Some Foundational Questions Concerning Language Studies: With a Focus on Categorial Grammars and Model Theoretic Possible Worlds Semantics

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Foundational Language-Studies Questions

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Abstract

There are four major goals for this paper. First, we demonstrate that the logical foundations of standard approaches to language studies involve an incoherence in their presuppositions. Second, we present an alternative approach that resolves this incoherence. Third, we discuss how this error manifests itself in categorial grammars and model theoretic possible worlds semantics. Fourth, we suggest some possible revisions in standard approaches to accommodate them to the alternative that we suggest. We arrive at a fundamentally functional, or pragmatic, conception - an interactive conception - of the nature of language and meaning.

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Contents

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Introduction

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The Incoherence Of Foundational Encodings

We begin by examining some issues of cognition and epistemology. In the standard empiricist view, perception encodes the world into the mind, cognition processes these encodings, and language recodes cognitive contents into utterances. As one consequence of this view, language is assumed to be based on cognition, and that assumption intrinsically commits the study of language to presuppositions about the nature of cognition - presuppositions that are themselves open to question. Furthermore, the critique of standard views of cognition that we will be presenting applies with little modification *directly* to standard views of language.

There is a tendency, stemming historically from Frege's (1918) rejection of "psychologism," to regard formal descriptions of language as independent of epistemological and psychological investigations of knowing - thus, such concerns about the nature of cognition would be considered to be irrelevant. Wittgenstein (1961), in the *Tractatus*, for example, relegated to psychology the problem of epistemically relating atomic propositions to atomic facts - outside the concern of philosophy. We contend, however, that such assumptions about cognition are unavoidable in the study of language, *and* that the assumptions about cognition built into standard approaches are untenable.

Cognition involves representation, and representation is usually assumed to be some form of encoding. What we wish to show is that assuming the *equivalence* of representations with encodings involves an internal contradiction - it is an incoherent conception of the nature of representation. As such, it cannot ground a valid approach to language. Our point is not to deny that encodings exist - they clearly do - but rather to show that they cannot be an epistemologically *fundamental* form of representation. There must be some alternative form of representation that provides a basis for defining encodings - and for studying language. Encodings can only be a derivative form of representation.

There are a number of relevant arguments (Bickhard, 1980, in press-a, in press-b; Bickhard and Richie, 1983; Bickhard and Terveen, in preparation), of which we will present only one - first rather intuitively and informally, and then with somewhat greater precision. Something is an encoding insofar as it represents something else. But "to represent" is a *functional* relationship that is relative to the epistemic agent for whom the representation is taking place - something may "represent" for one agent but not for another. Therefore, something is an encoding only insofar as some epistemic agent knows what it encodes. The essence of an encoding as a representation is constituted by some epistemic agent's knowing what it is that the encoding represents.[1]

This is unproblematic as long as "what the encoding represents" can be specified in terms of some other representation, e.g., "**X**" represents the same thing as "**Y**". It becomes incoherent if the encoding is presumed to be foundational - not defined in terms of, not derivative from, logically independent of, any other representation. If an encoding were foundational or logically independent, there would be no way to define or specify for the relevant epistemic agent what it represents, and therefore no way for it to exist as an encoded representation at all. For a foundational encoding, there would be no other representation in terms of which its representational content could be defined, in terms of which it could be constituted as an encoded representation. The best that could be done for a foundational encoding "**X**" would be "'**X**' represents (encodes) **X**" or perhaps "'**X**' represents (encodes) whatever '**X**' represents", neither of which succeeds in specifying a representational content for "**X**", and, therefore, either of which is *vacuous* as a definition of "**X**" as an encoding.

The incoherence of encodingism stems from the presupposition that there is a *non*-vacuous foundation of encodings. Encodingism assumes that encodings are the fundamental nature of *all* representation, but this assumption *requires* that there be some foundation of encodings in terms of which other encodings can be defined. This foundation must, in order to get the whole system off the ground at all, be non-vacuous in providing representational content. Without such a non-vacuous representational foundation, a purported encoding system will not contain any representations at all, and will at best constitute some formal system of representationally *empty* "symbols". Encodingism, then, is incoherent in that it presupposes an impossibility - the impossibility of a non-vacuous ground of encodings.

Such a ground is impossible, in turn, because it involves an intrinsic circularity: foundational encodings are supposed to provide representational content for other encodings, but such foundations cannot themselves exist without *prior* such representational contents as they themselves are supposed to provide. Foundational encodings require prior representational contents in order for those foundational encodings themselves to be defined as encodings at all, yet these contents are precisely what those foundations are presumed to provide to the rest of the system. Foundational encodings presuppose precisely what they are presumed to account for - a direct circularity. We now have a two step critique of encodingism: encodingism is incoherent because it presupposes something that is impossible - foundational encodings - and foundational encodings are impossible because the assumption of their existence involves an intrinsic circularity.

Foundational encodings cannot exist. They must be defined in terms of some other representation. Although this other representation might itself be an encoding, then it too would have to be defined in terms of still another representation. The regress continues until some truly foundational level of representation is reached, and that foundational level cannot itself be encodings. By nature, encodings are *stand-in* representations - they stand in for other representations and structures of representations - and so they must ultimately rest upon some non-encoding foundation.

Because foundational encodings are incoherent, there is no way for *new* foundational encodings to arise. New combinations of old encodings can occur, but for a new *foundational* encoding to arise there would have to be some way to specify what that new encoding element represented, and there is no such way. If it could be specified in terms of already existing encodings (or other representations), it would not be foundational. Any attempt to specify it independently of any other representations would collapse in the face of the incoherence problem. An alleged solution to this problem is to postulate that the foundational level of independent encodings is innate (e.g., Chomsky, 1975, 1980, 1988; Fodor, 1975, 1981, 1983; Piattelli-Palmarini, 1980). This maneuver just pushes the logical problem of how encodings could originate off into evolution, and it is no more solvable there than it is in the development of the individual (Bickhard, in press-a; Bickhard & Richie, 1983; Campbell and Bickhard, 1987).

We turn now to a somewhat more formal presentation. Begin with an encoding system. A single atomic element of such an encoding system is such an element only if at least two conditions are met: first, the element itself must be known by some particular epistemic agent,

and, second, what that element encodes must be known by that same epistemic agent. Consider both the knowing relationship from the agent to the encoding element, and the encoding relationship from the encoding element to what it encodes, to be maps. Then the special property of an encoding can be expressed by pointing out that the two maps must "compose" in the sense of generating a new knowing relationship directly from the epistemic agent to whatever it is that the element encodes. In other words, a knowing of an encoding element together with a knowing of the encoding relationship that makes that element an encoding generates a knowing of whatever that element encodes - generates a knowing of the resultant of that encoding map.

This is not enough, however. In a general encoding system, the atomic encoding elements are not isolated representational points. Whatever it is that those encoding elements represent will have various relationships with each other - spatial relationships, logical relationships, etc. - and those relationships among the encod*ed* elements must be represented by the relationships among the encod*ings*. Some such relationships may themselves be explicitly encoded - encodings of *relationships among* represented elements as well as encodings of elements themselves - but then there arises a new level of relationships among those encoded relationships which must themselves in turn be represented. At some point in the hierarchy of encodings of relationships among relationships among ... among atomic encoded elements, there must be a finite level at which the relationships are represented not by a new level of explicit encodings, but rather are represented directly by the relationships among the next lower level encodings. In a general encoding system, then, there must not only be knowledge of the encoding elements and of the encoding relationships in which they participate, there must also be knowledge of the relevant *intrinsic* relationships in which those encoding elements can participate with each other and of the encoded representational relationships in which those intrinsic encoding element relationships can participate (Palmer, 1978).

Finally, this whole structure of knowledge of encod*ing elements* and elemental relationships on the one hand, and of encod*ing relationships* of those elements and elemental relationships on the other, must compose in the sense of yielding for the epistemic agent knowledge of the encod*ed* elements *and* relationships. The three maps, then - 1) the *knowing* maps from the epistemic agent to the encoding system and 2) from the agent to what is encoded and 3) the *encoding* map from the encoding system to what is encoded - must be commuting morphisms with respect to the elements and relational organizations involved (this is diagram commutation in the sense of category theory, e.g., MacLane, 1971). The distinction between the encoding elements and the encoding relationships among them is a generalization of Wittgenstein's early distinction between saying and showing in the *Tractatus* (Bickhard, 1987; Fogelin, 1976; Kenny, 1973; Wittgenstein, 1961).

Formalization of "encoding" proceeds by explication of the intuitive concept of encoding. The situation for "representation" is a little more problematic in that the usual intuition of representation is that a representation *is* an encoding, and that presupposition is precisely what we wish to argue against. Representation, then, must be approached a little more indirectly. The key is the sense mentioned earlier in which representation is intrinsically *functional* in nature.

Consider first the concept of "control". A relationship of control exists between two processes when the outcome of one affects the course of the second. That is, the outcome of the first process exerts a selection among the various possibilities of the second. Among the simplest such control relationships is that of a switch: process **A** turns process **B** on and off. The mathematical concept of "information" is a measure of the amount of such selection among alternatives that is (or could be) exerted in such a control relationship (Khinchin, 1957; Shannon and Weaver, 1949). Information, then, is a measure of amount of (potential) control between processes (Bickhard, 1980). Note that these notions of control and information 1) do not involve or presuppose any epistemic notions, such as representation or interpretation, and 2) are emergent functional relationships that are instantiable in physical, chemical, or biological system processes - they are consistent with reasonable versions of materialism.

An organization of control relationships among various processes is called a control structure. A control structure of processes that interact with some physical or logical environment

is an interactive control structure or interactive system; and an interactive system that meets certain internal conditions is a goal directed system - specifically, those conditions for goal directedness are that the interactive system contains a test for an *internal* condition; that the outcome of that test determines whether or not some subordinate system will be executed (switched on: the internal condition exerts control - switching control - over the processes of the subordinate system); and that that subordinate system tends to yield or maintain the relevant internal condition. Note that "goal directed" is defined here solely in terms of internal conditions and functional organizations of the system - there is no dependence on environmental correspondents, nor, in particular, of *knowledge* of such correspondents, to those internal conditions. "Goal directed", in other words, as defined here, does not depend upon any epistemic concepts such as representation. It maintains the consistency with materialism.

Representation, however, can now be given a non-encoding explication. *A representation is a source of information (a source of selections among alternative process possibilities) that permits (increases the likelihood of) a goal directed (sub)system to reach its goal.*

This is not only a non-encoding explication of representation, it too presupposes only an underlying materialism, via the hierarchy of emergents of control, information, control organization, interactive system, and goal directedness. This is an explication of representation that avoids the Cartesian dualism of encodingism.

Note first that representation as explicated here is a functional concept that is relative to some particular goal directed system - a representation must be able to exert the relevant selections, and those selections must in fact be functional for that system. Note second that a system of encodings can in fact serve as a system of representations. If one set of elements contains information for the system, and a second set of elements is defined as encodings in terms of them, then that second set of elements, via those encoding relationships, can in turn provide information to the system, and thus constitute representations. But note that the foundational elements (and relationships) must already be representations themselves *in this strictly functional sense* in order for the encodings to exist. One must already know the possible elements and relationships that are to be "stood in for" in order for the encoding to be defined.

This is not a problem for actual encodings: they are always stand-in relationships in the sense that one element representationally stands in for some other already available (structure of) element(s). Thus, Morse code dots and dashes stand in for letters; written words stand in for spoken words; pulses in circuits stand in for characters; etc. The utility of encodings is that they change the *form* of information, and thus allow differing operations to be performed with it (and at differing speeds), but they do not ever constitute primordial representations. They cannot themselves cross epistemic boundaries: they cannot themselves cross the boundaries that define an epistemic agent - that differentiate the knower from the known, or the representing from the represented. In particular, encodings cannot by themselves epistemically cross from mind to world (e.g., perception), or world to mind (e.g., language). They cannot themselves provide *new* knowledge, for to do so would require that the encod*ed* elements be initially unknown - outside the epistemic boundary of the relevant epistemic agent - but then the encoding relationships could never be defined. To attempt to define them encounters the incoherence of foundational encodings: what is *to be* known via the encodings must be *already* known for the encodings to come into being.

Note that the source of information that constitutes a representation need not itself be *known*. It only needs to have the control structure selective effects, the functional properties, that have been specified. An *encoding* source of information *does* require that both sides of the encoding relationship (and the relationship itself) be known, and that is the reason that encodings cannot be a foundational form of representation. Some other form of representation is required to serve as a foundation, a form for which the informational selections can in fact occur without the sources of that information being already known. A form of representation is required that does not presuppose that what is to be represented be in fact already represented. An interactive form of representation that meets this requirement is introduced below.

General Implications for Language Studies

Standard approaches to language, from traditional grammar to Chomsky's several approaches to categorial grammar and model theoretic semantics, are permeated with encoding assumptions. In all standard approaches to language, utterances are taken to be some form of encoding of mental contents, and sentences are taken to be well-formed encoding types. Because they rely on an encoding view of representation, standard approaches to language are untenable. Foundational encodings do not exist, and, although derivative stand-in encodings do exist, they do not suffice to ground language as an encoding phenomenon. Derivative encodings cannot ground language because, as stand-ins, encodings can only exist within the boundaries of a given epistemic domain - a domain all of which is equally accessible by the relevant epistemic agent(s). Encodings cannot cross epistemic boundaries; they cannot constitute a *new* representation of something outside a domain that the knower already knows about. Encodings cannot be foundational for *any* type of knowledge. In particular, sentences and utterances cannot have any encoded epistemic access to mental contents (Wittgenstein, 1958; Bickhard and Richie, 1983; Bickhard, 1987).

Because of their encoding assumptions, standard approaches to language are inadequate and unacceptable, and an alternative approach is needed. Such an alternative will be sketched in the next section.

An Alternative Approach to Language

There are two major parts to an alternative non-encoding approach to language: first, a non-encoding ground for cognition and representation must be outlined, and second, an explication of language that is consistent with such an alternative must be indicated. Notwithstanding the contemporary split between linguistics and psychology, explanatory accounts of cognition and representation can impose *necessary* constraints on accounts of language.

A non-encoding form of representation, a form that we would contend underlies all encoding forms, derives from considering the interactive nature of any epistemic agent. An epistemic agent is not, and *cannot* be, a passive "agent" - it cannot be merely a passive bank of static encoded knowledge. Any view which permits such passive epistemics is inconsistent with the ontology of epistemic agents; such views are derived from and committed to encodingism. However sophisticated they may be in modern form, passive epistemics are nevertheless still committed to the encodingism of the blank waxed slate, on which representations are impressed by the world.

Agents actively, and interactively, engage the world. Agents are intrinsically interactive goal-directed systems. A representation *is* a representation only in terms of the consequences for, or the usefulness for, the goal-directed interactions of an epistemic agent. Representation is a functional concept. If a purported representation is not functional for an interactive epistemic agent, then it is not a representation at all. Attempts to understand representation in ways that do not acknowledge this interactive functional emergence of representation encounter deeply insoluble problems, of which the incoherence problem is only one (Bickhard, 1980, 1982, 1987, in press-a, in press-b; Bickhard and Richie, 1983; Campbell and Bickhard, 1989).

The interactive form of representation that ultimately derives from such considerations involves two dual concepts: *interactive implicit definition* and *differentiation*. Consider a goal-directed system in interaction with its environment. The course of that interaction will be jointly determined by the environment and by the (sub)system that is engaging in the interaction. In particular, the internal outcome of that interaction, the internal final state of that (sub)system at the end of the interaction, will depend upon the environment. Each possible internal outcome will "correspond" to the set of possible environmental conditions that would, if encountered in an interaction, yield that particular internal outcome. A potential internal outcome "picks out" a set of environmental conditions that would yield it. Actually having arrived at such an outcome indicates that some member of that environmental set has in fact been encountered. Note, however, that

the internal outcome contains no information about the *nature* of that set or its members except that they yield that internal outcome. The outcome defines a set of environmental states, but the definition is completely implicit. In fact, the relationship between the outcome and its environmental set is an interactive version of model theoretic implicit definition - the sense in which a logical system implicitly defines its class of models (Quine, 1966a).

The first of the dual concepts that form an interactive, non-encoding understanding of representation, then, is interactive implicit definition; the second is differentiation. If an interactive implicit definition is to contain any useful (selection) information for the system, then there must be more than one possible outcome to the relevant interaction. If there were only one possible outcome, then any interaction, and, thus, any environment, would yield the same outcome: such a single possible outcome would simply implicitly define "everything" or perhaps "anything". If there are two or more possible outcomes, then arriving at one of them serves to *differentiate* that particular environment from other possible classes of environments, and such a differentiation may be useful to the system in the course of further interactions. A particular possible outcome, then, interactively implicitly defines the class of environments that would yield that outcome. The set of mutually exclusive possible outcomes to which that particular one belongs - the *differentiation frame* which is formed by those possible outcomes - serves to differentiate the environments into types. Interactive implicit definition is the "correspondence" relation of an outcome to its environments; differentiation is the dual relationship among those implicitly defined sets of environments. Implicit definition and differentiation are simply dual perspectives on the same underlying explication, but the two perspectives highlight differing aspects of that explication, and are correspondingly useful in differing contexts.

A possible interactive outcome is not an encoding, and an interactive implicit definition relationship is not an encoding relationship, because nothing in the outcome specifies what about that class of environments makes them yield that outcome. The outcome implicitly defines, and dually differentiates, and in these senses represents, without any knowledge of what it is that is being represented. The fundamental character of an encoding is violated, and, thus, an interactive representation is not an encoding. Furthermore, it cannot be rendered in terms of encodings, precisely because of the lack of information about what is being represented. Interactive representation can, however, ground encodings: "**X**" can be defined as a stand-in for "**Y**" where "**Y**" is a differentiating outcome. Such interactively derivative encodings have some, but not all, of the properties of presumed foundational encodings and their derivatives (Bickhard and Richie, 1983).

An obvious question at this point is, "If knowledge of what is being represented is not present in an interactively differentiating outcome, what *is* the form and origin of such knowledge?" The form of such knowledge is in terms of the *uses* that other parts of the system can make of the outcomes that are in fact reached, in terms of the *information* provided to the system by those outcomes. That is, the differentiations of the environment can in general be used to differentiate the flow of other interactions; the differentiation frames for possible outcomes can be used to select among other possible interaction alternatives. For example, if the overall system is under the control of internal goal state **G**, and if internal outcome **A** of a relevant subsystem is obtained, then strategy **S22** should be selected, while if internal outcome **B** is obtained, then strategy **S196** should be selected. The internal outcomes indicate strategies, which constitutes indication of some of the interactive properties of the environments that yield those outcomes, and thereby constitutes representational content *for* those outcomes. The functional indicative relationships constitute functional representations of the *relevancies* of possible internal final states - such as **A** - to potential further system interactions - such as the potentiality of strategy **S22**. Furthermore, such functional indications constitute implicit predications: they predicate the availability and appropriateness of strategy **S22** of **A** type environments - **A** type environments *are* **S22** type environments. *Webs* of such interactive relevancies - such functional interactive indicative informational relationships - are the primary form of knowledge structures, and their origin is learning (Bickhard, 1980). Thus, something is an *interactive representation* insofar as it implicitly defines/differentiates something about the environment, and knowledge of what is being represented is constituted as relational

organizations - webs - of interactive relevancies concerning such implicit definitions/differentiations.

Precisely because *being a representation* is separated from *knowledge of what is represented*, interactive representations avoid the incoherence problem that afflicts foundational encodings. An interactive representation does not require any knowledge of what is being represented - that can come later when the usefulness of those differentiations is learned. Thus, the incoherence of needing to know *what* is being represented before it can *be* represented is not encountered.

There are two subsidiary points that we wish to mention here. The first concerns the relationship between the general interactive approach and the encoding approach. Not only can encodings be derivative from interactive representations, it can also be shown that the presumed foundational encoding approach is an asymptotic limiting case of the interactive approach. Briefly, if an interactive implicit definition were direct and one-to-one instead of one-to-many (perhaps, if the implicitly defined set were a unit set), and if that one-to-one "implicit" correspondence were in fact explicit - if there were knowledge *both* of the fact of the correspondence *and* of what the correspondence were with - then it would constitute a foundational encoding. But the restriction to one-to-one-ness can never be assured, and the explicitness of the correspondence is precisely what the incoherence problem prohibits, so this is truly an unreachable, asymptotic limiting case (Bickhard, 1980; Bickhard and Richie, 1983).

The second point concerns a potential rejoinder against the interactive approach. We cannot consider all possible objections, but there is one that we wish to address briefly: it may be that interactive implicit definition and differentiation can explicate knowledge of an *external* environment, in the sense of competence to interact successfully with that environment, but what about knowledge of abstractions, as in mathematics - where is the environment in which such abstractions could be known? Our answer is a generally Piagetian one (Piaget, 1977a). The properties of the interactions and of the control structures that engage in those interactions are more abstract than what those interactions are with, and those properties can themselves be known by a higher level interactive knower that knows the first level environmental system. This second level system could be known in turn by a third level system, and so on, generating an unbounded hierarchy of potential levels of knowing. The interactive perspective affords a very rich approach to abstract knowledge. This is not an ad hoc solution - it leads to a very powerful explication of cognitive development with many additional properties (Bickhard, 1980; Bickhard and Campbell, 1989; Campbell and Bickhard, 1986).

In this view, interactive knowledge consists of functional relationships among interactive control structures. One immediate implication for language studies is that those control structures cannot be encoded into utterances. Language cannot be an encoding of mental contents. Not only is it impossible to cross epistemic boundaries with new foundational encodings (see above), but interactive control structures do not correspond structurally from one person to another in the manner that such encodings would require (Bickhard, 1980, 1987). An alternative is necessary.

Within an interactive perspective, utterances must be some form of interaction. The problem is to characterize *what* form. We propose to characterize utterances in terms of what they interact with or operate on. A first candidate for the object of linguistic operations would be the knowledge structures of the individuals involved. This would imply that utterances operate on (change or transform) knowledge structures instead of encoding them. This would already involve deep changes from current views, and the view of utterances as operations on mental contents is not entirely wrong, but we suggest (the arguments must remain elsewhere; Bickhard, 1980) that the object of linguistic interactions is in fact the social situation that is constituted out of those individual representations. This social situation we call the *situation convention*. Situation conventions are a generalized form of Lewis' conventions (Lewis, 1969). Intuitively, situation conventions are what constitute a class-room situation *as* a class-room situation, a party *as* a party, or a discussion of *this topic* **as** a discussion of *this topic*. Situation conventions are constituted as the convergences of the *representations* of the situation among the *participants* in the situation: it *is* a party situation because everyone assumes that it is a party situation. Situation conventions

have deep affinities with the symbolic interactionist's concept of the definition of the situation, with Goffman's frames, with the reflexivities of the ethnomethodologists, with concerns about common knowledge that is presupposed in conversations, and so on. Arguments and explications are elaborated in Bickhard (1980) and Bickhard (1987), but they will not be developed here. Most of the consequences that we wish to consider here follow directly from the *operative character* of utterances, without regard for the specifically social nature of the *object* of those operations.

The interactive, operative character of utterances already involves a number of important consequences. For example, utterances are intrinsically context-dependent. The outcome of a linguistic interaction depends as much upon the contextual situation convention that is being operated on as it does on the operation performed. Context dependency is being increasingly discovered throughout language, but only an operative perspective makes context dependency a *necessary* characteristic of language, and thus explains why it is ubiquitous (Bickhard, 1980).[2]

The operative perspective provides a natural way of deriving, and of explaining the emergence of, the various illocutionary forms (e.g., declarative, imperative) in terms of the goals and consequences of the linguistic operations. The intuition, for example, that imperatives operate on goals, while declaratives are focused on representations *per se* can be given a straightforward explication within this framework (Bickhard, 1980). This is clearly preferable to the usual awkward attempts to derive all other illocutionary forms from the declarative. It is also preferable to simple typologies with little or no internal relational analysis (Bickhard, 1980).

A particularly broad consequence of the interactive, operative approach to language is that the standard subdivisions of language study--syntax, semantics, and pragmatics--are untenable and must be replaced. Syntax cannot be construed within this view as the study of well-formed encodings, because it is *operations* that are being dealt with, not encodings. Syntax must instead concern itself with structures and differentiations of operations (it is in this respect that we find categorial grammars suitable for an operative approach to language--see below). Semantics and pragmatics as usually defined are incoherent within this framework. Semantics is thought to be concerned with the *truth-conditions* of *sentences*, whereas pragmatics is thought to be concerned with the social, communicative *uses* of *utterances*. From the interactive standpoint, however, utterances are *operations* on situation conventions, which are, in turn, constituted out of individuals' representations. Such operations can *generate* new situation conventions, with new representations, which may have truth values, but neither the utterances themselves, nor the operative forms (sentences), are themselves bearers of truth values. They are *operations on* representations, not representations per se. On the other hand, a possible social, communicative *use* of an utterance might be precisely to construct a representation (with a truth value).

The interactive approach, then, removes properties of representations with truth values from sentences or utterances per se and locates them in the *contextual outcomes* of utterances, and it locates issues of potential operative use - of operative power - in sentences. In the interactive approach, the supposedly semantic issues of representations with truth values become part of the supposedly pragmatic issues of outcomes and usages of utterances, while the supposedly pragmatic issues of the social operative use of language becomes part of the supposedly semantic issues of the operative power - the meaning - of sentences. The standard conceptions of semantics and pragmatics, thus, divide up and group the properties of language in ways that are committed to the encoding approach. They are not theory-neutral ways of defining the subject matter (Bickhard, 1980); they are not mere descriptions. Similarly, and more specifically, the interactive approach has implications for categorial grammars.

Problems with Categorial Grammars

The considerations mentioned above are, if sound, fatal to virtually all contemporary approaches to language studies - language is universally presupposed to be an encoding phenomenon, and the problem of the incoherence of encodingism, for example, is not even addressed. At best, contemporary approaches are blind to deep objections; at worst - our claim -

they are incoherent in their foundations. The adjustments required to accommodate to the interactivist considerations, however, are not equal among all such contemporary approaches: some are more compatible, more easily adapted, than others. Approaches that merely provide some formalization (however complex and sophisticated) of a presumed encoding of a representational or abstract base (e.g., deep or logical structure) clearly receive no support from interactivism. One formal approach that *does* have significant properties in common with the interactivist perspective on language is the juncture of categorial grammars and model theoretic possible worlds semantics. These too, however, involve encodingist presuppositions in their contemporary versions.

Categorial grammars are recursive structures of definitions of grammatical categories, beginning with some basic generative set of categories (Ajdukiewicz, 1967). The principle of recursion is to define a new category in terms of its power, when a token of it is combined with a token of an already given category, to generate a token of some other specified category. The basic generative set typically consists of encoding categories for truth values and for entities, i.e., declarative sentences and names. There are variants of this (e.g., Montague, 1974, who does not make direct use of the entity category), but all involve a generative set of categories of presumed types of encodings, and this is clearly incompatible with the interactive approach. The generative set can be changed, however, while conserving the recursive structure of categorial grammars.

A Revision of the Categorial Approach

The principle of recursion involved in categorial grammars - the definition of a category in terms of the effects of its elements on elements of other categories - is compatible with the operative character of an interactive approach. Combining suboperations of given types can be expected to yield a resultant operation of some particular type, and the types could be recursively defined in terms of such "type operative" powers. The generative set, however, must be rethought to generate an *interactive* categorial approach to grammar.

The first element of the generative set is clear, and is nominally the same as for standard approaches - the sentence. But this is a sentence construed as a full transformational operation on situation conventions (thus including declaratives, imperatives, optatives, etc.). It is not an encoding with truth values.

The second generative element, or possible second elements, are not as immediately clear. The fully operative sentence is the root category toward which all others are constructively "aimed." The second generative category must in some sense be a "partial" operation, but the notion of a partial operation is difficult. It requires a more detailed analysis of how an operation that transforms a situation convention can occur at all. Without presenting the relevant arguments (see Bickhard, 1980), we state that there are two fundamental suboperations. One kind of suboperation differentiates, within the overall webs of knowledge relevancies (functional indications) that collectively constitute the situation convention, that part (or those parts) that are to be the focal object(s) of operative transformation. A second kind of suboperation differentiates, within the structure of possible (available) transformations, the transformation to be performed on that (those) focal point(s). Since differentiating the new focal point of operation is itself an operation on structures with prior focal points - an operation that changes focal points - the differentiation of the operations to be performed is the more fundamental suboperation, and is therefore more appropriate as the second generative category. This would roughly correspond with logical predicates in standard approaches (cf. Strawson, 1974).[3]

Regarding the construction of an utterance as the construction of an operator rather than as the construction of an encoded proposition introduces some new considerations into the grammatical task. It introduces distinctively *functional* considerations: the function of subtypes of partial operators (categories) is to contribute to the efficiency, effectiveness, and productivity of the construction of utterances as full operators on situation conventions. Furthermore, this is not just logical functionality; even the contribution of subtypes of encodings to the construction of a full propositional encoding could be said to be logically functional. Nor is it just mathematical

functionality; contributions towards functions (on functions) on structures of sets of objects, as in Montague grammars, could be said to be 'operatively' functional, in a mathematical sense. Instead, the contributions made by the categories in an interactive categorical grammar are interactively, *socially* functional by virtue of the nature of utterances as interactive operators on social realities (situation conventions).

Among existing approaches, functional grammars share with interactivism a strong functional motivation, and certainly take many functional considerations into account, but they do not involve a full social operativity, and they still posit a basic level of propositional encoding (e.g., Dik, 1978; Foley and van Valin, 1984; Silverstein, 1976). Categorial grammars have involved functionality in the mathematical sense, but not in the interactively social sense. As a system of operators, language has a definite formal character, and this is what most approaches to language have attempted to capture, albeit from the inadequate and incoherent encoding perspective. As a system of operators on situation conventions, language also has a definite social interactive character, and this has been ignored in attempts at formal treatment. Within the standard framework, the social aspects of language are merely among the various "pragmatic" uses to which encoded sentences can be put, and they are not in any way essential to language and its structure. By contrast, "semantic" truth conditions and entailments for encoded sentences are regarded as fundamental to language, and necessary for understanding its uses. Social uses are a part of pragmatics, which is derivative from and subordinate to syntax and semantics - pragmatics concerns the usages to which logically prior encoded propositions can be put. Social aspects of language, then, within the standard perspective, would have at best purely contingent influences on the structure of language, but would have no essential or constitutive bearing on formal considerations. The interactive perspective argues that these views of language, and corresponding approaches to grammar within them, are simply false (see Foley and van Valin, 1984, for related arguments), and incoherent in their encoding roots.

Problems with Model Theoretic Possible Worlds Semantics

Model theory arose from investigations of the semantics of formal logical systems (Tarski, 1956). It formalizes and extends encoding intuitions about the meanings of names and sentences in terms of mapping (satisfaction) relationships between sentences and underlying set-theoretic models. The primitive intuition of model theory is that a sentence is true if the relationships it encodes are true of the appropriate entities and sets that its terms encode. The conception of these static, epistemic, mapping relationships as given, and the lack of concern with how any knower could arrive at or establish such relationships, is deeply indicative of model theory's commitment to an encoding view of representation. One of the major insights of model theory was rendering quantifiers in satisfaction (encoding) terms, and a major accomplishment was rescuing the concept of truth from otherwise seemingly unavoidable paradox (Tarski, 1956).

The semantics of *modals*, however, required something more than models of sets of actual objects, and it came to be approached in terms of models of sets of possible worlds (Hughes & Creswell, 1968; Hintikka, 1969, 1975). The primitive intuition here is that a proposition is *necessarily* true if (and only if) it is true in *every possible* world. Model theoretic semantics and the special variant of possible worlds semantics were introduced as approaches to semantics for formal languages. Richard Montague, however, argued that these formal methods could be adapted to natural language and began to show how to do it (Montague, 1974), initiating the development of what are now known as Montague grammars (Cresswell, 1973; Partee, 1976; Reichl, 1982). Montague grammars combine a categorial approach to the development of grammatical categories with a treatment of semantics in terms of structures of, and functions on, possible worlds. It attempts to treat,within the framework of natural language, truth, modality, "intensions," and "propositional attitudes," among other topics.

Model-theoretic semantics, even when fortified with possible worlds, has encountered many difficulties in trying to capture ordinary intuitions about the reference of terms and the truth-conditions of sentences. Many of these difficulties are traceable to the use of set theory, which has often been considered to be a universally suitable formal framework (e.g., Cresswell, 1973). In fact, set theory per se lacks the conceptual resources for describing processes (Campbell &

Bickhard, 1986). (Even properties can only be captured in a cumbersome and unsatisfactory way, as functions from possible worlds to sets.) Specifically, model-theoretic possible worlds semantics lacks any means of characterizing mental processes. In consequence, "propositional attitudes" (such as knowledge, belief, and desire) have proven stubbornly intractable for this approach. Knowledge and beliefs must be externalized and reified - extensionalized - as sets of possible worlds or structures of possible worlds. Inconsistent beliefs have proven problematic, as has the fact that people do not know all of the logical consequences of their knowledge, or believe the logical consequences of their beliefs (Hintikka, 1969; Cresswell, 1973; Montague, 1974). The most recent attempts to deal with these problems (e.g., Barwise and Perry, 1983) continue to exclude mental processes from the ontology of their formal semantic framework.

The inability of model theoretic semantics to handle propositional attitudes is symptomatic of its "no epistemic process" encoding assumptions. The model theoretic approach to semantics is a formalization of static encoding assumptions, of the presumption of the passive epistemic agent, and possible worlds semantics is a particular version that attempts to deal with modality, intensions, etc. The incoherence problem emerges in these model theoretic approaches as soon as the attempt is made to account for the semantics of the *models* in terms of which the semantics of the *language* is supposedly rendered. Model theory is essentially a formalized, very sophisticated, version of the encoding stand-in relationship, and it suffers from exactly the same problem: the stand-in (satisfaction) relationship(s) must at some point have a foundation, and this foundation cannot itself consist of encodings (Bickhard and Terveen, in preparation). As such, it is clear that the interactive approach to language cannot be rendered in terms of standard model-theoretic satisfaction relationships. Utterances as operators do not encode anything, and certainly not structures of possible worlds.

A Revision of Semantics

Utterances do not encode structures of possible worlds, but they do operate on organizations of (representations of) possibilities - possible further interactions - and herein lies a potential similarity between Montague grammars and the interactive approach to language. Utterances operate on situation conventions, and situation conventions are constituted out of (relationships among) webs of indicated interactive potentialities - webs of interactive relevancies of the interactive implicit definitions of the persons involved in the situation. The key point here is that interactive relevancies indicate *possibilities for* ***potential*** *future interactions*. That is, implicit definitions can indicate interactive *possibilities*, and interactive relevancies functionally relate those interactive possibilities. In this sense, the realm of possibility is intrinsically present in the interactive approach, and, therefore, available to handle some of the broad concerns of Montague grammars, such as modality. Clearly, however, it will not do it in the same way.

Interactive Possibility and Possible Worlds. Unlike possible worlds semantics, the interactive model has no explicit encodings of possible worlds. Instead, there are implicit definitions of *realms* of possibility - realms of possible interaction - which realms become partially explicit in the web-like relational structures of interactive relevancies. Such relevancies function as partial characterizations of the implicitly defined realms of possibility represented by the interactive outcomes, but only as *partial* characterizations. There are never any *particular* 'worlds' represented here, only 'partial descriptions' (this term risks being misunderstood as a 'linguistically' encoded description) of *sets* of possible worlds. Furthermore, such 'sets of possible worlds' are given only implicitly, only in terms of the interactive 'intensions' or 'descriptions' of those sets, and never as explicit extensional sets. We share our emphasis on partial description (of implicitly defined and differentiated realms of interactive possibility) with Hintikka's conception of "model sets" (Hintikka, 1969). Unlike Hintikka, however, we do not represent possibilities via encoded sentences, nor do we impose logical consistency conditions on the implicit interactive representation of possibilities. In consequence, the realm of functional interactive possibility is different from the standard domain of "logical possibility."

The realms of possibility that are implicitly defined and partially characterized within the interactive model, unlike those in model theory, are not based on *objects* or *entities*. They are realms of *interactive* possibility - potentialities of further interaction. Representations of objects

and other entities of our more standardly familiar world are constituted as representations of various sorts of *invariances* of the patterns of such interactive potentialities (Bickhard, 1980; Piaget, 1954, 1971, 1977b, 1985, 1987), and, thus, are *derivative* forms of representation, not foundational forms. In this respect - the non-fundamentality of objects - the interactive approach induces a fundamental divergence from all of standard logic, including model theory, possible worlds semantics, and Hintikka's model set approach.[4]

Interactive possibilities, besides not being object-based, are organized differently than possible worlds. Interactive implicit definitions induce a progressive differentiation of the total realm of possible interactions. Implicit definitions are dual to differentiations (the differentiations among the various implicitly defined realms of potential interactions) and the induced differentiations are themselves recursively embedded within, crossed with, and indicative of, one another. The total realm of possible interaction constitutes the world, and the progressive differentiations constitute a differentiation structure within the representation of the world. The interactive perspective, then, entails an inherent organization of hierarchical differentiation relationships, and of indication-of-interactive-functional-accessability relationships, among the implicitly defined realms of interactive possibility. This organization *is* the organization of interactive 'accessibility' or 'reachability' or 'alternativeness' among realms of possibility. Possible worlds semantics makes use of such relations, without, however, providing any explication for them; they are just taken as given.

For possible worlds semantics, the current, actual, world is a particular privileged possible world. For the interactive approach the "actual world" is a current nexus within the web of interactive possibilities, and is only partially characterized by that web. The actual and the possible are differentiated *from* each other only *with respect to* each other in this view, rather than either one being foundational to the other. The *actual* is the current point in the reachability structure of the possible, while the possible is so only in terms of its reachability from the actual (Bickhard,1980).

Explicit Knowledge of Possibility and Necessity. For possible worlds semantics, how necessity and possibility come to be known is ignored. By contrast, the interactive perspective provides an intrinsic and natural approach to higher order considerations such as explicit knowledge of impossibility, necessity, relationships among extensions, and so on. That natural approach is in terms of the *levels of knowing*. The properties and relationships that are implicit in a given level of representation can be explicitly represented at the next higher level of knowing. The necessity of some particular relationship, for example, is a property of that relationship that cannot be represented at the same level as the relationship itself, but can be represented in the next level (Bickhard, 1988; Campbell and Bickhard, 1986). Similarly, the extension of a representation is a property of that representation that can be explicitly represented from the next level of knowing, and so on (Campbell & Bickhard, 1986).

In contrast, an encoding is *constituted* as *explicit* knowledge of what the encoding represents. The encoding has no other epistemic properties, and therefore certainly no implicit but inherent properties of its representational constitution (it may have contingent and epistemically irrelevant properties - e.g., color, weight, size, shape, location, or duration - derived from the particular 'thing' or 'event' that is being used as the encoding, but these have no bearing on the point being made; Bickhard and Campbell, 1989). Without such implicit and inherent properties of representation, encodings provide no foundation for anything like the levels of knowing: in the interactive perspective, the higher knowing level represents *explicitly* what was *implicit* in the next lower level, and if there is nothing implicit, as with encodings, then there is nothing for a next higher level to do. There is therefore no natural basis within an encoding perspective for distinctions between representing a relationship and representing the necessity of that relationship, or between knowing and knowing that one knows, etc. A classic illustration of this problem with encoding approaches is condition (C.KK*) of Hintikka's (1962, 1969) "epistemic logic," which equated knowing something and knowing that one knows it. By contrast, the interactive approach, with its hierarchy of levels of knowing, provides a totally natural way of modeling such distinctions as between knowing and knowing that one knows, and the representation and knowledge of such distinctions, both in cognition and in language.

Utterances as Operators on Possibilities. A fundamental difference between the interactive approach to language and the possible worlds approach is, of course, that, however represented and structured, the realms of possibility are not *encoded* by utterances but are instead *operated on* by utterances. The *realm* of possibility has a different character because of its being implicitly defined, based on interactive potentiality, inherently structured by differentiation, dualistically differentiated between actuality and possibility, and organized by knowing levels. The *relationship* between utterances and this realm of possibility is also different - it is a relationship of transformational operations rather than of encodings. That the operations are actually on situation conventions, which are constituted as *relationships* among individuals' interactive representations, introduces still another difference.

An Appropriate Formalism for Utterances as Operators. The consequences of these differences for the formal characterization of language are deep and complex. Yet they do not require a totally unrecognizable approach to that characterization. The interactive approach requires that utterances be considered as operations on underlying interactive representations. Those interactive representations are constituted as structures of indications of interactive relevancies. In the current literature, the formal approach most congenial to the interactive conception is algebraic logic. Model theory is based on the satisfaction encoding relationship to underlying sequences (of elements and sets, etc.). Algebraic logic is, in contrast, based on organizations of *operators* on such underlying sequences (Craig, 1974; Grandy, 1979; Henkin, Monk, & Tarski, 1971; Quine, 1966b). (It is interesting to note that Tarski, the originator of model theory, is also a major contributor to algebraic logic.) As such, it is a sort of operative version of standard encoding model theory. Just as standard possible worlds semantics is an extension of standard model theory, so also could there be an operative possible worlds semantics as an extension of algebraic logic. Such an extension would just begin to deal with the encoding to operative differences between the standard and interactive approaches. To accommodate to the further differences with respect to the representation of the realm(s) of possibility, the underlying *sequences* of elements and sets (of possible worlds) would have to be generalized to *relational structures* of indicators of interactive relevancies (among implicitly defined and differentiated realms of possibility). The algebraic operators on sequences of elements and sets of possible worlds, thus, become operators on relational structures of interactive relevancies (Bickhard, 1980).

The suggestion of the interactive perspective on language, then, is that the formal characterization of language would be better approached in terms of applications of algebraic logic instead of standard model theory, with the underlying sequences being replaced by underlying relational structures of indicators of interactive relevancies.[5] The incoherence of foundational encodings would thereby be avoided, while the power of the recursive constitution of categories and of the foundation of the realm of the possible would be retained. The extensions and revisions required are certainly non-trivial, but, unless the incoherence argument against foundational encodings is invalid, those extensions and revisions are necessary.[6]

Convergences and Applications

The discussion of the consequences of interactivism for language studies has of necessity been brief and mostly programmatic. There are, however, several illustrative applications of the interactive perspective to problems already in the literature and, similarly, convergences with points and perspectives already in the literature, that we would like to present.

Demonstratives. Studies of language are uncovering more and more characteristics that can be explained only when utterances are recognized as operations. The critical step of recognizing that it is the *essential* character of utterances to be operations is yet to be taken. An interesting example of this is provided by some analyses of demonstratives. To introduce this example, we will first develop a little further some of the concepts involved in the interactive model. Interaction *outcomes* differentiate the selection and course of *further* interactions within available *differentiation frames*. A differentiation frame is a hierarchically structured, possibly cross-connected, organization of potential differentiation *alternatives*. Differentiation frames may

be based on natural connections in the world, like the connection between the smell of smoke and the selection of actions appropriate to the possibility of a fire. They may be based on conventional connections in the world, like the connection between "dog" and the differentiation of a representation of a certain kind of animal. In Bickhard (1980) differentiation frames that participate in, and thereby constitute, situation conventions are called "semantic structures". The operative power of much of language functions with respect to and within such semantic structures. Some language elements, however, such as demonstratives, operate within differentiation frames that are more naturally present in the communication situation, e.g., "I" or "you." There *is* no (or very little) semantic structuring in these cases, in the sense that the differentiation frames are natural rather than conventional.

This lack of semantic structuring creates a problem for the propositional encoding approach. The presumed encoding "meaning" of a word is usually obtained by taking the operative differentiating effect - within a semantic structure differentiation-frame - to some context-independent encoding limit: by taking that which is context *de*pendently *differentiated* to be instead a context *in*dependent encoded element or concept. If "chair", for example, induces a differentiation with respect to "chairness" - e.g., picks out some representation in the current situation convention that is "chair-like" - then the standard approach will be to construe "chair" as *encoding* the "concept" of chair: the conventional principle of operative differentiation - the semantic structuring - is isolated and reified into an encoding. Without any semantic structuring, demonstratives would, in this sense, have no "meaning" - there is no determinate, context independent, semantic or representational outcome of the differentiation, therefore the move to an encoding rendering of that representational outcome is blocked. The meaning-as-differentiation-outcome of demonstratives is not capturable in terms of an encoded concept or a set of encoded properties. Because there is no semantic structuring to Procrusteanly render in encoding terms, the context *de*pendence of such words is unavoidable.

This has led, for example, to Kaplan's (1979) proposal to introduce two new varieties of Fregean sense: content and character. *Content*, roughly, is the situation convention consequence of a (demonstrative) utterance (though it is discussed in terms of encoded 'concepts'); it is the "proposition" determined by an utterance in a particular situation. *Character*, on the other hand, is that property of an expression which *determines* the content for a given context. Character is the *function* from context to content - character is the operative power.

Clearly, the operativity of language is forcing itself on Kaplan's analysis, but it is still seen only in limited ways: "The distinction between character and content was unlikely to be noticed before demonstratives came under consideration, because demonstrative-free expressions have a constant character, i.e., they express the same content in every context. Thus character becomes an uninteresting complication in the theory." (Kaplan, 1979, p. 404) From an interactive perspective, the assumption that *any* utterance has "a constant character" (a constant-function operative power) is in error: even "proper names", such as John or Mary, can easily be ambiguous and equivocal - they context *de*pendently differentiate; they do *not* encode.

This particular model of Kaplan's has been generalized (e.g., Richard, 1983), and similar partial encounters with context-dependent operativity are to be found throughout the literature (e.g., Cresswell, 1973), but they remain partial. The inherent operativity, and consequent inherent context-dependency, of language has not been generally recognized.

Pronouns. Another illustration of the power of the interactive, operative perspective on language can be derived from some problematic cases of pronouns. Utterances effect context dependent differentiat*ions* within a complex organization of context dependent differentiat*ors*. This double context dependency does not manifest itself in all language, but it does in some circumstances emerge explicitly. Partee (1972) presents a series of cases, each one deeply problematic for one or more apparent approaches to the meanings of pronouns, and collectively constituting counterexamples to all considered approaches. A serious problem for many models of pronouns arises with sentences in which the pronoun is not co-referential with its antecedent. From an encoding perspective, it is difficult to make sense of such deviance - reference is taken as the use of an encoding relation, and pronouns, at least superficially, would seem to involve re-

use of the same encoding. In this view, they should always be coreferential with their antecedents. An alternative approach treats pronouns as bound variables: when the antecedent contains a quantifier or a word like *only*, a bound variable approach seems difficult to avoid. For example,

> "*No one* would put the blame on *himself*."
>
> "If *anyone* had been there, I would have seen *him* (*them*)." (p. 430)

But there are cases that resist both a coreferentiality approach and a bound variable approach. One example is,

> "The man who gave *his paycheck* to his wife was wiser than the man who gave *it* to his mistress." (p. 434)

Partee concludes that such cases must be treated as pronouns of laziness - the pronoun simply substitutes for saying the antecedent again.

The intrinsic multiple context dependence of language, however, that interactivism reveals, is all that is being manifested here. "*his paycheck*" itself evokes a *context dependent* differentiator, which, in this context within the sentence, differentiates (a representation of; a differentiator of) the wise man's paycheck, while "*it*" evokes a differentiation of the *same* context dependent differentiat*or* as its antecedent, which, in this *differing context* within the sentence, now differentiates (a representation of) the less wise man's paycheck.

In cases in which coreference seems appropriate, e.g.,

> "John lost *a black pen* yesterday and Bill found *it* today." (p. 421)

the double context dependence is not manifest. Further examples that are quite natural in terms of the invoking of an antecedent context dependent differentiation operator are ubiquitous:

> "*My home* was once in Maryland, but now *it's* in Los Angeles."
>
> "John thinks *my home* is in Maryland, but Bill thinks *it's* in Los Angeles." (p. 425)
>
> "We need *a secretary* and we need *her* soon." (p. 429)
>
> "John couldn't catch *a fish* if *it* jumped into his lap." (p. 432)

In cases such as these, failure of coreferentiality is clear. The evoking of the antecedent's context dependent differentiator in the new context of the *pronoun* makes clear sense of the first two sentences, while the second two turn on the open endedness of the differentiations that are invoked in the *antecedents* - in effect, the antecedents involve generalized invocations of a differentiator without the context within which the differentiation could be taken as yielding a particular referent.[7]

Logics of Abilities and Actions. So far, our examples have all elaborated the context dependencies intrinsic to differentiation operators. The next examples connect with the sense in which interactivism provides a model in which *organizations of interactive possibilities* are indicated, and, thus, 'worlds' that satisfy those possibilities are implicitly defined, but the differentiations are in all cases only, and necessarily only, partial - the differentiations are of implicitly defined, partially characterized, *realms* of possibility, not of particulate or singular possible worlds, nor of classes of particulate possible worlds.

Brown (1988, 1990) provides an analysis of the logic of abilities. The critical point of this analysis for our purposes is that there are cases in the logic of abilities that cannot be defined with

respect to particulate possible worlds, but that *require* representation of realms of partially described possibilities. That is, there are cases in which it is necessary, even given standard approaches, to take "realms of possibility" or "clusters of possible worlds" as primitive, rather than individuated possible worlds per se. Consider, for example, John's ability in dart throwing. We may have "John can hit the dart board." and, therefore, "John can hit the red or the black.", but not at all have "John can hit the red or John can hit the black." Or, from a randomized face-down deck of cards, I certainly have the ability to draw a red or black card, but I do not have the ability to draw a red card nor do I have the ability to draw a black card. For still another example, consider a blocks-world robot that can distinguish "red or yellow" from other colors, but cannot discriminate red from yellow: this robot has the ability to pick up a "red or yellow" block, but does not have the ability to pick up a red block nor the ability to pick up a yellow block (Brown, 1990).

The ability does not distribute over the constituents; the realm of possibility cannot be arbitrarily further differentiated, and certainly not down to particular possible worlds. This shows that, at least in some cases, possibilities can *only* be partially differentiated, and cannot be treated in terms of (sets of) singular possible worlds. From another direction, Hintikka's model sets show that logics *requiring* specific, particulate or atomistic, possible worlds are never necessary. There is a convergence between these considerations in interactivism's notion of hierarchically and intersectingly differentiated realms of interactive possibility, in which the differentiations are necessarily finite, and, therefore, can never differentiate atomic possible worlds. The *fact* is that we never differentiate an atomic possible world; the *argument* is that that is an intrinsically necessary fact.

The convergence between Brown's logic of ability and the interactivist model is even closer than that they are both committed in a general sense to minimal models in terms of clusters or realms of possibility. Brown's formal semantics replaces the usual *accessibility* relationship between possible worlds with a *relevance* relationship from worlds to clusters of possible worlds. A *relevant* cluster corresponds to an action that the agent is *able* to perform; the action, if performed, will bring it about that the partial description of the cluster, what is common across the cluster, will be satisfied (Brown, 1990). This is *exactly* the notion of relevance of Bickhard (1980, and above).

Brown (1990) extends this logic of ability to a logic of action, which again requires a semantics in terms of clusters of possible words, realms of possibility, rather than in terms of atomic possible worlds. The move to clusters is required, for example, in order to be able to specify an action without having to specify all of its accidental accompaniments: the action is committed to the partial description that holds across the cluster - that is common to the cluster - but is not committed to any of the varying details within the cluster. Conversely, what is *reliable* about the action across the entire cluster of possibilities is that it accomplishes bringing it about that that partial description holds across, and in spite of, all of the varying details within the cluster.

Brown's logic is also for the first time able to model a critical relationship between action and ability: that if I *do* bring it about that **A**, then I am *able* to bring it about that **A**. The appropriateness of this derivation is dependent on the sense in which the logic of action - of *do* - involves a *reliability* of action, therefore, the exercising of an *ability*. Capturing that reliability of action is in turn dependent on the essential role of clusters of possibility in the semantics, as indicated above. Again, realms of possibility, instead of atomic possible worlds, are not only *sufficient* for standard applications, they are *necessary* for various critical cases. Both the *fact* of the reliance on partially described realms of possibility, and the *manner* of that reliance in Brown's formal semantics, are convergent with the interactive model.[8]

Broader Convergences. We turn now to some broader convergences between interactivism and other literature. For example, context-dependencies are not the only aspect of operativity that has been partially recognized in language studies. Speech act theory focuses on the 'action' inherent in an utterance (e.g., Austin, 1962; Searle, 1969), but it is still an action (a message transmission, not an *inter*action) based on an encoded proposition. Grice's model (1967, 1969, 1971; Shiffer, 1972) recognizes the intentionality, the goal-directedness, inherent in an utterance, but his model is not (socially) interactive or operative, and it too depends on the

concept of an encoded proposition (see Bickhard, 1980, for more elaborated discussions of Austin and Grice). Wittgenstein (1958) had deep insights into the flaws of encodingism, and he recognized that language was intrinsically social and functional, but there is no social operativity in his account either (Bickhard, 1987). Hermeneutics notes that understanding is historically context-dependent and interpretive in a non-encoding manner (the "hermeneutic circle"), but it has no sense of utterances as operators, nor of the distinction between such operators and the social structures of representations upon which they operate (Heidegger, 1962; Gadamer, 1975, 1976).

The convergence between interactivism and hermeneutics is even stronger when it is recognized that an utterance cannot *encode* an organization of operators any more than it can encode a proposition. Utterances must be apperceptively *interpreted* for the operations they select - potentially in an open, multiply constrained, non-algorithmic, problem solving manner: the hermeneutic circle. Timeless descriptive formalisms, then, even of suitably modified algebraic logic operators on relational structures of implicitly defined realms of interactive possibilities, can only be approximations. The realities intrinsically, not just contingently, involve the iterative and progressive variations and selections of mental process (Bickhard, 1988). Ultimately, language is intrinsically temporal and creative.

Such partial recognitions of the social operativity of language are widespread, but they cannot be fully developed and integrated without a full recognition of the interactive character of language, and that is not possible within standard approaches. In formal approaches to language, such interactive properties of language are walled off in "pragmatics", and either ignored, or treated as a separate component to be added to the encoding-based truth-conditional semantics. The types of suboperators that constitute utterances cannot be analyzed or understood except in terms of the (social structures of) interactive representations on which they operate; they cannot be understood except with respect to their operative power and potential consequences. That means that the usual subdomains of syntax, semantics, and pragmatics in language studies cannot be separated within the interactive perspective. The division of language studies into syntax, semantics, and pragmatics already makes a deep implicit commitment to the encoding approach, and, correspondingly, *any* approach to language that accepts that division cannot be compatible with the interactive perspective.

Conclusions

The encoding approach to representation is incoherent and ultimately untenable (Bickhard, 1980, in press-a, in press-b; Bickhard & Richie, 1983). This holds true for cognition and perception, and for the encoding assumptions inherent in standard approaches to language (Bickhard, 1987).

We have examined a number of the consequences of that incoherence, have outlined an alternative approach, and have traced several of the consequences of accommodating to the interactive alternative. In particular, we have argued that in developing a *formal* interactive approach to language, algebraic logic is a most relevant available technical tool. It provides a partial guide to the interactive characteristics of language, though in its standard form it still makes strong encoding assumptions about the objects of the algebraic operators. The formal relationships between algebraic logic and standard model theory are a partial formalization of some of the relationships between encoding and interactive perspectives, and in this sense these relationships provide even more of a guide to the development of valid encoding insights in an interactive direction. Interactivism encompasses encodingism as a special limiting case, both logically and formally (Bickhard & Richie, 1983). In this regard, it should be noted that even standard model theory can be rendered in interactive, differentiation-based, terms (Resnik, 1981).

The changes that interactivism suggests for categorial and possible worlds approaches to language are not trivial. They are radical, transforming the conceptual foundations of representations and utterances as encodings. The formal consequences of those foundational changes are not simple to foresee. One consequence that is clearly entailed by the interactive

approach, however, is that formal linguistics can no longer maintain its isolation from epistemology and from psychology. At the empirical level, details of the categories in an interactive categorial grammar may well depend on psychological theories about different kinds of mental representations, on psychological research about human cognition, and on both intrinsic and historically contingent properties of situation conventions. Similarly, the specifics of how interactive possibilities are differentiated is also an empirical psychological question. At the formal level, the exploration of the formal properties and possibilities of operators on functional webs of interactive relevancies is just beginning, and constitutes a formal analysis of knowledge and language as *psychological* phenomena. Contrary to popular opinion, including among psychologists, formal analysis of necessary characteristics and constraints *is* possible in psychology, is needed, is happening (Bickhard, 1980, in press-b; Campbell & Bickhard, 1986), and is highly productive. Understanding utterances as operators on situation conventions - as *essentially* pragmatic actions (Bickhard, 1987) - thus, ties linguistics to psychology, social psychology, and sociology, at both the empirical and the formal levels of analysis.

A rejoinder to our claims would, of course, be that formal approaches to language merely aim at the best formal *description* of some aspect of language, and explanatory concerns can be left to psychologists and perhaps philosophers. The relationship between descriptive and explanatory theories is too complex for us to address here (see Campbell & Bickhard, 1986). However, a descriptive approach that makes fundamentally false presuppositions about its subject matter cannot yield an adequate description, much less have any explanatory value. As we have shown, not only is standard formal linguistics permeated with incoherent assumptions about knowledge, but the very division of the field into syntax, semantics, and derivative pragmatics depends on these assumptions.

In psychology, rote memory tasks (paired associates, free recall, serial recall, etc.) were for a long time considered to assess a *unitary* psychological phenomenon. An entire subdiscipline of "verbal learning" arose to study the supposed unitary phenomenon of "rote memory." Investigations into the procedures used to do these tasks showed, however, that different individuals had radically different strategies that they used on these tasks, and the same individual could use different strategies on different tasks. From the standpoint of the mental processes involved, there was no such thing as rote memory. The study of "verbal learning" no longer exists. Even purely descriptive accounts have to attend to their presuppositions.

Whether the interactive approach to language that we have outlined should still be called a categorial approach is a matter of convention, depending on the similarities and differences that seem important to differentiate. What does seem clear is that encodingism, in all of its versions, involves a deep and ultimately fatal incoherence, and that a categorial flavored, algebraic logic inspired, approach to language in terms of "operators on webs of relevancies among realms of possibility" is the available formal approach that is most amenable to the interactive alternative. If the interactive approach is correct, then pragmatics is not only a matter of the *use* of language, and a source of *constraint* on syntax, but functional social operativity is the fundamental *nature* of language (Bickhard, 1987) - and, therefore, of language meaning *and* language structure.

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Footnotes

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[1] This is similar to C. S. Pierce's intrinsically triadic nature of representation. For Peirce, a sign (of which there are three kinds: the icon, the index, and the symbol) requires the triad of 1) the sign itself, 2) the object of the sign, and 3) the interpretant of the sign. Peirce's *interpretant* is the *idea* that a sign "determines" in a person's - an interpreter's - mind (Almeder, 1980; Hookway, 1985). The convergence with the interactive view of representation is - in our language, not Peirce's - a recognition of the *functional* character of representation, and, therefore, of the necessity that a representation be a representation *for an agent*.

Unfortunately, he develops this basic insight within his own unique version of the encodingist conception of representation that we wish to argue against. This criticism cannot be adequately developed here, but it is perhaps most clearly seen in Peirce's model of perception: He argued against sense data theories, but only in the sense that we do not *see* sense data - instead we "directly" see ostensible objects ("ostensible" in the sense that perceptual judgements are fallible, e.g., hallucinations). Such perceptions for Peirce, however, are still the products of unconscious inference on sense qualities. For Peirce, perceptions *obtrude* themselves on the person - rather than perceiving being an *activity* of the person (Bickhard and Richie, 1983). This is clearly a sensory encoding model of perception.

More broadly, Peirce's encodingism manifests itself, for example, in his focus on representational *elements* - signs - that represent by virtue of the interpreter *knowing what they represent* - having an interpretant. As stated in the main text, this conception of representation is *definitional* of encodingism. Recognizing that pragmatic functionality is the ground of representation is a shared convergence between interactivism and Peirce, but the respective models of *how* representation emerges from that functional ground are radically different.

[2] From the interactive perspective, the standard encoding perspective makes two related errors. First, it takes the operativity of language to the limit of a constant function - a function which gives the same result for all possible arguments - thereby eliminating all context dependency, which then must be put back in with various ad hoc devices. Second, it identifies the function, or operation, with its result, thereby making that functional operator an encoding of that result. This misidentification of an operator with its result is, of course, much easier to make if the function is taken as constant in the first place - then there is only one possible result with which to identify it.

[3] In this view, constructing an utterance is similar to constructing a function in recursive function theory (Rogers, 1967; Eilenberg and Elgot, 1970). The basic generating relationship, however, is inverted. Recursive function theory deals with the recursive construction of an infinite class of functions out of a small set of generating functions. The construction of an utterance involves the composition of an element of one single class of full operators - the class of sentences - out of a potentially very large set of differentiated suboperators. There is also the difference that the recursive composition of new functions in recursive function theory generates new functions of one single type, while the recursive differentiation of categories of sub- or partial operators generates new (sub)types of operators, as does, dually, the composition of elements of those subtypes in the construction of a sentence. Thus, there is recursion involved both in the differentiation of the suboperator types, and in the composition of utterances out of elements of those types.

[4] There are strong arguments, in fact, that such an object-based ontology as is found in standard logic inherently commits one to an encoding epistemology. An object-based ontology requires a

representational foundation that is based on representations of those basic objects - there is no 'deeper' level that could be represented - and those basic representations must be encodings because they must be defined in terms of their property of representing those basic objects, and any representation defined in terms of what it represents *is* an encoding.

Note that this non-object based ontology of the interactive approach makes standard correspondence theories of truth *impossible* within the interactive perspective.

[5] A further consequence of the interactive approach is that the elements of such structures of indicators cannot themselves be considered to be encoded, but rather exist only as points in the functional organization of the indications. Operators on such structures, then, cannot simply "name" a point or region of the relevancy structure to be operated on - since they have no properties other than those functional locations, no properties by which they could be "named" - but must instead *differentiate* such a point or region beginning with the current focus within that organization as the initial state from which such differentiations can proceed. The fundamental form of operations, then, will be various types of such differentiations (Bickhard, 1980).

[6] One deep consequence of encodingism is the unavoidability of scepticism. The incoherence argument, in fact, is just a version of the classic sceptic's argument. The failure to resolve or dissolve the problem of scepticism over centuries of attempts certainly suggests that encodingism is fatally flawed. The interactive approach, on the other hand, does not lead to scepticism. Interactivism holds that the sceptic's argument is fully valid - but it only applies to foundational encodings, not to interactive representations (Bickhard, 1987, in press-b).

[7] *Hilbert's Calculus*. A similar open endedness of differentiation or selection is found in Hilbert's ‰ calculus. For a definition, we have: If (Ex)Fx, then '‰xFx' selects one from amongst the F's, while if -(Ex)Fx, then '‰xFx' selects one thing from the universe at large (Slater, 1988). Note that '‰xFx' is guaranteed to have a reference - to select something. Using this notion of ‰, the quantifiers can be defined as follows:

Since F(‰xFx) iff (Ex)Fx, therefore

-F(‰xFx) iff (x)-Fx and

F(‰x-Fx) iff (x)Fx.

Thus, the ‰ calculus provides a foundation for quantificational logic.

Slater argues that ‰ reference provides philosophically powerful and superior solutions to several problems of reference, including those involved in definite descriptions and fictions. Among other positive characteristics, Slater shows that the ‰ approach to definite description does not make the error of "The properties mentioned in a definite description are the properties that constitute an ontological guise, whereas the properties mentioned in a predicate are external to the guise. This distinction is not made in Russell's analysis" (Castañeda, 1977, p. 317).

We do not agree with all aspects of Slater's analyses, but, for our purposes, the critical points to note are 1) that ‰ reference is powerful, philosophically interesting, and potentially a solution to a number of difficult problems, and 2) ‰ reference functions exactly as a *context dependent differentiator*, one that differentiates within a specified category if such is available, but that differentiates, that operates, and yields *something*, in whatever context it is invoked.

[8] *Predicates of Varying Degree*. A still different sort of convergence between the interactive approach and current literature derives from an aspect of the differentiating character of operators other than their intrinsic context dependence - in fact, from a kind of "extended context" *openness* or *in*difference. Since language operators differentiate 'from the top down', and are indifferent to the numbers of *further* possibilities (further alternatives) involved in a given differentiation possibility, so long as the differentiating operator *can* in fact function in the relevant differentiation frame - since, in other words, the differentiation operators operate *locally* - it follows that the interactive approach, in a strict sense, yields predicates and relations of *varying degree*. That is, predicates and relations that can take *varying numbers* of subjects and relata, instead of the classical case in which each distinct integer n must correspond to a different class or *type* of predicate or relation. Still further, since it is possible for arbitrary further differentiation frames to be embedded 'within' any particular option in a given differentiation frame, the model does not yield in any natural way a syntactically typed theory - although the knowing levels do yield a complex type theory based on 'semantics', representational power (Campbell and Bickhard, 1986).

Such characteristics converge strongly with Grandy's anadic logic (Grandy, 1979) and Mundy's generalization of categorial logic (Mundy 1989), both of which allow predicates and relations of variable degree. The convergence is stronger with Mundy's system in that it also allows open ended *set* and *sequence* relations, as does interactivism, since, as indicated above, arbitrary set and sequence representations might be embedded within a given differentiation frame. Mundy provides strong arguments both for the naturalness of the variable-degree generalization of categorial logic per se, including some notes concerning the historical accidents that have yielded some of the generalizations of classical logic that are more familiar, and for the power of this generalization in the formulation of a logic of quantity.