MATH 22 PRACTICE EXAM ON SERIES COMPARISON TESTS

Name:

Section:

Instructions:

(1) Books, notes, and calculators are not permitted.
(2) Simplify as instructed. On some problems, unsimplified correct answers will not receive full credit.
(3) Show your work as instructed. On some problems, correct answers without justification will not receive full credit.

(1)

(2)

(3)

(4)

(5)

Total

Problem 1. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Consider the following statements about the series

$$\sum_{n=1}^{\infty} \frac{1}{n^2}, \qquad \sum_{n=2}^{\infty} \frac{1}{n^2-1}.$$

I. $\sum \frac{1}{n^2-1}$ diverges because $\frac{1}{n^2-1} > \frac{1}{n^2}$.
II. $\sum \frac{1}{n^2-1}$ converges because $\frac{1}{n^2-1} > \frac{1}{n^2}$.
III. $\sum \frac{1}{n^2-1}$ converges because $\frac{1}{n^2-1} < \frac{1}{n^2}$.
IV. $\sum \frac{1}{n^2-1}$ converges because $\lim_{n\to\infty} \frac{n^2}{n^2-1} = 1$.

Which of the above statements are false?

A. I only.

B. I and II only.

C. I, II, and III only.

D. All four statements are false.

E. None of these.

Problem 2. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let $(a_0, a_1, a_2, \dots)$ be a sequence of positive numbers which satisfies

$$\lim_{n\to\infty} a_n = 0, \qquad \sum_{n=0}^{\infty} a_n = 6,$$

and let $(b_0, b_1, b_2, \dots)$ be a sequence of numbers which satisfies $b_n \geq a_n$ for all n. Which of the following best describes our knowledge about of the convergence or divergence of the series

$$\sum_{n=1}^{\infty} b_n?$$

A. It diverges to ∞.

B. It either converges to a positive number or diverges to ∞.

C. It converges to a positive number greater than 6.

D. It converges to 6.

E. None of these.

Problem 3. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let $(a_0, a_1, a_2, \dots)$ and $(b_0, b_1, b_2, \dots)$ be sequences of numbers which satisfy

(1) $-10 \leq b_n \leq 5$ for $n \leq 100$,
(2) $-5 \leq a_n \leq 5$ for $n \leq 100$,
(3) $0 \leq a_n \leq b_n$ for $n \geq 101$,
(4) $\sum_{n=0}^{\infty} b_n = 25$.

Which of the following best describes our knowledge about of the convergence or divergence of the series

$$\sum_{n=0}^{\infty} a_n?$$

A. It diverges to ∞.

B. It either converges to a positive number or diverges to ∞.

C. It converges to a positive number less than 25.

D. It converges.

E. None of these.

Problem 4. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let $(a_0, a_1, a_2, \dots)$ and $(b_0, b_1, b_2, \dots)$ be sequences of positive numbers which satisfy

$$\lim_{n\to\infty} \frac{b_n}{a_n} = \frac{1}{2}, \qquad \sum_{n=0}^{\infty} b_n = 100.$$

Which of the following best describes our knowledge about of the convergence or divergence of the series

$$\sum_{n=0}^{\infty} a_n?$$

A. It either converges to a positive number or diverges to ∞.

B. It converges to 200.

C. It converges to 50.

D. It converges.

E. None of these.

Problem 5. (15 pts.) Decide if the series

$$\sum_{n=0}^{\infty} ne^{-n}$$

converges. Explain your reasoning.

Problem 6. (15 pts.) Decide if the series

$$\sum_{n=0}^{\infty} \frac{n \sin^2(n)}{n^3 + 1}$$

converges. Explain your reasoning.

Problem 7. (15 pts.) Decide if the series

$$\sum_{n=0}^{\infty} \frac{n+2^n}{n+3^n}$$

converges. Explain your reasoning.

Problem 8. (15 pts.) Decide if the series

$$\sum_{n=1}^{\infty} \frac{4^{n+1}}{3^n - 2}$$

converges. Explain your reasoning.

Problem 9. (15 pts.) Decide if the series

$$\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{n+2}$$

converges. Explain your reasoning.

Problem 10. (15 pts.) Decide if the series

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + n^2}$$

converges. Explain your reasoning.