

MATH 22 PRACTICE EXAM ON PARAMETRIC EQUATIONS

Name:

Section:

Instructions:

- (1) Books, notes, and calculators are not permitted.
- (2) Simplify as instructed.
- (3) Show your work as instructed. On some problems, correct answers without justification will not receive full credit.

(1)

(2)

(3)

(4)

(5)

Total

Problem 1. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

A particle moves along a curve according to the parametric equation

$$(x, y) = (f(t), g(t)).$$

Which of the following parametric equations describes the motion of a second particle traveling backwards along the same curve?

A. $(x, y) = (-f(t), -g(t)).$

B. $(x, y) = (f^{-1}(t), g^{-1}(t)).$

C. $(x, y) = (g(t), f(t)).$

D. $(x, y) = (f(-t), g(-t)).$

E. None of these.

Problem 2. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

A particle moves around the unit circle $x^2 + y^2 = 1$ according to the parametric equation

$$(x, y) = (\cos(\pi\theta), \sin(\pi\theta)), \quad 0 \leq \theta \leq 3.$$

How many times does the particle visit the point $(-1, 0)$?

A. 1.

B. 2.

C. 3.

D. 6.

E. None of these.

Problem 3. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

A particle moves around the ellipse $\frac{x^2}{36} + \frac{y^2}{4} = 1$ according to the parametric equation

$$(x, y) = (6 \sin(\theta), 2 \cos(\theta)), \quad 0 \leq \theta \leq 2\pi.$$

Which of the following describes motion of the particle?

- A. Clockwise, starting and ending at $(0, 2)$.
- B. Counterclockwise, starting and ending at $(0, 2)$.
- C. Clockwise, starting and ending at $(6, 0)$.
- D. Counterclockwise, starting and ending at $(6, 0)$.
- E. None of these.

Problem 4. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

A curve is given by the parametric equation

$$(x, y) = (t + 1, t^2 + t).$$

Which of the following is the slope of the line tangent to the curve at the point $(2, 2)$?

- A. 1.
- B. 2.
- C. 3.
- D. 5.
- E. None of these.

Problem 5. (15 pts.) A particle moves along a curve according to the parametric equation

$$(x, y) = (\cos \theta, \cos^2 \theta).$$

Express the curve as an equation in x and y , and describe in words the motion of the particle.

Problem 6. (15 pts.) Set up an integral which gives the distance traveled by a particle moving according to the parametric equation

$$(x, y) = (e^t, t^2), \quad 0 \leq t \leq 4.$$

Use Simpson's Rule with $n = 4$ (two parabolas) to write an approximation of this integral.

Problem 7. (15 pts.) Consider the parametric curve given by

$$(x, y) = (8 - t^2, t^3 - 6t).$$

(a) Find the values of t and the coordinates of the points on the curve where the tangent line is horizontal or vertical. Show your work and simplify your answer.

(b) Find the values of t for which the curve is concave down. Show your work and simplify your answer.