MATH 242 PRACTICE EXAM ON LINEAR SYSTEMS OF EQUATIONS

Name:

Section:

Instructions:

(1) Books, notes, and calculators are not permitted.
(2) Simplify as instructed. On some problems, unsimplified correct answers will not receive full credit.
(3) Show your work as instructed. On some problems, correct answers without justification will not receive full credit.

(1)

(2)

(3)

(4)

(5)

(6)

Total

Problem 1. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

The linear system

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ 3 & 6 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 15 \end{bmatrix}$$

has how many solutions?

A. 0.

B. 1.

C. 3.

D. Infinitely many.

E. None of these.

Problem 2. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Consider the linear system

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 1 \\ -1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 7 \end{bmatrix}.$$

Which of the following solves the system?

A. $(x_1, x_2, x_3) = (2, 1, 1)$.

B. $(x_1, x_2, x_3) = (1, -1, 2)$.

C. $(x_1, x_2, x_3) = (1, 1, 2)$.

D. $(x_1, x_2, x_3) = (2, 1, -1)$.

E. None of these.

Problem 3. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let A be an $n \times n$ matrix and consider the homogeneous linear system

$$A\mathbf{x} = \mathbf{0}. \tag{1}$$

Which of the following statements best describes the number of solutions of (2)?

A. It could have one, zero, or infinitely many solutions.

B. It could have one or zero solutions.

C. It could have one or infinitely many solutions.

D. It could have zero, or infinitely many solutions.

E. None of these.

Problem 4. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let A be an $m \times n$ matrix with $m < n$, let $\mathbf{b}$ be a nonzero element of $\mathbb{R}^m$, and consider the nonhomogeneous linear system

$$A\mathbf{x} = \mathbf{b}. \tag{2}$$

Which of the following statements best describes the number of solutions of (2)?

A. It could have one, zero, or infinitely many solutions.

B. It could have one or zero solutions.

C. It could have one or infinitely many solutions.

D. It could have zero, or infinitely many solutions.

E. None of these.

Problem 5. (15 pts.) Use Gaussian elimination to solve the linear system

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 24 \\ 40 \end{bmatrix},$$

or to show that it has no solutions. Show your work by labeling row operations in the notation of your choice.

Problem 6. (15 pts.) Use Gaussian elimination to solve the linear system

$$\begin{bmatrix} 1 & 1 & 1 \\ 6 & -4 & 5 \\ 5 & 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 31 \\ 13 \end{bmatrix},$$

or to show that it has no solutions. Show your work by labeling row operations in the notation of your choice.

Problem 7. (15 pts.) Use Gaussian elimination to solve the linear system

$$\begin{bmatrix} -1 & 1 & -1 & 3 \\ 3 & 1 & -1 & -1 \\ 2 & -1 & -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 6 \end{bmatrix},$$

or to show that it has no solutions. Show your work by labeling row operations in the notation of your choice.

Problem 8. (15 pts.) Use Gaussian elimination to solve the linear system

$$\begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 3 \end{bmatrix}.$$

Show your work by labeling row operations in the notation of your choice.

Problem 9. (15 pts.) Find nonzero constants a, b, c, d such that the linear system

$$\begin{bmatrix} 2 & 10 & 3 \\ a & b & c \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ d \end{bmatrix}$$

has no solution.

Problem 10. (15 pts.) Find nonzero constants b_1, b_2, b_3 such that the linear system

$$\begin{bmatrix} 2 & 1 \\ 1 & 3 \\ 5 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

has no solution.

Problem 11. (10 pts.) Complete the following claim, whose proof is given.

Claim: Let A be an $m \times n$ matrix and let $\mathbf{c}$ and $\mathbf{d}$ be vectors in $\mathbb{R}^n$. If $\mathbf{x} = \mathbf{c}$ is a solution of the system

and $\mathbf{x} = \mathbf{d}$ is a solution of the system

then $\mathbf{x} = \mathbf{c} + \mathbf{d}$ is a solution of the system

Proof. Suppose that we have $A\mathbf{c} = \mathbf{b}$ and $A\mathbf{d} = \mathbf{0}$. By the distributive law we then have

$$A(\mathbf{c} + \mathbf{d}) = A\mathbf{c} + A\mathbf{d} = \mathbf{b} + \mathbf{0} = \mathbf{b}.$$

$\square$