MATH 242 PRACTICE EXAM ON MATRIX INVERSION

Name:

Section:

Instructions:

(1) Books, notes, and calculators are not permitted.
(2) Simplify as instructed. On some problems, unsimplified correct answers will not receive full credit.
(3) Show your work as instructed. On some problems, correct answers without justification will not receive full credit.

(1)

(2)

(3)

(4)

(5)

(6)

Total

Problem 1. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let A, B be $n \times n$ invertible matrices. Which of the following equations are false?

A. $(AB)^{-1} = B^{-1}A^{-1}$.

B. $(A^{\top})^{-1} = (A^{-1})^{\top}$.

C. $(A + B)^{-1} = A^{-1} + B^{-1}$.

D. $(A^{\top}B^{\top})^{-1} = (A^{-1}B^{-1})^{\top}$.

E. None of these: the four equations hold for all invertible $n \times n$ matrices.

Problem 2. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let A be an invertible matrix. Which of the following statements are false?

A. $A^{\top}$ and A^{-1} are both invertible.

B. If A has only integer entries, then A^{-1} has only integer entries.

C. If A is diagonal, then A^{-1} is diagonal.

D. If A is upper triangular, then A^{-1} is upper triangular.

E. None of these: the four statements are true.

Problem 3. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let c be a nonzero real number and define the matrices

$$A = \begin{bmatrix} 1 & c & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Which of these matrices are invertible?

A. A only.

B. B only.

C. A and B only.

D. A, B, and C.

E. Other.

Problem 4. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let a, b, c be nonzero real numbers and define the matrices

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}, \quad B = \begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}, \quad C = \begin{bmatrix} -a & 0 & 0 \\ 0 & -b & 0 \\ 0 & 0 & -c \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & a \\ 0 & b & 0 \\ c & 0 & 0 \end{bmatrix}.$$

Which of the following is equal to A^{-1}?

A. A.

B. B.

C. C.

D. D.

E. None of these.

Problem 5. (15 pts.) Invert the matrix

$$\begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix}$$

or show that it is not invertible. Show your work by labeling row operations in the notation of your choice.

Problem 6. (15 pts.) Invert the matrix

$$\begin{bmatrix} 1 & 4 & 3 \\ 2 & 0 & 2 \\ 1 & 12 & 7 \end{bmatrix}$$

or show that it is not invertible. Show your work by labeling row operations in the notation of your choice.

Problem 7. (15 pts.) Define the matrix and vector

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}.$$

Solve the system

$$A\mathbf{x} = \mathbf{b} \tag{1}$$

by inverting A and left-multiplying both sides of (1) by A^{-1}.

Problem 8. (15 pts.) Define the matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Use row operations to invert A, assuming that $ad - bc \neq 0$.

Problem 9. (10 pts.) Complete the missing parts of the claim, whose proof is given.

Claim: Let A be an $n \times n$ matrix with

Then

Proof. Assume that for some k, every entry in row k of A is 0. If A is invertible, then there is a matrix B such that $AB = I$. It follows that the dot product of row i of A with column j of B is

$$\begin{bmatrix} a_{i,1} & a_{i,2} & \cdots & a_{i,n} \end{bmatrix} \begin{bmatrix} b_{1,j} \\ b_{2,j} \\ \vdots \\ b_{n,j} \end{bmatrix} = \begin{cases} 1 & \text{if } i = j, \\ 0 & \text{otherwise.} \end{cases}$$

But when $i = j = k$, the dot product is 0, contradicting the above. It follows that the matrix B cannot exist, and that A is not invertible.

Now assume that for some k, every entry in column k of A is 0. If A is invertible, then its transpose is invertible as well. But $A^{\top}$ contains a row of zeros and by the above argument it can not be invertible. It follows that A is not invertible. $\square$