MATH 22 PRACTICE EXAM ON TAYLOR SERIES

Name:

Section:

Instructions:

(1) Books, notes, and calculators are not permitted.
(2) Simplify as instructed. On some problems, unsimplified correct answers will not receive full credit.
(3) Show your work as instructed. On some problems, correct answers without justification will not receive full credit.

(1)

(2)

(3)

(4)

(5)

Total

Problem 1. (10 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

The series

$$-x^2 + \frac{x^4}{3!} - \frac{x^6}{5!} + \frac{x^8}{7!} \pm \cdots$$

is the Taylor series of which of the following functions?

A. $-\sin x$.

B. $-\cos x$.

C. $-x \sin x$.

D. $-x \cos x$.

E. None of these.

Problem 2. (10 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

The series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \pm \cdots$$

converges to what?

A. 0.

B. $1/e$.

C. $\ln 2$.

D. e.

E. None of these.

Problem 3. (10 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

The limit

$$\lim_{x\to 0} \frac{1-\cos(x^2)}{x^4}$$

is equal to which of the following?

A. 0.

B. $\frac{1}{4}$.

C. $\frac{1}{2}$.

D. 1.

E. None of these.

Problem 4. (10 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

The series

$$\sum_{n=0}^{\infty} \frac{(-1)^n t^{4n+3}}{(4n+3)(2n+1)!} = \frac{t^3}{3\cdot 1!} - \frac{t^7}{7\cdot 3!} + \frac{t^{11}}{11\cdot 5!} - \frac{t^{15}}{15\cdot 7!} \pm \cdots$$

is equal to which of the following integrals?

A. $\int_0^t \sin x\, dx$.

B. $\int_0^t \sin(x^2)\, dx$.

C. $\int_0^t \sin(x^3)\, dx$.

D. $\int_0^t \sin(x^4)\, dx$.

E. None of these.

Problem 5. (15 pts.) Find the first four nonzero terms of the Taylor series of $e^x \cos x$.

Problem 6. (15 pts.) Find the Taylor series of $\cos x$ centered at $\pi/2$.

Problem 7. (15 pts.) Evaluate the indefinite integral

$$\int x^2 \cos(x^2)\, dx$$

as an infinite series.

Problem 8. (15 pts.) Use the Taylor series

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \pm \cdots$$

to express $\ln 2$ as an infinite series. Explain why this series converges.

Problem 9. (15 pts.) Find the sum of each series.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!}$.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{n5^n}$.

(c) $3 + \dfrac{9}{2!} + \dfrac{27}{3!} + \dfrac{81}{4!} + \cdots$.