

**MATH 205 PRACTICE EXAM ON LINEAR DIFFERENTIAL
EQUATIONS WITH CONSTANT COEFFICIENTS**

Name:

Section:

Instructions:

- (1) Books, notes, and calculators are not permitted.
- (2) Simplify as instructed. On some problems, unsimplified correct answers will not receive full credit.
- (3) Show your work as instructed. On some problems, correct answers without justification will not receive full credit.

(1)

(2)

(3)

(4)

(5)

(6)

Total

Problem 1. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Consider the differential equation

$$y'' + y = 0.$$

Which of the following functions is a solution of the differential equation?

A. $y = \sin(x) + 3\cos(x)$.

B. $y = \sin(3x) + \cos(x)$.

C. $y = e^x + 3e^{-x}$.

D. $y = e^{3x} + e^{-x}$.

E. None of these.

Problem 2. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Consider the differential equation

$$y'' - y = 0.$$

Which of the following functions is a solution of the differential equation?

A. $y = \sin(x) + 3\cos(x)$.

B. $y = \sin(3x) + \cos(x)$.

C. $y = e^x + 3e^{-x}$.

D. $y = e^{3x} + e^{-x}$.

E. None of these.

Problem 3. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Consider the differential equation

$$y'' + 100y = 36 \cos(8x) + 36 \sin(8x).$$

Which of the following functions is a solution of the differential equation?

- A. $y = \cos(8x) + \sin(8x).$
- B. $y = 36 \cos(8x) + 36 \sin(8x).$
- C. $y = \cos(10x) + \sin(10x).$
- D. $y = 36 \cos(10x) + 36 \sin(10x).$
- E. None of these.

Problem 4. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Consider the differential equation

$$y'' - 15y' + 56y = 6e^{5x}.$$

Which of the following functions is a solution of the differential equation?

- A. $y = e^{4x}.$
- B. $y = e^{5x}.$
- C. $y = e^{6x}.$
- D. $y = e^{7x}.$
- E. None of these.

Problem 5. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Consider the differential equation

$$y'' - 15y' + 56y = (2 + 2x)e^{8x}.$$

Which of the following functions is a solution of the differential equation?

- A. $y = e^{8x}$.
- B. $y = xe^{8x}$.
- C. $y = x^2e^{8x}$.
- D. $y = x^3e^{8x}$.
- E. None of these.

Problem 6. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Consider the differential equation

$$y'' - 15y' + 56y = (10 - 10x)e^{7x} + (10 + 10x)e^{8x}.$$

Which of the following functions is a solution of the differential equation?

- A. $y = 5x^2(e^{7x} + e^{8x})$.
- B. $y = 5x^3(e^{7x} + e^{8x})$.
- C. $y = (10 - 10x)e^{7x} + (10 + 10x)e^{8x}$.
- D. $y = (10x - 10x^2)e^{7x} + (10x + 10x^2)e^{8x}$.
- E. None of these.

Problem 7. (20 pts.) Consider the differential equation

(1)
$$y'' - 20y' + 100y = 6e^{10x}.$$

(a) Find all solutions of the homogeneous differential equation corresponding to (1).

(b) Find one particular solution of (1).

(continued $\rightarrow$)

(c) Find all solutions of (1).

(d) Which solution of (1) satisfies $y(0) = 2$ and $y'(0) = -5$?

Problem 8. (20 pts.) Consider the differential equation

(2)
$$y'' + 16y = 27 \cos(5x).$$

(a) Find all solutions of the homogeneous differential equation corresponding to (2).

(b) Find one particular solution of (2).

(continued $\rightarrow$)

(c) Find all solutions of (2).

(d) Which solution of (2) satisfies $y(0) = -1$ and $y'(0) = -12$?

Problem 9. (20 pts.) Give the general solution to the following differential equations:

(a) $y'''' + 2y'' + y = 0$.

(b) $(D - 3)(D^2 - 2D + 5)^2 y = 0$.

Problem 10. (20 pts.) Consider the following nonhomogeneous differential equation:

$$y''' + 11y'' + 7y' - 147y = -120x^2e^{-7x} + 81e^{2x},$$

which can be equivalently written as $P(D)y = F$:

$$(D + 7)^2(D - 3)y = -120x^2e^{-7x} + 81e^{2x}.$$

The solution of the homogeneous equations is given by

$$y_c(x) = C_1e^{3x} + C_2e^{-7x} + C_3xe^{-7x}.$$

Find one particular solution of the nonhomogeneous equation.