Time Value of Money

Basis for the course
Power of compound interest
$3,600 each year into a 401(k) plan yields $2,390,000 in 40 years

First some technical stuff

- You will use your financial calculator in every single module
- The time value of money is the concept that binds the whole course together
- If you do not have your financial calculator yet, turn off the PC and buy one now at Staples or Office Max or find a former Fin 225 student and borrow his or hers
- HP12C or HP10BII is recommended

HP10B II users

- To change number of decimal places, press DISP key followed by an integer 0 to 9
  - Always internally to 9 places
- If "BEGIN" indicator ever appears, press BEG/END key to toggle it off
  - END is the default but there is no "END" indicator
- Before any new calculation, **clear the entire calculator with CLEAR ALL key**
  - Pressing the "C" key only erases the display

More HP10BII prep

- Need to make sure your calculator is set for one period per year. Press and hold down your CLEAR ALL key and it should say 1 P_YR. If yours says 12 P_YR (set this way at the factory) you need to fix it. Press 1 and then the orange or green function key followed by the P/YR key on the top row (above PMT). Then retry the CLEAR ALL and it should now say 1 P_YR. You're now good to go.

HP12C users

- To change the number of displayed decimals, press yellow f followed by an integer
- If "BEGIN" indicator ever appears, press blue g followed by the END key to toggle it off
  - END is the default but there is no "END" indicator
- Before any new calculation, **clear the entire calculator with yellow f and REG key**
  - Pressing the "CL*X*" key only erases the display
  - CL*X* is for fixing typos
  - f and REG makes it factory fresh ready for new problem

Power of compound interest

- If the Native Americans had taken the $24 worth of beads and trinkets they received from the sale of Manhattan Island in 1626 and invested it at 8%, today their investment would be worth **$130 trillion!**
- They could buy back New York plus a couple of other major cities

Our symbols

- PV_0 = present value at time 0 (today)
- FV_n = future value at time n (n **periods** from today)
- i = interest rate **per period** (like .06 or 6%)
- n = number of **periods**

Compound interest

- Invest $100 ($PV_0$) today at an interest rate of 6%/yr for 1 year
  - $FV_1 = 100 + 100(.06) = 100(1+.06) = \$106.00$
- Leave it all in for a second year and earn 6% on the original $100 again plus 6% on the first year's $6.00 of interest
  - $FV_2 = 106 + 106(.06) = 106(1+.06) = \$112.36$
  - $FV_2 = 100(1+.06)(1+.06) = 100(1+.06)^2$
  - $FV_n = 100(1+.06)^n$ and $\mathbf{FV_n = PV_0(1+i)^n}$

Most important equation in finance

- $\mathbf{FV_n = PV_0(1+i)^n}$
- $FV_2 = 100(1+.06)^2$
  - 100=>PV 6=>i 2=>n solve for FV = -112.36
  - Interest rate is entered as 6 and not .06
    - For now disregard the negative sign
- Invest $2,000 for 40 years at i=8%
- $FV_{40} = 2,000(1.08)^{40}$
  - 40=>n 8=>i 2000=>PV solve for FV=-43,449
    - Negative sign is a convention used by HP & Excel

Same problem only different

- You invest $2,000 for 40 years and emerge at the end with $43,449
- What was the annual growth (interest) rate?
- **$FV_n = PV_0(1+i)^n$**
- $43,449 = 2,000(1 + i)^{40}$
  - 43,449=>FV 2,000=>PV 40=>n solve for i
    - "Error 5" or "No solution" Now the minus sign convention matters PV & FV must have opposite signs (use CHS or +/-)
  - -43,449=>FV 2,000=>PV 40=>n solve for i=8%

Discounting - PV

- What's a future sum worth today?
- Investment promises lump-sum payoff of $10,000 in 20 years; what's it worth today?
  - How much would you be willing to pay today for this promise?
  - How much would you have to invest today to amass $10,000 in 20 years?
    - Need to know the interest rate – expected rate of return – let's assume it's 6% a year

Same formula

- **$FV_n = PV_0(1+i)^n$**
- Now we know FV_n and are looking for PV_0
- **$PV_0 = FV_n / (1+i)^n$**
- $PV_0 = 10,000 / (1+.06)^{20}$
  - 10000=>FV 6=>i 20=>n solve PV=-3118.05
  - $3,118.05 invested today at 6% will grow to $10,000 in 20 years
  - You'd be willing to pay $3,118.05 today for the promise if you wanted a 6% annual return

One formula – three ways

- Given a PV today, you can find what it'll be worth at some point in the future by
  - **$FV_n = PV_0(1+i)^n$**
- Given a FV at some point in the future, you can find what it is worth today by
  - **$PV_0 = FV_n / (1+i)^n$**
- Given both the FV and PV, you can find the interest rate with either version
  - Just remember to switch one of the signs

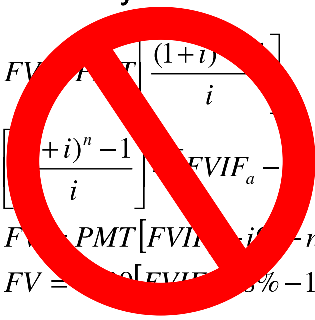
Finding i

- You can buy an insurance policy today for \$3,000 and then redeem it in 20 years for \$10,000. To find your rate of return
- $10{,}000 = 3{,}000(1+i)^{20}$
  - -3,000=>PV 20=>n 10,000=>FV solve i = ?
  - Gotta be higher than 6%
    - At 6% it took \$3,118.05 to grow to \$10,000
    - Starting with only \$3,000 so rate must be higher
    - i = 6.20% (6.2047%)

Annuities

- Series of **equal** payments at **equal** intervals
- On your child's 1st birthday deposit \$4,000 in investment earning 8% and continue to invest \$4,000 thru her 18th birthday. What's the final amount you have saved for college?
- Could just tediously add up all the FV's
  - First deposit + second dep +…+ last dep
  - $FV_{18} = 4000(1.08)^{17} + 4000(1.08)^{16} + \ldots + 4000$
  - First is compounded only 17 and last one not at all
- Annuity simplifies calculations
  - Annuity of \$4,000 => equal amts, regular fixed (annual) intervals for 18 years

FV of annuity math and notation



$$FV = PMT\left[\frac{(1+i)^n - 1}{i}\right]$$

$$\left[\frac{(1+i)^n - 1}{i}\right] = \left[FVIF_a - i\% - n\right]$$

$$FV = PMT\left[FVIF_a - i\% - n\right]$$

$$FV = 4000\left[FVIF_a - 8\% - 18\right]$$

FV of annuity math and notation

$$FV = PMT\left[\frac{(1+i)^n - 1}{i}\right]$$

$$\left[\frac{(1+i)^n - 1}{i}\right] = \left[FVIF_a - i\% - n\right]$$

$$FV = PMT\left[FVIF_a - i\% - n\right]$$

$$FV = 4000\left[FVIF_a - 8\% - 18\right]$$

FV of annuity on the calculator

- **FV_n = PMT [$FVIF_a$-i%-n]**
- FV_{18} = 4,000 [$FVIF_a$-8%-18]
  - 4,000=>PMT 8=>i 18=>n FV=$149,800.98
  - Notice that 18x4,000=$72,000
  - More than half of the savings is from interest
  - Compounding at work
  - Incidentally

$$FV = 4000\left[\frac{(1.08)^{18} - 1}{.08}\right] = 4000[37.4502] = 149{,}800.98$$

Present value of an annuity

- An investment (life insurance policy) promises to pay you $10,000 a year for 20 years starting one year from today. What's the investment worth now? What's its present value? How much would you be willing to pay for it today? How much would you have to deposit today to be able to withdraw 20 payments of $10,000?
- All questions have same answer

Enter the star of the show

- To answer any of these questions, need to know:
  - The required growth rate
  - The going rate of return
  - The **interest rate** you can earn
  - Let's assume an interest rate of 8%
    - Dependent on risk
    - Dependent on expected inflation

Restate the problem

- Receive $10,000 a year for 20 years starting one year from now
- Find the present value of the annuity discounted at i=8%
- Could just tediously add up all the PV's
  - First payment + second payment +…+last payment
  - $PV_0=10000/(1.08)^1 + 10000/(1.08)^2 + \ldots + 10000/(1.08)^{20}$
  - First is discounted 1 and last one is discounted 20
- Annuity simplifies calculations
  - Annuity of $10,000 => equal amts, regular intervals for 20 years
- Since the $10,000 is constant and interval is regular (once a year) can use the PV of annuity formula

PV of annuity math and notation

$$PV_0 = PMT\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = \left[PVIF_a - i\% - n\right]$$

$$PV_0 = PMT\left[PVIF_a - i\% - n\right]$$

$$PV_0 = 10000\left[PVIF_a - 8\% - 20\right]$$

PV of annuity on the calculator

- **PV_0 = 10,000 [$PVIF_a$- 8% - 20]**
  - 10,000=>PMT 8=>i 20=>n PV=98,181.47
  - Notice that you receive 20x10,000=$200,000
  - More than half of the benefits is from interest
  - Compounding at work
  - Incidentally

$$PV = 10000\left[\frac{(1.08)^{20} - 1}{.08(1.08)^{20}}\right] = 10000[9.818147] = 98{,}181.47$$

What's the 98,181.47 mean?

- You could deposit $98,181 today into an investment earning 8%/yr and be able to withdraw $10,000 each year for 20 years
- If someone or some investment promises you that for $98,181 today, you would receive $10,000 a year for 20 years, you'd be making an 8% annual return

Recap and some examples

$$Lump-sum\ (one\ payment)$$

$$FV_n = PV_0(1+i)^n$$

$$PV_0 = \frac{FV_n}{(1+i)^n}$$

$$Annuity\ (series\ of\ payments)$$

$$FV_n = PMT(FVIF_a - i\% - n) = PMT\left[\frac{(1+i)^n - 1}{i}\right]$$

$$PV_0 = PMT(PVIF_a - i\% - n) = PMT\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

Nothing special about a year

- Formulas work even if not annual compounding
- Let n=number of **periods** and i=interest rate per **period**
- Lots of very common examples

Application	Frequency	Periods per year
Bonds	Semi-annually	2
Saving accounts	Quarterly	4
Mortgages & car loans	Monthly	12
Visa & MC credit cards	Daily	365

Car loan example #1

- You can afford $300 monthly car payment
- Take out a 4-year loan (change to 48 months)
- How much can you spend on a car today, not including the down payment?
- Interest rate = 12%/yr compounded monthly = 12%/12 = 1%/month
- PV_0=PMT($PVIF_a$- i% - n)
- PV_0=300($PVIF_a$-1%-48)
- 300=>PMT 1=>i 48=>n solve PV = $11,392.19

Car loan example #2

- Dream car costs $25,000, you put $5,000 down
- Borrow remaining $20,000 from dealer
- Int rate = 8%/yr comp month=> 8/12 = .667%/mo
- 4 year loan (48 months)
- PV_0 = PMT($PVIF_a$-i%-n)
- 20,000 = PMT($PVIF_a$-.667%-48)
- 20,000=>PV 8/12=.667=>i 48=>n PMT=488.26
- Make 48 monthly payments of $488.26 and car is yours

Car loan example #3

- Bank will lend you the 20,000 but requires 36 monthly payments of $613.89
- Find bank's interest rate
- PV_0 = PMT($PVIF_a$- i% - n)
- 20,000 = 613.89($PVIF_a$-i%-36)
- 613.89=>PMT -20,000=>PV 36=>n solve i = .55%/month or .55x12=6.6%/year
- Bank has lower rate than dealer

Bond example #1

- Bond (corporate IOU) maturing in 15 years promises a $70 coupon payment every six months plus $1,000 at maturity
- PV_0 = PMT($PVIF_a$-i%-n) + $FV_n/(1+i)^n$
- PV_0 = 70($PVIF_a$-i%-30) + $1000/(1+i)^{30}$
- Let's say we're given **i=12% a year**
- PV_0 = 70($PVIF_a$-6%-30) + $1000/(1.06)^{30}$
- 70=>PMT 6=>i 30=>n 1000=>FV PV = -$1,137.65 (bond's price today)

Bond example #2

- What if the same bond sells for \$794.53? Find the interest rate or yield to maturity
- PV_0 = PMT($PVIF_a$-i%-n) + $FV_n/(1+i)^n$
- PV_0 = 70($PVIF_a$-i%-30) + $1000/(1+i)^{30}$
- 794.53 = 70($PVIF_a$-i%-30) + $1000/(1+i)^{30}$
- 794.53=>PV -70=>PMT -1000=>FV 30=>n (careful of the signs)
- Solve i=9.00% per period or 18% per year

Big 4

$$Lump-sum\,(one\ payment)$$

$$FV_n = PV_0(1+i)^n$$

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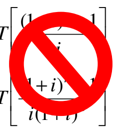
$$Annuity\,(series\ of\ payments)$$

$$FV_n = PMT(FVIF_a - i\% - n) = PMT\left[\frac{(1+i)^n - 1}{i}\right]$$

$$PV_0 = PMT(PVIF_a - i\% - n) = PMT\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$



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Big 4 – **all you need**

- Mortgages
  - Monthly payments, maximum loan, 15-years vs. 30-years, bank or credit union
- Car loans
  - Monthly payments, maximum loan, 3, 4, 5 or even 6 years, bank or dealer
- Retirement plans
  - Monthly contributions, how soon can you retire, term vs. whole life insurance
- Get the exact answers, not approximations

All the math of the course

- **You don't need any more math than what we've covered in this module**
- **All you need is PV and FV of single payments and of annuities**
- **But PLEASE do not go on to any other modules until you feel comfortable with this one**