions is O(n).

Interesting Research Directions

- Back to cylindrical / analytic cell decompositions? (Canny,…)
- Beyond persistence? (Chacholski, ...)
- Towards sheaf cohomology? (Ghrist, …)
- Human Robot Interaction?

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20w5194 Home

Confirmed Participants

ATDA2019

Topological Complexity and Motion Planning (20w5194)

Organizers

Daniel Cohen (Louisiana State University)

Jesus Gonzalez (Centro de Investigacion y de Estudios Avanzados del Instituto Politecnico Nacional)

Lucile Vandembroucq (Universidade do Minho)

Description

The Casa Matemática Oaxaca (CMO) will host the "Topological Complexity and Motion Planning" workshop in Oaxaca, from May 24 to May 29, 2020.

ATDA2019: Applications of Topological Data Analysis

International Workshop on Applications of Topological Data Analysis

In conjunction with ECML PKDD 2019

Würzburg, Germany, Monday 16th September 2019

The emergent area of Topological data analysis (TDA) aims to uncover hidden structure in a wide variety of data sets, combining methods from algebraic topology and other tools of pure mathematics to study the shape of data. Though the pure mathematical foundation of TDA is a major research topic on its own, TDA has been applied to a wide variety of real world problems, among which image compression, cancer research, and shape or pattern recognition are only a few of the many examples.

As TDA is generally not a well-known topic to the data mining and machine learning community, this workshop aims to address the flow of information between the different communities. By illustrating some of its recent and new applications, we will discuss the potential of TDA to active researchers within the fields of data science and machine learning. Furthermore, this workshop provides new and young TDA researchers a chance to present their work to a new community in an interesting and creative way, emphasizing the many possible applications of TDA in real-world data sets.

Topics of interest to the workshop include, but are not limited to, the following:

- TDA and applications in biology, economics, computer science, medicine, physics, geology, . . .
- TDA in machine learning
- Clustering