LINEAR MODELS AND DECISIONS

10) $E(U) = E(Y_1 - 2Y_2 + Y_3) = E(Y_1) - 2E(Y_2) + E(Y_3) = 1 - 2\cdot 2 + (-1) = -4$

$$V(U) = V(Y_1 - 2Y_2 + Y_3) = V(Y_1) + 4V(Y_2) + V(Y_3) + 2(1)(-2)\,Cov(Y_1, Y_2) + 2(1)(1)\,Cov(Y_1, Y_3) + 2(-2)(1)\,Cov(Y_2, Y_3)$$

$$= 1 + 4(3) + 5 + (-4)(-4) + 2\ (1/2) + (-4)\ (2)$$

$$= 1 + 12 + 5 + 16 + 1 - 8 = 27$$

11) $E(Y) = E(XB + \tilde{E}) = E(XB) + E(\tilde{E}) = XB + 0 = XB$

12) $$V(\underline{X}) = E\left[(\underline{X} - E(\underline{X}))(\underline{X} - E(\underline{X}))'\right] = E\left[(\underline{X} - E(\underline{X}))(\underline{X}' - E(\underline{X})')\right] = E\left[\underline{X}\underline{X}' - \underline{X}E(\underline{X})' - E(\underline{X})\underline{X}' + E(\underline{X})E(\underline{X})'\right]$$

$$= E(\underline{X}\underline{X}') - E\left[\underline{X}E(\underline{X})'\right] - E\left[E(\underline{X})\underline{X}'\right] + E\left[E(\underline{X})E(\underline{X})'\right] = E(\underline{X}\underline{X}') - E(\underline{X})E(\underline{X})' - E(\underline{X})E(\underline{X}') + E(\underline{X})E(\underline{X})'$$

$$= E(\underline{X}\underline{X}') - E(\underline{X})E(\underline{X}') = E(\underline{X}\underline{X}') - E(\underline{X})E(\underline{X})'$$

13) $$Cov(\underline{X}_i, \underline{X}_j) = E\left[(\underline{X}_i - E(\underline{X}_i))(\underline{X}_j - E(\underline{X}_j))'\right] = E\left[\underline{X}_i\underline{X}_j' - \underline{X}_iE(\underline{X}_j)' - E(\underline{X}_i)\underline{X}_j' + E(\underline{X}_i)E(\underline{X}_j)'\right] = E(\underline{X}_i\underline{X}_j') - E(\underline{X}_i)E(\underline{X}_j)' - E(\underline{X}_i)E(\underline{X}_j)' + E(\underline{X}_i)E(\underline{X}_j)'$$

$$= E(\underline{X}_i\underline{X}_j') - E(\underline{X}_i)E(\underline{X}_j)'$$

$$V(\underline{Y}) = E(\underline{Y}\underline{Y}') - E(\underline{Y})E(\underline{Y})' = E\left[\left(\sum_{i=1}^N A_i\underline{X}_i\right)\left(\sum_{i=1}^N A_i\underline{X}_i\right)'\right] - E\left(\sum_{i=1}^N A_i\underline{X}_i\right)E\left(\sum_{i=1}^N A_i\underline{X}_i\right)' = E\left[\left(\sum_{i=1}^N A_i\underline{X}_i\right)\left(\sum_{i=1}^N A_i\underline{X}_i'\right)\right] - E\left(\sum_{i=1}^N A_i\underline{X}_i\right)E\left(\sum_{i=1}^N A_i\underline{X}_i'\right)$$

$$= E\left[\sum_{i=1}^N\sum_{j=1}^N A_iA_j\underline{X}_i\underline{X}_j'\right] - \left[\sum_{i=1}^N A_iE(\underline{X}_i)\right]\left[\sum_{i=1}^N A_iE(\underline{X}_i')\right] = \sum_{i=1}^N\sum_{j=1}^N A_iA_jE(\underline{X}_i\underline{X}_j') - \sum_{i=1}^N\sum_{j=1}^N A_iA_jE(\underline{X}_i)E(\underline{X}_j')$$

$$= \sum_{i=1}^N\sum_{j=1}^N A_iA_j\left[E(\underline{X}_i\underline{X}_j') - E(\underline{X}_i)E(\underline{X}_j)'\right] = \sum_{i=1}^N\sum_{j=1}^N A_iA_j\,Cov(\underline{X}_i, \underline{X}_j)$$

$$= \sum_{i=1}^N A_i^2\,Cov(\underline{X}_i, \underline{X}_i) + \sum_{i=1}^N\sum_{\substack{j=1\\ j\neq i}}^N A_iA_j\,Cov(\underline{X}_i, \underline{X}_j) = \sum_{i=1}^N A_i^2\,V(\underline{X}_i) + 2\sum_{i=2}^N\sum_{j<i} A_iA_j\,Cov(\underline{X}_i, \underline{X}_j)$$

14) $$E(\underline{t}) = E\left[\tfrac{1}{N}(\underline{X}_1 + \dots + \underline{X}_N)\right] = \tfrac{1}{N}\left(E(\underline{X}_1) + \dots + E(\underline{X}_N)\right) = \frac{1}{N}(\underline{\mu} + \dots + \underline{\mu}) = \frac{1}{N}N\underline{\mu} = \underline{\mu}$$

$$V(\underline{t}) = V\left(\tfrac{1}{N}\sum_{i=1}^N \underline{X}_i\right) = \tfrac{1}{N^2}V\left(\sum_{i=1}^N \underline{X}_i\right) = \tfrac{1}{N^2}\left[\sum_{i=1}^N V(\underline{X}_i) + 2\sum_{i=2}^N\sum_{j<i} Cov(\underline{X}_i, \underline{X}_j)\right] = \tfrac{1}{N^2}\left[\sum_{i=1}^N \sigma^2 I + 0\right]$$

$$= \frac{1}{N^2}\sigma^2\sum_{i=1}^N I = \frac{1}{N^2}N\sigma^2 I = \sigma^2/N\ I$$