

MATH 21 PRACTICE EXAM ON THE DEFINITION OF LIMIT

Name:

Section:

Instructions:

- (1) Books, notes, and calculators are not permitted.
- (2) Simplify as instructed. On some problems, unsimplified correct answers will not receive full credit.
- (3) Show your work as instructed. On some problems, correct answers without justification will not receive full credit.

(1)

(2)

(3)

(4)

(5)

(6)

Total

Problem 1. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let $f(x)$ be a function and let a, b be numbers. Suppose that for all $\epsilon > 0$ there exists a $\delta > 0$ (which can depend on ϵ) such that the inequality

$$a - \delta < x < a + \delta$$

implies the inequality

$$b - \epsilon < f(x) < b + \epsilon.$$

Which of the following is true?

A. $\lim_{x \rightarrow a} f(x) = b.$

B. $\lim_{x \rightarrow b} f(x) = a.$

C. $\lim_{x \rightarrow a} f(x) = b$ and $f(a) = b.$

D. $\lim_{x \rightarrow b} f(x) = a$ and $f(b) = a.$

E. None of these.

Problem 2. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let $f(x)$ be a function. Suppose that for all large numbers $M > 0$ there exists a large number $N > 0$ (which can depend on M) such that the inequality

$$x > N$$

implies the inequality

$$f(x) < -M.$$

Which of the following is true?

A. $\lim_{x \rightarrow \infty} f(x) = \infty.$

B. $\lim_{x \rightarrow \infty} f(x) = -\infty.$

C. $\lim_{x \rightarrow -\infty} f(x) = \infty.$

D. $\lim_{x \rightarrow -\infty} f(x) = -\infty.$

E. None of these.

Problem 3. (20 pts.) Consider the assertion

$$\lim_{x \rightarrow 4} (5x - 7) = 13.$$

(a) If $\epsilon > 0$ is fixed, then how small must the difference between x and 4 be in order to guarantee that the difference between $5x - 7$ and 13 is smaller than ϵ ?

(b) Assume that $|x - 4|$ is less than the expression you gave in part (a). Show that this indeed implies $|(5x - 7) - 13|$ to be less than ϵ .

Problem 4. (20 pts.) Consider the assertion

$$\lim_{x \rightarrow 2} (3x^2 - 4x + 5) = 9.$$

(a) Factor $(x - 2)$ out of $(3x^2 - 4x + 5) - 9$.

(b) Assuming that $|x - 2|$ is less than 1, find an upper bound for the remaining factor of $(3x^2 - 4x + 5) - 9$.

(c) How small must $|x - 2|$ be in order to guarantee that $|(3x^2 - 4x + 5) - 9|$ is smaller than ϵ ?

Problem 5. (20 pts.) Consider the function

$$f(x) = x^2 + x + 1.$$

(a) Use the ϵ , δ definition of limit to prove that

$$\lim_{x \rightarrow 2} f(x) = 7.$$

(b) Use the M , N definition of infinite limits to prove that

$$\lim_{x \rightarrow \infty} f(x) = \infty.$$

(Hint: use the fact that $f(x) > x^2$.)