Jordan Canonical Form: Application to Differential Equations

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ABSTRACT

Jordan Canonical Form (JCF) is one of the most important, and useful, concepts in linear algebra. In this book we develop JCF and show how to apply it to solving systems of differential equations. We first develop JCF, including the concepts involved in it–eigenvalues, eigenvectors, and chains of generalized eigenvectors. We begin with the diagonalizable case and then proceed to the general case, but we do not present a complete proof. Indeed, our interest here is not in JCF per se, but in one of its important applications. We devote the bulk of our attention in this book to showing how to apply JCF to solve systems of constant-coefficient first order differential equations, where it is a very effective tool. We cover all situations–homogeneous and inhomogeneous systems; real and complex eigenvalues. We also treat the closely related topic of the matrix exponential. Our discussion is mostly confined to the 2-by-2 and 3-by-3 cases, and we present a wealth of examples that illustrate all the possibilities in these cases (and of course, a wealth of exercises for the reader).

KEYWORDS

Jordan Canonical Form, linear algebra, differential equations, eigenvalues, eigenvectors, generalized eigenvectors, matrix exponential