MATH 22 PRACTICE EXAM ON SEQUENCES

Name:

Section:

Instructions:

(1) Books, notes, and calculators are not permitted.
(2) Simplify as instructed. On some problems, unsimplified correct answers will not receive full credit.
(3) Show your work as instructed. On some problems, correct answers without justification will not receive full credit.

(1)

(2)

(3)

(4)

(5)

Total

Problem 1. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Assume that the limit

$$\lim_{n\to\infty} a_n$$

exists and is nonnegative. Then the limit

$$\lim_{n\to\infty} \sqrt{a_n}$$

is equal to which of the following?

A. $\lim_{n\to\infty} a_n$.

B. $\sqrt{\lim_{n\to\infty} a_n}$.

C. $\lim_{n\to\infty} \sqrt{-a_n}$.

D. All of the above.

E. None of these.

Problem 2. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Consider the sequence $(a_0, a_1, a_2, \ldots,)$ defined by $a_n = 1 + (\frac{1}{3})^n$, having limit 1. Is there a number N such that for each index $n > N$, the term a_n must be within $\frac{1}{100}$ of 1?

A. Yes, and $N = 1$.

B. Yes, and $N = 2$.

C. Yes, and $N = 3$.

D. Yes, and $N = 4$.

E. None of these.

Problem 3. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Consider the sequence $(a_0, a_1, a_2, \ldots,)$ defined by $a_n = 3 - 2^n$, having limit $-\infty$. Is there a number N such that for each index $n > N$, the term a_n is less than -1000?

A. Yes, and $N = 7$.

B. Yes, and $N = 8$.

C. Yes, and $N = 9$.

D. Yes, and $N = 10$.

E. None of these.

Problem 4. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Consider the sequence $(a_1, a_2, \ldots)$ whose nth term is

$$\frac{10n^2 + 100n + 1000}{n! + \ln(n)}.$$

How does the sequence behave as as n approaches infinity?

A. It converges to 0.

B. It converges, but not to 0.

C. It diverges to ∞.

D. It diverges, but not to ∞.

E. None of these.

Problem 5. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Which of the functions

$$n\ln(n), (\ln(n))^2, \ln(n), \ln(\ln(n))$$

approach infinity as n approaches infinity?

A. $n\ln(n)$ only.

B. $n\ln(n)$ and $(\ln(n))^2$ only.

C. $n\ln(n)$, $(\ln(n))^2$, and $\ln(n)$ only.

D. All of them.

E. None of these.

Problem 6. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

For what values of c does the sequence $(1, c^1, c^2, c^3, \dots)$ converge to zero?

A. $-1 \leq c \leq 1$.

B. $-1 < c < 1$.

C. $0 \leq c < 1$.

D. $-1 < c \leq 0$.

E. None of these.

Problem 7. (15 pts.) We say that $f(n)$ *grows faster than* $g(n)$ if

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} > 1.$$

Order the functions $n!$, $2n!$, $(2n)!$, $(n!)^2$ by fastest growth, from fastest to slowest. Justify your answer.

Problem 8. (15 pts.) Find
$$\lim_{n\to\infty} \frac{\ln(2n)}{\ln(n)}.$$
Show your work.

Problem 9. (15 pts.) Show that the sequence $(a_0, a_1, a_2, \dots)$ defined by

$$a_n = \frac{1-n}{2+n}$$

is decreasing and bounded.

Problem 10. (15 pts.) Assume that

$$\lim_{n\to\infty} a_n = L$$

for some positive number L. What can we say about $\lim_{n\to\infty} a_{2n}$ and $\lim_{n\to\infty} a_{2n+1}$? Justify your answer.