MATH 22 PRACTICE EXAM ON ALTERNATING SERIES 11.5

Name:

Section:

Instructions:

(1) Books, notes, and calculators are not permitted.
(2) Simplify as instructed.
(3) Show your work as instructed. On some problems, correct answers without justification will not receive full credit.

(1)

(2)

(3)

(4)

(5)

Total

Problem 1. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Consider the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}.$$

Applying the Alternating Series Test to this series leads to which of the following conclusions?

A. This series diverges because $\sqrt{n} = n^{1/2}$ and $1/2 < 1$.

B. The test shows that the series diverges.

C. The test is inconclusive and we therefore have no information.

D. The test shows that the series converges.

E. None of these.

Problem 2. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Consider the alternating series

$$\sum_{n=1}^{\infty} a_n = 1 - \frac{1}{2^1} + \frac{1}{2} - \frac{1}{2^2} + \frac{1}{3} - \frac{1}{2^3} + \frac{1}{4} = \frac{1}{2^4} \pm \cdots .$$

Applying the Alternating Series Test to this series leads to which of the following conclusions?

A. The series converges because it satisfies $\lim_{n\to\infty} a_n = 0$.

B. The series converges because it satisfies $\lim_{n\to\infty} a_n = 0$ and $|a_n| > |a_{n+1}|$.

C. The series might converge, but does not satisfy $\lim_{n\to\infty} a_n = 0$ and $|a_n| > |a_{n+1}|$.

D. The series diverges because it does not satisfy $\lim_{n\to\infty} a_n = 0$ and $|a_n| > |a_{n+1}|$.

E. None of these.

Problem 3. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Consider the series

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} \pm \cdots .$$

Which of the following statements best describes this series?

A. The series is $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$ and converges.

B. The series is $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$ and diverges.

C. The series is $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{2n+1}$ and converges.

D. The series is $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{2n+1}$ and diverges.

E. None of these.

Problem 4. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Consider the alternating series

$$\frac{1}{2} - \frac{2}{3} + \frac{3}{4} - \frac{4}{5} + \frac{5}{6} \pm \cdots .$$

Which of the following statements best describes this series?

A. The series is $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+1}$ and converges.

B. The series is $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n+1}$ and diverges.

C. The series is $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{n+1}$ and converges.

D. The series is $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{n+1}$ and diverges.

E. None of these.

Problem 5. (15 pts.)

(a) Decide if the series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(\ln(n))}$$

converges. Explain your reasoning.

(b) Suppose we approximate the above infinite sum by the finite sum

$$\sum_{n=2}^{10} \frac{(-1)^n}{\ln(\ln(n))}.$$

Find an upper bound for the magnitude of the error

$$\left| \sum_{n=11}^{\infty} \frac{(-1)^n}{\ln(\ln(n))} \right|.$$

Simplify your answer.

Problem 6. (15 pts.) Find the smallest integer t such that the approximation

$$\sum_{n=0}^{t} \frac{(-1)^n}{(2n)!}$$

of the infinite sum

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}$$

has error less than .002. You may use the table of factorials

$1! = 1, \quad 2! = 2, \quad 3! = 6, \quad 4! = 24, \quad 5! = 120, \quad 6! = 720, \quad 7! = 5040.$

Show your work.

Problem 7. (15 pts.) Decide if the series

$$\sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{\pi}{n}\right)$$

converges. Explain your reasoning.

Problem 8. (15 pts.) Decide if the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\arctan n}$$

converges. Explain your reasoning.