MATH 243 PRACTICE EXAM 5 ON SUBGROUPS AND COSETS

Name:

Instructions:

(1) This is a closed-book, closed-notes, no-calculator exam.
(2) Show your work as instructed.

Do not write below this line.

(1)

(2)

(3)

(4)

(5)

(6)

(7)

(8)

Total

Problem 1. (5 pts.) Circle the correct answer.

Let H, K be subgroups of G. Which of the following must also a subgroup of G?

A. $H \cup K$.

B. $H \cap K$.

C. $H \setminus K$.

D. HK.

E. None of these.

Problem 2. (5 pts.) Circle the correct answer.

Let s_1 be a standard generator of the symmetric group S_n and define the subgroup $H = \{e, s_1\}$. How many cosets of H are there in S_n?

A. 1.

B. n.

C. $n!/2$.

D. $n!$.

E. None of these.

Problem 3. (5 pts.) Circle the correct answer.

Let the cyclic group C_8 have elements $\{g, g^2, \ldots, g^8 = e\}$ and let H be the subgroup generated by g^2. How many elements belong to the coset g^3H?

A. 2.

B. 4.

C. 6.

D. 8.

E. None of these.

Problem 4. (5 pts.) Circle the correct answer.

Let G be a 12-element group and let H be a subgroup of G. Which of the following cannot be the cardinality of H?

A. 2.

B. 4.

C. 6.

D. 8.

E. None of these.

Problem 5. (15 pts.) Let D_8 be the dihedral group of symmetries of the square.

(a) State a description of D_8 in terms of generators and relations.

(b) Draw a diagram which shows all of the subgroups of D_8, ordered by inclusion.

Problem 6. (15 pts.) Briefly answer the following questions. You do not need to write in claim-proof form.

(a) Name a finite group G and a subgroup H of G satisfying $[G : H] = 2$.

(b) Name an infinite group G and a subgroup H of G satisfying $[G : H] = 2$.

Problem 7. (15 pts.) Briefly answer the following questions. You do not need to write in claim-proof form.

(a) Name a group G, a subgroup $H \subseteq G$, and an element $b \in G$ satisfying $bH \neq Hb$.

(b) Name a nonabelian group G, a subgroup $H \subseteq G$, and an element $b \in G$ satisfying $bH = Hb$.

Problem 8. (20 pts) Let H be a subgroup of G. Show that for $a, b \in G$, the cosets aH, bH satisfy exactly one of the conditions

(1) $aH = bH$,
(2) $aH \cap bH = \emptyset$.

Problem 9. (15 pts.) Let U be the unit circle in the complex plane,

$$U = \{a + bi \mid a^2 + b^2 = 1\}$$

(a) Show that $(U, \cdot)$ is a subgroup of $(\mathbb{C} \setminus \{0\}, \cdot)$.

(b) What is the relationship of U to the matrix group $\mathrm{SO}_2(\mathbb{R})$?

Problem 10. (15 pts) Show that for every finite group G and every subgroup H of G, the number $|H|$ divides the number $|G|$.