MATH 22 PRACTICE EXAM ON APPROXIMATING INTEGRALS

Name:

Section:

Instructions:

(1) Books, notes, and calculators are not permitted.
(2) Simplify as instructed. On some problems, unsimplified correct answers will not receive full credit.
(3) Show your work as instructed. On some problems, correct answers without justification will not receive full credit.

(1)	
(2)	
(3)	
(4)	
(5)	
(6)	
(7)	
Total	

Problem 1. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Which of the following approximations of the area under the line $y = mx + b$ between $x = x_0$ and $x = x_1$ is the most accurate?

A. The midpoint rule with 1 rectangle.

B. The trapezoid rule with 1 trapezoid.

C. Simpson's rule with 1 parabola.

D. All of these give the same approximation.

E. None of these.

Problem 2. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Which of the following approximations of the area bounded by the curve $y = 25 - x^2$ and the x-axis is the most accurate?

A. Simpson's rule with 1 parabola.

B. Simpson's rule with 5 parabolas.

C. Simpson's rule with 10 parabolas.

D. All of these give the same approximation.

E. None of these.

Problem 3. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Consider approximating the area under $y = \sin x$ using the trapezoid rule with 1 trapezoid, the midpoint rule with 1 rectangle, and Simpson's rule with 1 parabola. Which of the following statements best describes the three approximations?

A. The midpoint rule overestimates, the trapezoid rule underestimates, and Simpson's rule is pretty close.

B. The midpoint rule underestimates, the trapezoid rule overestimates, and Simpson's rule is pretty close.

C. The midpoint rule is pretty close, the trapezoid rule overestimates, and Simpson's rule underestimates.

D. The midpoint rule and trapezoid rule both overestimate, and Simpson's rule is pretty close.

E. None of these.

Problem 4. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let $f(x)$ be a continuous function on $a \leq x \leq b$, partition the interval $[a, b]$ into n subintervals of width Δx with endpoints $a = x_0, x_1, \ldots, x_n = b$, and consider the sums

$$\text{I. } \sum_{i=1}^{n} \frac{f(x_{i-1}) + f(x_i)}{2} \Delta x, \qquad \text{II. } \sum_{i=1}^{n} f\Big(\frac{x_{i-1} + x_i}{2}\Big) \Delta x,$$

$$\text{III. } \sum_{i=1}^{n} \Big(\frac{1}{6} f(x_{i-1}) + \frac{4}{6} f\Big(\frac{x_{i-1} + x_i}{2}\Big) + \frac{1}{6} f(x_i)\Big) \Delta x.$$

For which of the above sums is the limit as n approaches ∞ equal to the integral

$$\int_a^b f(x)\, dx?$$

A. I only.

B. II only.

C. I and II only.

D. I, II, and III.

E. None of these.

Problem 5. (15 pts.) Use the midpoint rule with 3 rectangles, the trapezoid rule with 3 trapezoids, and Simpson's rule with 3 parabolas to find two different approximations of the integral

$$\int_2^8 \frac{1}{\ln x}\, dx.$$

You do not need to calculate decimal values for the approximations.

Problem 6. (15 pts.) Use the midpoint rule with 4 rectangles, the trapezoid rule with 4 trapezoids, and Simpson's rule with 4 parabolas to write three different approximations of

$$\int_0^8 e^{x^2}\, dx.$$

You do not need to calculate decimal values for the approximations.

Problem 7. (15 pts.) Approximate the integral

$$\int_1^3 8x^3\,dx$$

using Simpson's rule with 1 parabola, and compare this to the actual value of the integral. Show your work and simplify your answer.