MATH 21 PRACTICE EXAM ON THE DEFINITION OF DERIVATIVE

Name:

Section:

Instructions:

(1) Books, notes, and calculators are not permitted.
(2) Simplify as instructed. On some problems, unsimplified correct answers will not receive full credit.
(3) Show your work as instructed. On some problems, correct answers without justification will not receive full credit.

(1)

(2)

(3)

(4)

(5)

(6)

Total

Problem 1. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let $f(x) = x^2$. Which of the following is equal to $f'(3)$?

A. $\lim_{h\to 0} \dfrac{(3+h)^2 - 3^2}{h}$.

B. $\lim_{h\to 3} \dfrac{(x+h)^2 - x^2}{h}$.

C. $\lim_{h\to 3} \dfrac{(x-h)^2 - x^2}{h}$.

D. $\lim_{h\to 0} \dfrac{(3-h)^2 - 3^2}{h}$.

E. None of these.

Problem 2. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

The limit

$$\lim_{h\to 0} \frac{\sin(\pi + h)}{h}$$

is equal to which of the following?

A. The derivative of $\sin(x)$ at $x = \pi$.

B. The derivative of $\sin(x)$ at $x = -\pi$.

C. The derivative of $\cos(x)$ at $x = 0$.

D. The derivative of $\cos(x)$ at $x = h$.

E. None of these.

Problem 3. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let $f(x)$ be a differentiable function and define the function $g(x) = f'(x)$. What is the significance of the number $g(2)$?

A. It is the limit of the value of $f(x)$ when x approaches 2.

B. It is the slope of the line tangent to $y = f(x)$ at $x = 2$.

C. It is the limit of the slope of the line intersecting $y = f(x)$ at $x = 0$.

D. It is 2 times the slope of the line tangent to $y = f(x)$ at $x = 0$.

E. None of these.

Problem 4. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

The limit

$$\lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3}{h}$$

is equal to which of the following?

A. The derivative of $3x^2 + 3x + 1$.

B. The derivative of $3x^2 + 3x$.

C. The derivative of $3x^3 + 3x^2$.

D. The derivative of x^3.

E. None of these.

Problem 5. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let $f(x) = x^4$. Which of the following describes the relationship between $f'(-1)$, $f'(0)$, and $f'(1)$?

A. $f'(-1) < f'(0) < f'(1)$.

B. $f'(-1) = f'(0) = f'(1)$.

C. $f'(0) < f'(-1) = f'(1)$.

D. $f'(-1) = f'(1) < f'(0)$.

E. None of these.

Problem 6. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Consider the semicircle $f(x) = \sqrt{1 - x^2}$, defined for $-1 \leq x \leq 1$. For what values of x do we have $0 < f'(x) < \infty$?

A. $-1 < x < 1$.

B. $0 < x < 1$.

C. $-1 < x < 0$.

D. $x = 0$ only.

E. None of these.

Problem 7. (20 pts.) Use the definition of derivative to find $f'(x)$ for each of the following functions.

(a) $f(x) = 3x^2 - 2x + 1$.

(b) $f(x) = 1 - x^2$.

Problem 8. (20 pts.) Use the definition of derivative to find the equation of the line tangent to the curve

$$y = \frac{1}{x}$$

at $x = 2$.

Problem 9. (20 pts.) Let $f(x) = |x|$.

(a) Use the definition of the derivative to determine whether $f'(0)$ exists.

(b) Compute $f'(x)$ for $x \neq 0$ using the definition.

Problem 10. (15 pts.) Compute the limit

$$\lim_{x \to -2} \frac{2 - |x|}{2 + x}$$

if it exists, or explain why it does not exist. Show your work.

Problem 11. (15 pts.) Compute the limit

$$\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{x^2 + x} \right)$$

if it exists, or explain why it does not exist. Show your work.

Problem 12. (20 pts.) Consider the assertion

$$\lim_{x \to 4}(5x - 7) = 13.$$

(a) If $\epsilon > 0$ is fixed, then how small must difference between x and 4 be in order to guarantee that the difference between $5x - 7$ and 13 is smaller than ϵ?

(b) Assume that $|x - 4|$ is less than the expression you gave in part (a). Show that this indeed implies $|(5x - 7) - 13|$ to be less than ϵ.

Problem 13. (20 pts.) Consider the assertion

$$\lim_{x\to 2}(3x^2 - 4x + 5) = 9.$$

(a) Factor $(x - 2)$ out of $(3x^2 - 4x + 5) - 9$.

(b) Assuming that $|x-2|$ is less than 1, find an upper bound for the remaining factor of $(3x^2 - 4x + 5) - 9$.

(c) How small must $|x-2|$ be in order to guarantee that $|(3x^2-4x+5)-9|$ is smaller than ϵ?