MATH 22 PRACTICE EXAM ON TAYLOR POLYNOMIALS

Name:

Section:

Instructions:

(1) Books, notes, and calculators are not permitted.
(2) Simplify as instructed. On some problems, unsimplified correct answers will not receive full credit.
(3) Show your work as instructed. On some problems, correct answers without justification will not receive full credit.

(1)

(2)

(3)

(4)

(5)

Total

Problem 1. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let $T_3(x)$ be the third Taylor approximation of $f(x)$ for x near a. Which of the following derivatives do $T_3(x)$ and $f(x)$ have in common?

A. $T_3'(a) = f'(a)$.

B. $T_3''(a) = f''(a)$.

C. $T_3'''(a) = f'''(a)$.

D. All of the above.

E. None of these.

Problem 2. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Suppose that we approximate $\sin(\frac{1}{2})$ by the finite sum

$$\frac{1}{2} - \frac{1}{3!\,8} + \frac{1}{5!\,32}.$$

Which of the following best describes this approximation and the corresponding error?

A. The approximation is an underestimate with error at most $\dfrac{1}{6!\,64}$.

B. The approximation is an overestimate with error at most $\dfrac{1}{6!\,64}$.

C. The approximation is an underestimate with error at most $\dfrac{1}{7!\,128}$.

D. The approximation is an overestimate with error at most $\dfrac{1}{7!\,128}$.

E. None of these.

Problem 3. (15 pts.) Consider the function

$$y = \ln(x)$$

and its values when x is near 1.

(a) Find the equation of the first degree Taylor polynomial for $y = \ln(x)$ when x is near 1. (This is also the line tangent to $y = \ln(x)$ at $x = 1$.)

(b) Find the equation of the third degree Taylor polynomial which approximates $\ln(x)$ when x is near 1. Show your work.

(c) Use the third degree Taylor polynomial to write an expression which approximates $\ln(1/2)$. Show your work and express your answer as a finite sum of integers and/or reduced fractions.

Problem 4. (15 pts.) Consider the function

$$y = x^{1/3}$$

and its values when x is near 8.

(a) Find the equation of the first degree Taylor polynomial for $y = x^{1/3}$ when x is near 8. (This is also the line tangent to $y = x^{1/3}$ at $x = 8$.)

(b) Find the equation of the third degree Taylor polynomial which approximates $x^{1/3}$ when x is near 8. Show your work.

(c) Use the third degree Taylor polynomial to write an expression which approximates $7^{1/3}$. Show your work and express your answer as a finite sum of integers and/or reduced fractions.

Problem 5. (15 pts.) Use a 6th degree Taylor polynomial for e^{x^2} to approximate the integral

$$\int_0^1 e^{x^2} dx$$

as a sum of four integers and/or reduced fractions. Show your work.