

MATH 21 PRACTICE EXAM ON CONTINUITY

Name:

Section:

Instructions:

- (1) Books, notes, and calculators are not permitted.
- (2) Simplify as instructed. On some problems, unsimplified correct answers will not receive full credit.
- (3) Show your work as instructed. On some problems, correct answers without justification will not receive full credit.

(1)

(2)

(3)

(4)

(5)

(6)

Total

Problem 1. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Which of the following functions is *not* continuous at $x = 0$?

- A. 1 over the cosine of x .
- B. The absolute value of x .
- C. The square root of the absolute value of x .
- D. The greatest integer less than or equal to x .
- E. None of these. (All are continuous at $x = 0$.)

Problem 2. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

The function $f(x) = \frac{x-3}{x^2-9}$ has a removable discontinuity at which value of x ?

- A. $x = 0$.
- B. $x = 3$ only.
- C. $x = -3$ only.
- D. $x = 3$ and $x = -3$.
- E. None of these.

Problem 3. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Suppose that f is continuous on the closed interval $[0, 5]$ with $f(0) = -2$ and $f(5) = 7$. By the Intermediate Value Theorem, which of the following is guaranteed?

- A. f achieves a maximum value of 7 and a minimum value of -2 on $[0, 5]$.
- B. There exists a number c in the open interval $(0, 5)$ with $f(c) = 0$.
- C. f is increasing on the open interval $(0, 5)$.
- D. f is differentiable at every point in the open interval $(0, 5)$.
- E. None of these.

Problem 4. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let

$$f(x) = \begin{cases} 2x + k & \text{if } x \leq 1, \\ x^2 + 3 & \text{if } x > 1. \end{cases}$$

For what value of k is f continuous at $x = 1$?

- A. $k = -2$.
- B. $k = 2$.
- C. $k = 4$.
- D. $k = -4$.
- E. None of these.

Problem 5. (20 pts.) For each of the following functions, state that it is continuous on $(-\infty, \infty)$ or name one value of x at which the function is not continuous.

(a) $f(x) = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0, \\ 1 & \text{if } x = 0 \end{cases}.$

(b) $x^3 - x^2 - x - 1.$

(c) $\frac{1}{\sin x}.$

(d) $\tan x.$

(e) $\frac{x+2}{x-2}.$

(f) $\frac{1}{e^x}.$

(g) $\sqrt{x^2 + 1}$

Problem 6. (20 pts.) Define the function

$$f(x) = \begin{cases} -x + 3 & \text{if } x < 2, \\ -1 & \text{if } x = 2, \\ -x^2 + 4x - 3 & \text{if } x > 2. \end{cases}$$

(a) Find $\lim_{x \rightarrow 2^-}$.

(b) Find $\lim_{x \rightarrow 2^+}$.

(c) Show that $f(x)$ is or is not continuous at $x = 2$.

Problem 7. (15 pts.) Let

$$g(x) = \frac{\sqrt{x^2 - 9}}{x^2 - 2}.$$

(a) Where is $g(x)$ defined?

(b) Where is $g(x)$ continuous?