

MATH 22 PRACTICE EXAM ON DIFFERENTIAL EQUATIONS

Name:

Section:

Instructions:

- (1) Books, notes, and calculators are not permitted.
- (2) Simplify as instructed. On some problems, unsimplified correct answers will not receive full credit.
- (3) Show your work as instructed. On some problems, correct answers without justification will not receive full credit.

(1)

(2)

(3)

(4)

(5)

(6)

Total

Problem 1. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

How many solutions of the differential equation

$$\frac{dy}{dx} = \frac{x}{y}$$

pass through the point $(0, -3)$?

- A. None of them.
- B. Exactly 1 of them.
- C. Exactly 2 of them.
- D. All of them.
- E. None of these.

Problem 2. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

The differential equation

$$y' = x + y$$

has which of the following properties?

- A. It is neither separable nor linear.
- B. It is separable, but not linear.
- C. It is linear, but not separable.
- D. It is both linear and separable.
- E. None of these.

Problem 3. (15 pts.) A tank contains 1000 liters of salt water, initially containing 20kg of dissolved salt. Pure water enters the tank at a rate of 10 liters per minute. The solution is kept thoroughly mixed and drains from the tank at the same rate of 10 liters per minute.

(a) Let $y(t)$ be the amount of salt (in kg) in the tank after t minutes. Set up a differential equation which relates $y(t)$ to $y'(t)$.

(b) Find an explicit formula for $y(t)$. If your answer to part (a) is not correct, then you may not receive much credit for this part. Show your work and simplify your answer.

Problem 4. (15 pts.) Solve (find all solutions of) the differential equation

$$\frac{dy}{dx} = 1 - x + y - xy.$$

Show your work and simplify your answer.

Problem 5. (15 pts.) Solve (find all solutions of) the differential equation

$$\frac{dy}{dx} + e^{x+y} = 0.$$

Show your work and simplify your answer.

Problem 6. (15 pts.) Which particular solution of the differential equation

$$xy' - y = x \ln x$$

passes through the point $(1, 2)$? Show your work and simplify your answer.