MATH 21 PRACTICE EXAM ON LIMITS

Name:

Section:

Instructions:

(1) Books, notes, and calculators are not permitted.
(2) Simplify as instructed. On some problems, unsimplified correct answers will not receive full credit.
(3) Show your work as instructed. On some problems, correct answers without justification will not receive full credit.

(1)

(2)

(3)

(4)

(5)

(6)

Total

Problem 1. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Which of the following is equal to

$$\lim_{x\to 3} \frac{x^2-9}{x-3}?$$

A. 0.

B. 3.

C. 6.

D. The limit does not exist.

E. None of these.

Problem 2. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let $f(x) = \dfrac{|x-2|}{x-2}$. Which of the following best describes $\lim_{x\to 2} f(x)$?

A. The limit equals -1.

B. The limit equals 0.

C. The limit equals 1.

D. The limit does not exist because the one-sided limits differ.

E. None of these.

Problem 3. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Suppose that $\lim_{x\to 4} f(x) = 5$, $\lim_{x\to 4} g(x) = -2$, and $\lim_{x\to 4} h(x) = -1$. Which of the following is equal to

$$\lim_{x\to 4}[f(x)g(x) + 4h(x)]?$$

A. -28.

B. -14.

C. 6.

D. This cannot be determined without more information.

E. None of these.

Problem 4. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Suppose that $g(x)$ is defined for all x and that $\lim_{x\to 2} g(x) \geq 0$. Which of the following statements is true?

A. $\lim_{x\to 2} \sqrt{g(x)} < \sqrt{\lim_{x\to 2} g(x)}$

B. $\lim_{x\to 2} \sqrt{g(x)} = \sqrt{\lim_{x\to 2} g(x)}$

C. $\lim_{x\to 2} \sqrt{g(x)} > \sqrt{\lim_{x\to 2} g(x)}$

D. This cannot be determined without more information.

E. None of these.

Problem 5. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let $f(x)$ be a function and let a, b be numbers. Suppose that for all $\epsilon > 0$ there exists a $\delta > 0$ (which can depend on ϵ) such that the inequality

$$a - \delta < x < a + \delta$$

implies the inequality

$$b - \epsilon < f(x) < b + \epsilon.$$

Which of the following is true?

A. $\lim_{x \to a} f(x) = b$.

B. $\lim_{x \to b} f(x) = a$.

C. $\lim_{x \to a} f(x) = b$ and $f(a) = b$.

D. $\lim_{x \to b} f(x) = a$ and $f(b) = a$.

E. None of these.

Problem 6. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Let $f(x) = |x - 3|$. Which of the following best describes the behavior of $f(x)$ when x is near 3?

A. Neither of the limits $\lim_{x \to 3^-} f(x)$ and $\lim_{x \to 3^+} f(x)$ exists.

B. Exactly one of the limits $\lim_{x \to 3^-} f(x)$ and $\lim_{x \to 3^+} f(x)$ exists.

C. The limits $\lim_{x \to 3^-} f(x)$ and $\lim_{x \to 3^+} f(x)$ exist, but they are not equal.

D. The limits $\lim_{x \to 3^-} f(x)$ and $\lim_{x \to 3^+} f(x)$ exist and are equal.

E. None of these.

Problem 7. (20 pts.) Compute the following trigonometric limits or write "Does not exist" if appropriate. Showing your work is not necessary but may be worth partial credit.

(a) $\lim_{x\to 0} \dfrac{1}{\sin x} =$

(b) $\lim_{x\to 0} \cos x =$

(c) $\lim_{x\to 0} \dfrac{\cos x}{x} =$

(d) $\lim_{x\to 0} \tan x =$

(e) $\lim_{x\to 0} x \sin \frac{1}{x} =$

(f) $\lim_{x\to \infty} \sin \frac{1}{x} =$

(g) $\lim_{x\to \infty} \dfrac{\sin x}{x^2} =$

Problem 8. (20 pts.) Define the function

$$f(x) = \begin{cases} 0 & \text{if } x < -2, \\ 4 - x^2 & \text{if } -2 \leq x < 2, \\ x - 1 & \text{if } x \geq 2. \end{cases}$$

(a) Sketch the graph of $y = f(x)$.

(b) For which numbers a does the limit $\lim_{x \to a} f(x)$ exist?

Problem 9. (15 pts.) Compute the limit

$$\lim_{x \to -2} \frac{2 - |x|}{2 + x}$$

if it exists, or explain why it does not exist. Show your work.

Problem 10. (15 pts.) Compute the limit

$$\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{x^2 + x} \right)$$

if it exists, or explain why it does not exist. Show your work.

Problem 11. (20 pts.) Consider the assertion

$$\lim_{x \to 4}(5x - 7) = 13.$$

(a) If $\epsilon > 0$ is fixed, then how small must difference between x and 4 be in order to guarantee that the difference between $5x - 7$ and 13 is smaller than ϵ?

(b) Assume that $|x - 4|$ is less than the expression you gave in part (a). Show that this indeed implies $|(5x - 7) - 13|$ to be less than ϵ.

Problem 12. (20 pts.) Consider the assertion

$$\lim_{x\to 2}(3x^2 - 4x + 5) = 9.$$

(a) Factor $(x - 2)$ out of $(3x^2 - 4x + 5) - 9$.

(b) Assuming that $|x-2|$ is less than 1, find an upper bound for the remaining factor of $(3x^2 - 4x + 5) - 9$.

(c) How small must $|x-2|$ be in order to guarantee that $|(3x^2-4x+5)-9|$ is smaller than ϵ?