MATH 205 PRACTICE EXAM ON OSCILLATIONS OF A MECHANICAL SYSTEM

Name:

Section:

Instructions:

(1) Books, notes, and calculators are not permitted.
(2) Simplify as instructed. On some problems, unsimplified correct answers will not receive full credit.
(3) Show your work as instructed. On some problems, correct answers without justification will not receive full credit.

(1)

(2)

(3)

(4)

(5)

(6)

Total

Problem 1. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Consider the differential equation

$$my'' + cy' + ky = 0$$

for a spring-mass system with mass $m > 0$, damping constant $c > 0$, and spring constant $k > 0$. Which of the following statements about the system is false?

A. Motion is always oscillating, regardless of the values of m, c, k.

B. The motion is underdamped if $c^2 < 4km$.

C. The motion is critically damped if $c^2 = 4km$.

D. The motion is overdamped if $c^2 > 4km$.

E. None of these: all four statements are true.

Problem 2. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Consider the differential equation

$$my'' + ky = A\cos(\alpha t) + B\sin(\alpha t)$$

for an undamped spring-mass system with mass $m > 0$, spring constant $k > 0$, and forcing function $A\cos(\alpha t) + B\sin(\alpha t)$, where A, B, α are real constants. If resonance occurs in the system, then which of the following is the general solution of the differential equation?

A. $y(t) = C_1 \cos\left(\sqrt{\frac{k}{m}}t\right) + C_2 \sin\left(\sqrt{\frac{k}{m}}t\right) + \frac{1}{2m}\sqrt{\frac{m}{k}}Bt\cos\left(\sqrt{\frac{k}{m}}t\right) + \frac{1}{2m}\sqrt{\frac{m}{k}}At\sin\left(\sqrt{\frac{k}{m}}t\right)$.

B. $y(t) = C_1 \cos\left(\sqrt{\frac{k}{m}}t\right) + C_2 \sin\left(\sqrt{\frac{k}{m}}t\right) - \frac{1}{2m}\sqrt{\frac{m}{k}}Bt\cos\left(\sqrt{\frac{k}{m}}t\right) + \frac{1}{2m}\sqrt{\frac{m}{k}}At\sin\left(\sqrt{\frac{k}{m}}t\right)$.

C. $y(t) = C_1 \cos\left(\sqrt{\frac{k}{m}}t\right) + C_2 \sin\left(\sqrt{\frac{k}{m}}t\right) + \frac{1}{2m}\sqrt{\frac{m}{k}}At\cos\left(\sqrt{\frac{k}{m}}t\right) + \frac{1}{2m}\sqrt{\frac{m}{k}}Bt\sin\left(\sqrt{\frac{k}{m}}t\right)$.

D. $y(t) = C_1 \cos\left(\sqrt{\frac{k}{m}}t\right) + C_2 \sin\left(\sqrt{\frac{k}{m}}t\right) - \frac{1}{2m}\sqrt{\frac{m}{k}}At\cos\left(\sqrt{\frac{k}{m}}t\right) + \frac{1}{2m}\sqrt{\frac{m}{k}}Bt\sin\left(\sqrt{\frac{k}{m}}t\right)$.

E. None of these.

Problem 3. (20 pts.) A spring-mass system has an object of mass $2\,\mathrm{kg}$, which hangs from a spring with spring constant $5\,\mathrm{kg/s^2}$, and which is subject to damping with constant $6\,\mathrm{kg/s}$. Let $y(t)$ be the position in meters below equilibrium position of the object at time t seconds. At time 0 the object is stretched $2\,\mathrm{m}$ below equilibrium position and released with an initial velocity of $1\,\mathrm{m/s}$ upward.

(a) Write an initial value problem consisting of a differential equation which relates $y(t)$ and its derivatives, and initial conditions which describe the initial position and initial velocity of the object.

(b) Solve the above initial value problem to give an explicit formula for the position of the object at time t.

(continued $\to$)

(c) Describe the long-term behavior of the object as t approaches infinity.

Problem 4. (20 pts.) Consider a spring-mass system whose motion is governed by the differential equation

$$y'' + 16y = 60\cos(2t) + 60\sin(2t). \tag{1}$$

(a) Find the complementary solution of (1), that is, the general solution of the homogeneous differential equation corresponding to (1).

(b) Use the method of undetermined coefficients to find a particular solution of (1).

(continued →)

(c) Find the general solution of (1).

(d) The object in the spring-mass system has a period of motion which does not depend upon any initial conditions. Find this period of motion.

Problem 5. (20 pts.) Consider a spring-mass system whose motion is governed by the differential equation and initial conditions

$$y'' + 5y' + 6y = 0, \qquad y(0) = -1, \quad y'(0) = 4. \tag{2}$$

(a) Solve the initial value problem (2).

(continued →)

(b) Is the system (2) underdamped, critically damped, or overdamped?

(c) Find the time at which the mass passes through the equilibrium position.

(d) Determine the maximum displacement of the object from its equilibrium position.