Right angle bends in light pipes: analysis

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An expression for the fraction of incident radiation transmitted by a right angle bend in a rectangular light pipe is derived. This expression is used to show that, in a planar light pipe structure with right angle bends, light attenuation is due only to reflection losses at the walls if the launching angle is 45°.

I. Introduction

The transmission characteristics of rectangular light pipes with specularly reflecting walls have been studied earlier by Poehler and Turner[1] and Powell.[2] However, both have dealt only with straight pipes, and the former have not considered skew rays. In many signal processing and optical communication problems, bends in the light pipes are unavoidable. The effect of bends on the flow of light energy through the pipes has not been reported thus far. It seems natural to define bend efficiency as the fraction of incident radiation transmitted through the bend. An explicit expression for the efficiency of a right angle bend with perfectly reflecting walls is derived in this paper. It is found that among other factors, it also depends on the launching angle. Therefore, the expressions derived in this paper can be used to choose the optimum launching angle.

II. Analysis

Consider a right angle bend in the light pipe as shown in Fig. 1. a and b are the widths of the entrance and exit legs of the pipe. The only possible paths of a ray $PQR(S)$ whose last reflection from the wall OPB is at P are sketched in this figure. It is obvious from the inspection of these paths that $\alpha \geq 2a$ is a necessary and sufficient condition for the ray to be reflected in the exit leg of the bend.

For the purpose of analysis, we attach a Cartesian coordinate frame to the entrance of the pipe as shown in Fig. 2. We assume that the entrance of the pipe is uniformly illuminated with parallel rays and try to find the fraction of this illumination that passes through the bend.

We shall first assume that the rays at the entrance of the tube are inclined away from the exit leg of the tube. It will be shown later that the other case of rays inclined toward the exit leg gives rise to identical formulae with minor changes.

Using Powell's method,[2] the opening of the tube can be imaged on a plane $z = L_o + 2b$, where L_o is the z coordinate of point B. As all the rays are parallel and travel with a direction specified by angles θ and ϕ, a point (x_o, y_o, O) will be projected to

$$[(L_o + 2b)\tan\theta\cos\phi + x_o,\ (L_o + 2b)\tan\theta\sin\phi + y_o,\ (L_o + 2b)]$$

in the plane $z = L_o + 2b$. If the last reflection of this ray, which enters the tube at (x_o, y_o, O) on wall OPB, is its

$$t_{x_o}\text{th}$$

reflection (on OPB and its parallel wall), then it is easy to see that

$$\alpha = (L_o + 2b)\tan\theta\cos\phi + x_o - t_{x_o}a, \tag{1}$$

where t is given a suffix x_o to emphasize its dependence on x_o.

To find

$$t_{x_o},$$

we note that it has to be an even integer as the rays are inclined away from wall OPB at the entrance. If we image again the opening of the tube on plane $z = L_o$, the x coordinate of the projection of (x_o, y_o, O) is $L_o \tan\theta \cos\phi + x_o$. The largest even integer not larger than $(L_o \tan\theta \cos\phi + x_o)/a$ will therefore be the value of

$$t_{x_o}.$$

We uniquely define an integer n by

$$n \leq \frac{L_o \tan\theta \cos\phi}{2a} < (n + 1) \tag{2}$$

The totality of rays entering the pipe can now be divided into the two cases shown below. Case 1 is all the rays that start from (x_o, y_o, O), where

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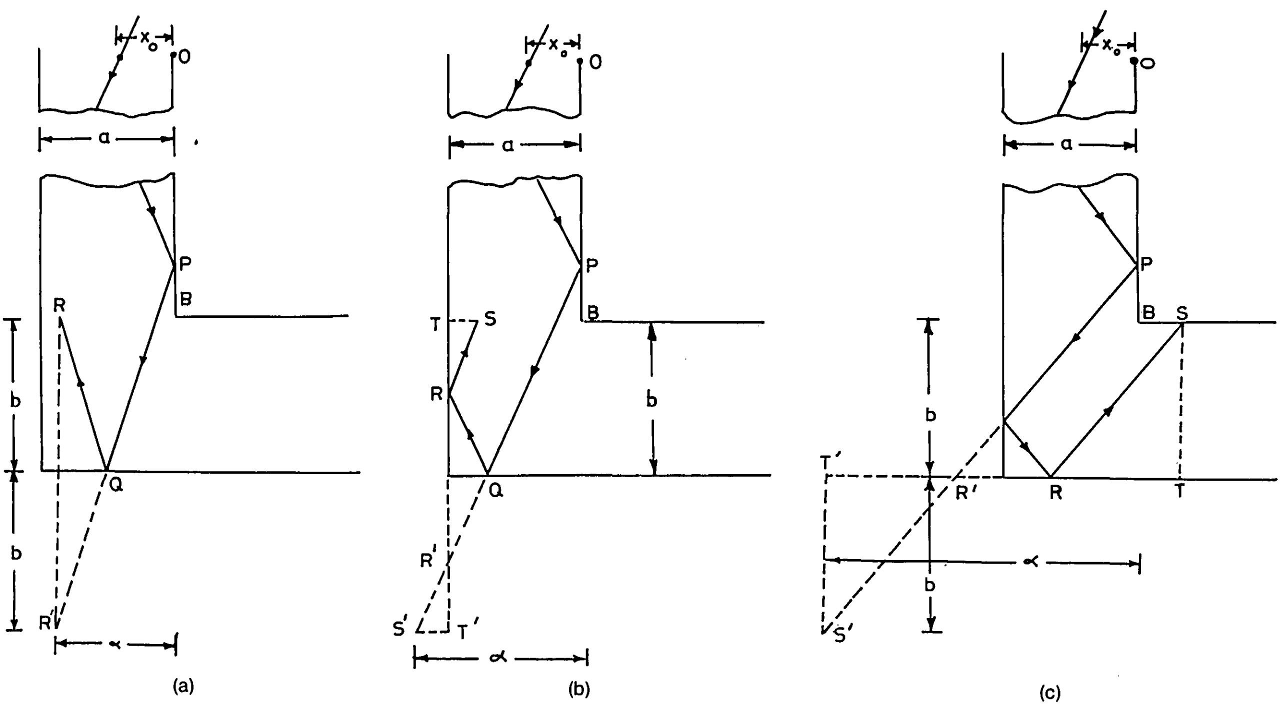

Fig. 1. Three possible paths of a light ray whose last reflection on wall OPB is at P.

$$2na \leq L_o \tan\theta \cos\phi + x_o < 2(n + 1)a. \quad (3)$$

For these rays,

$$t_{x_o} = 2n.$$

Therefore, from Eq. (1), the condition $\alpha \geq 2a$ can be translated to

$$x_o \geq 2(n + 1)a - (L_o + 2b) \tan\theta \cos\phi. \quad (4)$$

Conditions (3) and (4) can be written together as

$$L_1 \leq x_o < L_2, \quad (5)$$

where $L_1 = L_2 - 2b \tan\theta \cos\phi$ and $L_2 = 2(n + 1)a - L_o \tan\theta \cos\phi$. Since $O \leq x_o \leq a$, $L_1 < L_2$, and $L_2 > O$ [from Exp. (2)], the range of values of x_o satisfying Exp. (5) is

$$T_1 = \max[O, \min(L_2, a) - \max(L_1, O)]. \quad (6)$$

Case 2 is all the rays not satisfying condition (3), i.e., rays that start from (x_o, y_o, O), where

$$2(n + 1)a \leq L_o \tan\theta \cos\phi + x_o < 2(n + 2)a. \quad (7)$$

For these rays,

$$t_{x_o} = 2(n + 1);$$

and, as before, condition $\alpha \geq 2a$ can be translated to

$$x_o \geq 2(n + 2)a - (L_o + 2b) \tan\theta \cos\phi. \quad (8)$$

Conditions (7) and (8) can be written together as

$$x_o \geq L_2 \text{ and } x_o \geq L_3, \quad (9)$$

where $L_3 = L_1 + 2a$. Again, since $L_2 > O$, the range of values of x_o between O and a satisfying Exp. (9) is

$$T_2 = \max[O, a - \max(L_2, L_3)]. \quad (10)$$

Equations (6) and (10) give two disjointed [as conditions (3) and (7) are mutually exclusive] ranges of x_o for which $\alpha \geq 2a$, i.e., the ray starting from (x_o, y_o, O) is reflected into the exit leg of the bend. We can now define the efficiency η of the bend as the fraction of the incident radiation that is transmitted across the bend. Clearly,

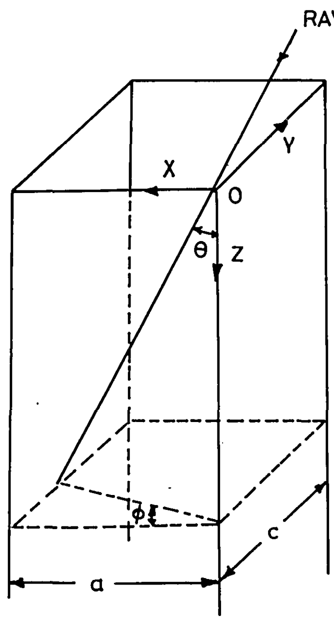

Fig. 2. Light pipe geometry, coordinate frame, and the light ray characterization.

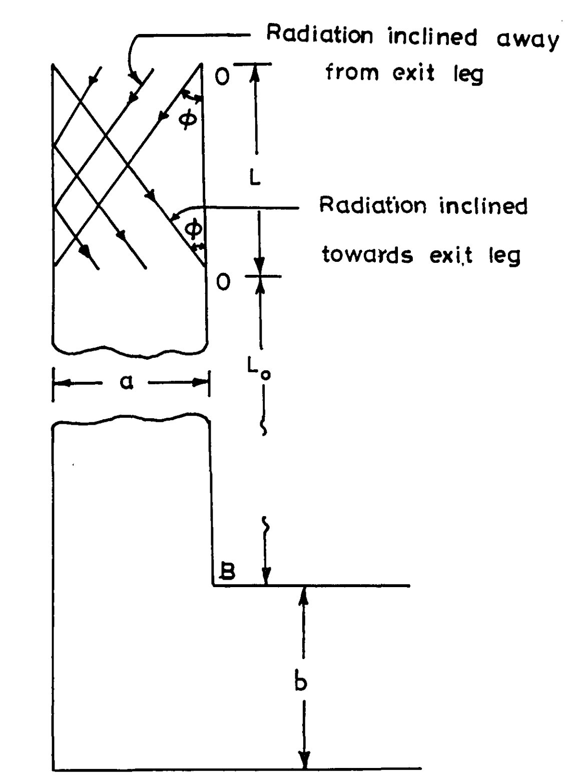

Fig. 3. Increasing the length of the entrance leg to change the inclination of the incident rays.

$$\eta = \frac{1}{a}(T_1 + T_2)$$
$$= \frac{1}{a}\{\max[O,\ \min(L_2, a) - \max(L_1, O)] + \max[O, a - \max(L_2, L_3)]\}. \quad (11)$$

If the incident radiation is inclined toward the exit leg, then

$$t_{x_o}$$

is odd; and the results developed so far do not apply. However, an inspection of Fig. 3 reveals that if we increase the tube entrance leg length by L so $L \tan\theta \cos\phi = a$, then this new tube can be treated with methods developed earlier because now the incident radiation is inclined away from the exit leg.

Thus for incident radiation inclined toward the exit leg, Eq. (11) can still be used to find out the bend efficiency, provided L_o is replaced by $L_o + L$, where

$$L \tan\theta \cos\phi = a. \quad (12)$$

We have to determine n in this case from

$$n \le \frac{L_0 \tan\theta \cos\phi}{2a} + \frac{1}{2} < n + 1 \quad (13)$$

and L_2 from

$$L_2 = (2n + 1)a - L_o \tan\theta \cos\phi. \quad (14)$$

The expressions for L_1, L_3, and η do not contain L_o explicitly and, therefore, need not be modified. It must be noted that though the expression for η does not change in this case, its value may change because of the changes in L_1, L_2, and L_3.

III. Discussion and Conclusions

Equation (11) can be used to evaluate the performance of any right angle bend in a light pipe. Unfortunately, in a general case, this performance is dependent not only on the direction of the incident radiation and physical dimensions of the bend but also on the distance of the bend from the incident end, L_o. A closer inspection of this equation, however, yields a special case where η is independent of L_o. If the dimensions of the light pipe are such that

$$b \tan\theta \cos\phi \ge a \quad (15)$$

then

$$L_1 < O \text{ and } L_3 < L_2.$$

Equation (11) can then be simplified as

$$\eta = \frac{1}{a}[\min(a, L_2) + \max(O, a - L_2)]$$
$$= \frac{1}{a}[\min(a - L_2/2, L_2/2) + \max(L_2/2, a - L_2/2)]$$
$$= 1.$$

Thus, if the dimensions of the light pipe satisfy condition (15), efficiency of bend, η, is 1 and is independent of L_o.

In a practical situation where $a = b$ and $\phi = O$, equivalent of condition (15) is

$$45° \le \theta < 90°.$$

After the bend, light travels into the exit leg with new angles $\theta' = 90 - \theta$ and $\phi' = \phi$. If θ is chosen to be 45° and $\phi = 0°$, there is no change in θ or ϕ after the bend; and a right angle bend in the same plane encountered by this radiation will also be traversed without any attenuation. In general, therefore, a light beam will travel through a planar structure of light pipes with right angle bends without any attenuation if the launching angles are $\theta = 45°$ and $\phi = 0°$.

For the analysis in this paper, it is assumed that the walls of the tube are perfectly reflecting and the medium in the tube is perfectly transparent. In practice, these conditions may not be satisfied. A small calculation shows that for a length L of the tube with launching angles θ and ϕ, the optical pathlength for each ray is $L \sec\theta$. Powell[2] has shown that the number of reflections from the walls in this tube is also proportional to $L \tan\theta$. We thus find that if the walls of the tube are not perfectly reflecting and the medium in the tube is absorbing the radiation, then increasing the launching angle θ increases the attenuation of the beam. Thus launching angles θ satisfying condition (15) can be favored only for low-loss tubes.

References

1. T. O. Poehler and R. Turner, Appl. Opt. 9, 971 (1970).
2. W. R. Powell, Appl. Opt. 13, 952 (1974).