Differential Forms: A Complement to Vector Calculus

Errata

Page 3  
k or ℓ

Page 4  
Ex. 2  
a) $3\varphi_3 - 4\varphi_4$  
b) $x\varphi_3 + y\varphi_4$

Ex. 5  
a) $x\psi_3 + y\psi_4$  
b) $2y\psi_3 + \psi_4$

Ex. 6  
d') $\psi_2\varphi_2$

Page 5  
line 12  
$(z + 1)e^zdz$

line -1  
d$(x^4 + y^3 - z^2)dz$

Page 10  
line 15  
$\psi = x^5y^2z^3$

Page 17  
line -15  
Let $\varphi =$

line -8  
$\frac{\partial}{\partial y}(x^2y^3 + x^4 + c(y))$

Page 18  
line 1  
Let $\varphi =$

line 6  
Then $x^2z^3 +$

line 7  
$= x^2z^3 + 2xy +$

line -14  
$x^2y^3 + xy^2 + 4xz + 2x + 2yz^3 - y - 2z^2 + c$

Page 19  
line -12  
$\varphi =$

Page 20  
line 15  
Let $\varphi =$

Page 21  
line 14  
Let $\varphi =$

line 17  
Then $\psi = (x^2y^2z^3 + 2x^3y^3z - x^4yz^2)dydz$

Page 30  
line 15  
DEFINITION 3.12: $(d?_1)^* = \varepsilon d?^1$, $\varepsilon = \pm 1$, where $(d?_1)(\varepsilon d?^1) = dx dy dz$.

Page 31  
line 13  
$dx_1, ..., dx_n$

Page 32  
Ex. 4 b)  
f = $2xy^3$

Page 33  
Ex. 7e)  
$-4xy^2zdxdy$

Ex. 10  
$(d?_1)^* = \varepsilon d?^1$ where $(d?_1)(\varepsilon d?^1) = dx dy$.

Page 38  
line 3  
every

Page 54  
line 11  
then

Page 57  
line -3 of footnote  
because that analogy lets us use
\begin{align*}
&= x_1 x_2 \varphi(i, i) + x_1 y_2 \varphi(i, j) + x_2 y_1 \varphi(j, i) + y_1 y_2 \varphi(j, j) \\
&= x_1 x_2(0) + x_1 y_2 \varphi(i, j) + x_2 y_1 (-\varphi(i, j)) + y_1 y_2(0) \\
&= (x_1 y_2 - x_2 y_1) \varphi(i, j)
\end{align*}
\[ + C(f(t), g(t), h(t)) \, dt \]

\[ \int_{C_3} \varphi_1 = \frac{16}{15} \quad \int_{C_3} \varphi_2 = \frac{14}{15} \]

\[ \partial C = \{q\} \cup -\{p\} \]

\[ (1 \cdot 1 - 0 \cdot 0) \]

\[ dy = \frac{-4uvdu + 2(-v^2 + u^2 + 1)dv}{(u^2 + v^2 + 1)^2} \]
Since $k_2 = k_1 \circ \ell$, $k^*_2(\varphi) = \ell^*(k^*_1(\varphi))$ by proposition III.3.61.

For simplicity we complete the proof when $n = 2$. As $k^*_1(\varphi)$ is a 2-form on a region in $\mathbb{R}^2$, we may write $k^*_1(\varphi) = f(x, y)dxdy$ for some function $f(x, y)$. Then

$$\ell^*(k^*_1(\varphi)) = \ell^*(f(x, y)dxdy) = f(\ell(u, v))J(\ell)(u, v)dudv \quad (5.17)$$

by proposition 3.34.

Then the conclusion of the theorem becomes, by definition 3.2,

$$\pm \int_{T_1} f(x, y)dA_{xy} \pm \int_{T_2} f(\ell(u, v))J(\ell)(u, v)dA_{uv}. \quad (5.18)$$

We are assuming the orientations are all compatible. This implies that either $T_1$ and $T_2$ both have the same orientation as surfaces in $\mathbb{R}^2$, so both sides of (5.18) get the same sign, and that $J(\ell)(u, v)$ is always positive, or else $T_1$ and $T_2$ have opposite orientations as surfaces in $\mathbb{R}^2$, so the two sides of (5.18) have opposite signs, and that $J(\ell)(u, v)$ is always negative. In either case, then, (5.18) is just the standard change-of-variable formula for double integrals.

The case of general $n$ is similar. (For $n = 3$, use 4.8 and 4.2 instead of 3.34 and 3.2.)
Ex. 13 should be flush with left margin

Page 194 footnote line 1  invertible

Page 197 line 18
\[ a_2 \leq x_2 \leq b_2, \ldots, a_k \leq x_k \leq b_k \]

Page 216 line -3  involve $dx_m$.

Page 245 line -5  $Jq$ should be $Jg$

Page 247 I.1 1 a) $(4x^2 - x)dx + 3xdy$
   b) $3x^2dx + (-x^2 + 2xy + x + y)dy$

I.1 2 a) $(3x^3 - 4y^2z)dx + (3yz + 4xz)dy$
   \[-(3x^2 + 3y^2 + 3z^2 + 8x + 4)dz \]
   b) $(x^4 + y^3z)dx + (-x^3 - xy^2 - xz^2 + 2xy + y)dz$

Page 248 I.2 2 d) $xdydz + (y^2 - 2)dzdx + (-z - 2yz)dx\,dy$

Page 249 I.3 2 d) $xi + (y^2 - 2)j + (-z - 2yz)k$

Page 251 III.1 2 e) 34 2 f) -17

III.2 2 d) 27
   3 a) 85

Page 252 IV.2 7 a) $x^3 - 2xy + 2y^2$

Page 253 IV.4 2) 1/15

Page 254 V.3 13) $-79/5$

V.4 2) 1/15