MATH 22 PRACTICE EXAM ON THE INTEGRAL TEST

Name:

Section:

Instructions:

(1) Books, notes, and calculators are not permitted.
(2) Simplify as instructed. On some problems, unsimplified correct answers will not receive full credit.
(3) Show your work as instructed. On some problems, correct answers without justification will not receive full credit.

(1)

(2)

(3)

(4)

(5)

Total

Problem 1. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Which of the following series converges?

A. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$.

B. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.

C. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$.

D. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n}}$.

E. None of these.

Problem 2. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let $f(x)$ be a function which is positive and continuous on the interval $1 \le x < \infty$, and which is decreasing on the interval $10 \le x < \infty$. Suppose that $f(x)$ satisfies

$$\int_1^{\infty} f(x)\,dx = 6, \qquad \int_{10}^{\infty} f(x)\,dx = 1.$$

Which of the following best describes our knowledge about of the convergence or divergence of the series

$$\sum_{n=1}^{\infty} f(n)?$$

A. It either converges to a positive number or diverges to ∞.

B. It certainly converges to a positive number.

C. It certainly converges to 6.

D. It certainly converges to a positive number, and $f(10) + f(11) + \cdots = 1$.

E. None of these.

Problem 3. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let $(a_0, a_1, a_2, \ldots,)$ be a sequence of positive and negative numbers and consider the following statements about the corresponding series $a_0 + a_1 + a_2 + \cdots$.

I. The series converges to a negative number.
II. The series diverges to $-\infty$.
III. The series converges to a positive number.
IV. The series diverges to ∞.
V. The series diverges, but not to ∞ or $-\infty$.

Which of the above statements could possibly apply to the series?

A. I and III only.

B. I, II, III, and IV only.

C. II, IV, and V only.

D. All of them could apply.

E. None of these.

Problem 4. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let $f(x)$ be a function which is positive, continuous, and decreasing on the interval $1 \leq x < \infty$, and consider the integral and series

$$\int_1^{\infty} f(x)\,dx, \qquad \sum_{n=1}^{\infty} f(n).$$

If the integral converges to a number L, then which of the following best desribes what we know about the series?

A. It converges to a positive number.

B. It converges to a positive number greater than L.

C. It converges to a positive number less than L.

D. It converges to L.

E. None of these.

Problem 5. (15 pts.) Decide if the series

$$\sum_{n=1}^{\infty} \frac{n}{e^{n^2}}$$

converges. Explain your reasoning.

Problem 6. (15 pts.) Find all values of p for which the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$$

converges. Explain your reasoning.

Problem 7. (15 pts.) Suppose that we estimate the infinite series

$$\sum_{n=0}^{\infty} \frac{1}{n^3}$$

by the finite sum

$$c = 1 + \frac{1}{8} + \frac{1}{27} + \cdots + \frac{1}{1000}.$$

(a) Find a lower bound ℓ and an upper bound u for the error

$$\sum_{n=11}^{\infty} \frac{1}{n^3}.$$

Express ℓ and u as reduced fractions.

(b) If we now use the improved estimate

$$c + \frac{\ell + u}{2}$$

for the infinite series, how big can the error be? Express your answer as a reduced fraction.