MATH 242 PRACTICE EXAM ON MATRIX ALGEBRA

Name:

Section:

Instructions:

(1) Books, notes, and calculators are not permitted.
(2) Simplify as instructed. On some problems, unsimplified correct answers will not receive full credit.
(3) Show your work as instructed. On some problems, correct answers without justification will not receive full credit.

(1)

(2)

(3)

(4)

(5)

(6)

Total

Problem 1. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let A, B, C be $n \times n$ matrices. Which of the following equations is not always true?

A. $A + B = B + A$.

B. $A(B + C) = AB + AC$.

C. $AB = BA$.

D. $(AB)C = A(BC)$.

E. None of these: the four equations hold for all $n \times n$ matrices.

Problem 2. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let A, B, C be $n \times n$ matrices. Which of the following equations is not necessarily true?

A. $A^\top + B^\top = (A + B)^\top$.

B. $A(B^\top + C^\top) = AB^\top + AC^\top$.

C. $(AB)^\top = B^\top A^\top$.

D. $(AB)^\top C = A(BC)^\top$.

E. None of these: the four equations hold for all $n \times n$ matrices.

Problem 3. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let m, n and p be three distinct positive integers. Let A be an $m \times n$ matrix, let B be an $n \times p$ matrix, and let C be a $p \times m$ matrix. Which of the following products is not defined?

A. AB.

B. AC.

C. BC.

D. ABC.

E. None of these: the four matrices are all defined.

Problem 4. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let m, n and p be three distinct positive integers. Let A be an $m \times n$ matrix, let B be an $n \times p$ matrix, and let C be a $p \times m$ matrix. Which of the following expressions is not defined?

A. $AA^{\top} + ABC$.

B. $ABC + C^{\top}C$.

C. $A^{\top}A + BB^{\top}$.

D. $ABC + (CBA)^{\top}$.

E. None of these: the four matrices are all defined.

Problem 5. (15 pts.) Find nonzero 2×2 matrices A, B, C with the property that $AB = AC$ but $B \neq C$.

Problem 6. (15 pts.) Let $A = (a_{i,j})$ be an $m \times n$ matrix and define the $m \times m$ matrix $B = (b_{i,j})$ by $B = AA^{\top}$.

(a) Find a formula for $b_{i,j}$ in terms of the entries of A.

(b) Explain why $b_{i,j} = b_{j,i}$ for all i, j.

Problem 7. (15 pts.) Define the matrix

$$A = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}.$$

(a) Compute A^2.

(b) Find a formula for A^n, where n is any positive integer.

Problem 8. (10 pts.) Fill in the missing parts of the claim, whose proof is given.

Claim: Every $n \times n$ matrix A can be written as

where B and C satisfy

Proof. Given an $n \times n$ matrix $A = (a_{i,j})$, define matrices $B = (b_{i,j})$ and $C = (c_{i,j})$ by

$$b_{i,j} = \frac{a_{i,j} + a_{j,i}}{2}, \qquad c_{i,j} = \frac{a_{i,j} - a_{j,i}}{2}.$$

It is easy to see that we have $b_{i,j} + c_{i,j} = a_{i,j}$, i.e.,

$$A = B + C.$$

Furthermore, the matrices B and C satisfy

$$b_{j,i} = \frac{a_{j,i} + a_{i,j}}{2} = b_{i,j}, \qquad c_{j,i} = \frac{a_{j,i} - a_{i,j}}{2} = -c_{i,j},$$

i.e., $B^\top = B$ and $C^\top = -C$. $\square$