

MATH 22 PRACTICE EXAM ON CYLINDRICAL SHELLS

Name:

Section:

Instructions:

- (1) Books, notes, and calculators are not permitted.
- (2) Simplify as instructed. On some problems, unsimplified correct answers will not receive full credit.
- (3) Show your work as instructed. On some problems, correct answers without justification will not receive full credit.

(1)

(2)

(3)

(4)

(5)

(6)

Total

Problem 1. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Which of the following integrals gives the volume of the solid obtained by rotating the region bounded by the curves $y = e^{x^2}$ and $y = ex^2$, $0 \leq x \leq 1$ about the y -axis?

- A. $\pi \int_0^1 (xe^{x^2} - ex^3) dx.$
- B. $2\pi \int_0^1 (xe^{x^2} - ex^3) dx.$
- C. $\pi \int_0^1 x^2(e^{x^2} - ex^2) dx.$
- D. $2\pi \int_0^1 (e^{2x^2} - e^2x^4) dx.$
- E. None of these.

Problem 2. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Which of the following is equal to the volume of the solid obtained by rotating the region bounded by the curves $y = x^2$ and $y = \sqrt{x}$ about the y -axis?

- A. $3\pi/10.$
- B. $3\pi/5.$
- C. $9\pi/100.$
- D. $9\pi/50.$
- E. None of these.

Problem 3. (15 pts.) Sketch the region bounded by the y -axis and the curves $y = x^2 - 3$, $y = 5 - x^2$. Find the volume of the solid obtained by rotating this region about the line $x = -1$. Show your work and simplify your answer. (An expression such as $13 \cdot 51/7\pi$ is simple enough.)

Problem 4. (15 pts.) Sketch the region bounded by the curve segment $y = \cos^2 x$, $-\pi/2 \leq x \leq \pi/2$, and the line $y = 1/4$. Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating this region about the line $x = \pi/2$. Show your work.

Problem 5. (15 pts.) Describe the solid whose volume is given by the integral

$$\int_0^4 2\pi(6-y)(4y-y^2) dy.$$

Explain your reasoning.

Problem 6. (15 pts.) A circle with center $(p, 0)$ and radius $r \leq p$ is rotated about the y -axis. Show that the volume of the resulting solid is $2\pi p \cdot \pi r^2$. Explain your reasoning.

Problem 7. (15 pts.) Let D be the region in the x, y plane bounded by the curve $y = \sec^2(x)$ and the lines

$$y = 0, \quad x = 0, \quad x = \pi/4.$$

Let E be the solid obtained by rotating D around the line $x = \pi/2$. Write an integral to compute the volume of E . Do not evaluate the integral.