Coupling between two cylindrical light pipes: a design

M. D. Wagh

An expression for the transmission characteristics of a conical coupling is derived. This expression is shown to be useful for choosing an optimum coupling length.

I. Introduction

Cylindrical light pipes have been in use for the last few years in the transmission of ir radiation for absorption spectroscopy. They also find use in optical information processing and communications. Light pipes generally have specularly reflecting walls and have dimensions that are large compared to the wavelength of radiation carried by them. Geometrical optics is therefore good enough to describe the transmission characteristics of these pipes. These characteristics were discussed first by Ohlmann and his co-workers[1] in 1958 and later by Lin and Sparrow[2] in 1965. The problem of coupling two cylindrical light pipes of different diameters by means of a simple conical coupling with specularly reflecting walls is discussed in this paper. It should be mentioned here that although the conical geometry of the light pipe has been studied as early as 1952 by Williamson[3] for concentrating radiation into a small area, his studies have not extended either to skew rays or to cones with wall reflectivities less than unity. Recently Powell[4] calculated the transmission characteristics of light pipes by taking into account the wall reflectivities, but he has not extended his analysis to conical couplings.

II. Analysis

The analysis in this paper is restricted to the radiation parallel to the optical axis of the system shown in Fig. 1. In practice the radiation angle with the axis rarely exceeds a few degrees, and therefore the results obtained here are directly applicable to practical cases.

It is further assumed that the entrance of the coupling is uniformly illuminated. Let the entrance and exit diameters of the coupling be $2a$ and $2b$, respectively, and its semiangle be denoted by ψ. If $b > a$, all the incident radiation will cross the coupling, and no analysis is needed. If, however, $b < a$, some radiation may be reflected back. The fraction of the incident radiation that is carried by the coupling to the other end is defined as the efficiency η of the coupling, and an explicit expression for calculating it is derived in terms of the coupling wall reflectivity ρ.

From the circular symmetry of the structure and the incident radiation, we get

$$\eta = \frac{1}{\pi a^2} \int_0^a \delta_r 2\pi r dr, \tag{1}$$

where $\delta_r = \rho^n$ if a ray entering the coupling at a distance r from the axis crosses the coupling in n reflections. $\delta_r = 0$ if the ray is not able to cross the coupling.

From the similarity with an earlier analysis[5] one finds that the ray is able to cross the coupling in n reflections iff the maximum axial distance it is able to cover in n reflections, z_{n+1}, is related to the coupling dimensions through

$$z_{n+1} \geq (a - b)/\tan\psi, \tag{2}$$

where

$$z_{n+1} = \frac{a}{\tan\psi} \left((1 - \frac{1}{T_n} \right) + \frac{z_1}{T_n}, \tag{3}$$

$$T_n = \sin(\overline{2n + 1}\psi)/\sin\psi, \tag{4}$$

$$\text{maximum value of } n = [\pi/4\psi], \tag{5}$$

and z_1 is the axial distance of the first reflection of the ray from the coupling entrance. From elementary geometry z_1 can be obtained as

$$z_1 = (a - r)/\tan\psi. \tag{6}$$

Equations (2), (3), and (6) together give the condition for the ray to cross the coupling in n reflections as

$$r \leq bT_n. \tag{7}$$

Therefore for r satisfying

$$bT_{n-1} < r \leq bT_n,\ \delta_r = \rho^n. \tag{8}$$

Substituting Eq. (8) in Eq. (1), one gets

The author is with the Indian Institute of Technology, Department of Electrical Engineering, Bombay 400076, India.

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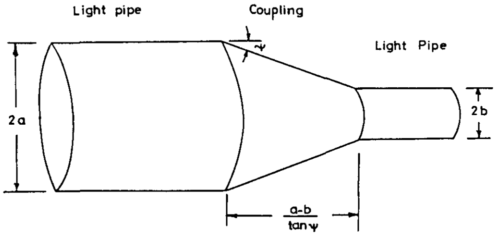

Fig. 1. Conical coupling between two cylindrical light pipes.

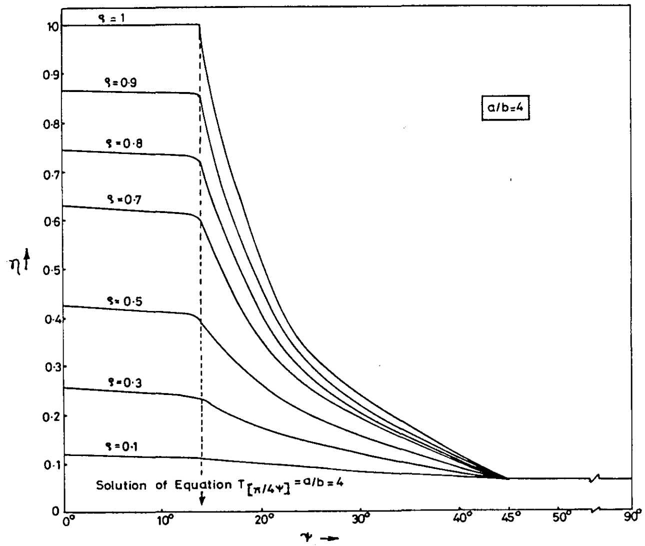

Fig. 2. Plot of efficiency of the coupling η against the semiangle of the coupling ψ for various values of wall reflectivity ρ.

$$\eta = (b/a)^2 + \sum_{n=1}^{[\pi/4\psi]} A_n \cdot \rho^n, \tag{9}$$

where A_n is the fraction of the entrance area of the coupling. Rays starting from area A_n undergo exactly n reflections, before coming out of the coupling. An expression for A_n is given by

$$\begin{aligned} A_n &= (b/a)^2(T_n{}^2 - T_{n-1}{}^2) \text{ if } bT_n \le a \\ &= (b/a)^2(a^2 - T_{n-1}{}^2) \quad \text{if } bT_n > a \ge bT_{n-1} \\ &= 0 \qquad\qquad \text{if } bT_{n-1} > a. \end{aligned} \tag{10}$$

Using Eqs. (4), (5), (9), and (10), efficiency of the coupling, η can be calculated. Figure 2 gives a plot of η against the coupling semiangle ψ for various values of wall reflectivities.

III. Conclusions

If $\rho = 1$, one gets from Eqs. (9) and (10)

$$\eta = \min\{1, (b/a)^2 T^2{}_{[\pi/4\psi]}\}. \tag{11}$$

So that if ψ satisfies the condition

$$bT_{[\pi/4\psi]} \ge a, \tag{12}$$

$\eta = 1$. It has been shown[5] that $T_{[\pi/4\psi]}$ is a monotonically decreasing function of ψ, and it tends to ∞ as $\psi \to 0$. For a given pipe dimensions ratio a/b, it is therefore always possible to satisfy Eq. (12) by choosing ψ small enough. Choosing ψ smaller than the solution of

$$bT_{[\pi/4\psi]} = a \tag{13}$$

does not increase the efficiency η when $\rho = 1$. Moreover, even when $\rho < 1$, decreasing ψ beyond the solution of Eq. (13) increases η only marginally.

The solution of Eq. (13) thus gives an ideal coupling semiangle ψ and thereby its length $(a - b)/\tan\psi$. This solution for various a/b ratios is plotted in Fig. 4 ($\phi = 0$ curve) of Ref. 5.

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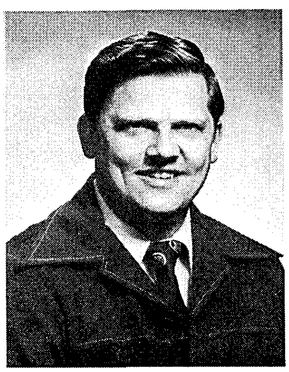

R. B. Jaeger
Galileo Electro-Optics Corporation