MATH 243 PRACTICE EXAM 8 ON RINGS

Name:

Instructions:

(1) This is a closed-book, closed-notes, no-calculator exam.
(2) Show your work as instructed.

Do not write below this line.

(1)

(2)

(3)

(4)

(5)

(6)

(7)

(8)

Total

Problem 1. (5 pts.) Circle the correct answer.

Let R be a ring satisfying $0_R = 1_R$. Then R is guaranteed to have which of the following properties?

A. $|R| = 1$.

B. R has no units.

C. R has infinitely many units.

D. R is not commutative.

E. None of these.

Problem 2. (5 pts.) Circle the correct answer.

Let R be a nonzero ring and let $a \in R$ be a zero-divisor. Which of the following statements about a is true?

A. a must be invertible.

B. a cannot be invertible.

C. a must have multiplicative order 2.

D. The multiplicative order of a divides the number $|R| - 1$.

E. None of these.

Problem 3. (5 pts.) Circle the correct answer.

Which of the following is the set of units in $\mathbb{Z}[i]$?

A. $\{1\}$.
B. $\{1, -1\}$.
C. $\{i, -i\}$.
D. $\{1, -1, i, -i\}$.
E. None of these.

Problem 4. (5 pts.) Circle the correct answer.

Which of the following is the set of units in $\mathbb{Z}/12\mathbb{Z}$?

A. $\{1\}$.
B. $\{1, 5, 7, 11\}$.
C. $\{1, 3, 5, 7, 9, 11\}$.
D. $\{1, 2, 3, 4, 6, 8, 9, 10\}$.
E. None of these.

Problem 5. (15 pts.) Let $\mathbb{R}[x]$, $\mathbb{R}[x, x^{-1}]$, $\mathbb{R}(x)$, $\mathbb{R}[[x]]$ be the rings of polynomials in x, Laurent polynomials in x, rational functions in x and power series in x, all with real coefficients.

(a) Name an element of $\mathbb{R}[x]$ which does not belong to $\mathbb{R}$.

(b) Name an element of $\mathbb{R}[x, x^{-1}]$ which does not belong to $\mathbb{R}[[x]]$.

(c) Name an element of $\mathbb{R}[[x]]$ which does not belong to $\mathbb{R}[x, x^{-1}]$.

(d) Name an element of $\mathbb{R}(x)$ which does not belong to $\mathbb{R}[[x]]$.

(e) Name a unit in $\mathbb{R}[[x]]$ which belongs to $\mathbb{R}[x]$ but is not a unit in $\mathbb{R}[x]$.

Problem 6. (10 pts.) Find the residue of

$$88882 \cdot 64007 \cdot 8003 \cdot (-25) \cdot 32003 + 2488 \cdot (-1601) \cdot 7202,$$

modulo 8. Show your computations.

Problem 7. (15 pts.) Let R be a nonzero ring. Show that $R[x]$ is an integral domain if and only if R is.

Problem 8. (15 pts.) Let R be an integral domain. Show that the units of $R[x]$ are just the units of R.

Problem 9. (15 pts.) Find an inverse in $\mathbb{Z}/3\mathbb{Z}[[x]]$ for the polynomial $1+x+x^2$. Compute the first four coefficients and describe the rest using a recurrence relation.