

MATH 205 PRACTICE EXAM ON LAPLACE TRANSFORMS

Name:

Section:

Instructions:

- (1) Books, notes, and calculators are not permitted.
- (2) Simplify as instructed. On some problems, unsimplified correct answers will not receive full credit.
- (3) Show your work as instructed. On some problems, correct answers without justification will not receive full credit.

(1)

(2)

(3)

(4)

(5)

(6)

Total

Problem 1. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Which of the following statements best explains the Laplace transform $L[1]$ of the constant function 1?

- A. We can think of it as $L[t^0]$ or $L[e^{0t}]$.
- B. We can think of it as $L[t^0]$, but not as $L[e^{0t}]$.
- C. We can think of it as $L[e^{0t}]$, but not as $L[t^0]$.
- D. We can not think of it as $L[t^0]$ or $L[e^{0t}]$.
- E. None of these.

Problem 2. (5 pts.) Circle the correct answer. There will be no partial credit. You do not need to show your work.

Which of the following functions does not have a Laplace transform?

- A. $2t$.
- B. t^2 .
- C. e^{2t} .
- D. e^{t^2} .
- E. All of these functions have Laplace transforms.

Problem 3. (15 pts.) Find Laplace transforms of the following functions.

(a) $(2 + e^{5t})t^3$.

(b) $e^{\alpha t}(k \cos(\beta t) + m \sin(\beta t))$, where α, β, k, m are real numbers.

(c) $f(t) = \begin{cases} 0 & \text{when } 0 \leq t < 1, \\ t^2 - 2t + 2 & \text{when } t \geq 1. \end{cases}$

Problem 4. (15 pts.) Find inverse Laplace transforms of the following functions.

(a) $\frac{3}{s^2 + 4}$.

(b) $\frac{2s - 3}{s^2 - 4}$

(c) $\frac{1 - 2s}{s^2 + 4s + 5}$

Problem 5. (15 pts.) Use the method of Laplace transforms to solve the initial value problem

$$y'' - 9y' + 20y = 0, \quad y(0) = 3, \quad y'(0) = 10.$$

Problem 6. Use the method of Laplace transforms to solve the initial value problem

$$y'' - y = 2 \sin(t), \quad y(0) = -1, \quad y'(0) = 0.$$

Problem 7. (15 pts.) Use the method of Laplace transforms to solve the initial value problem

$$y'' + 2y' + 5y = 6e^{-t} \cos(t), \quad y(0) = 1, \quad y'(0) = 1.$$

Hint: use the fact that

$$\frac{6(s+1)}{((s+1)^2+1)((s+1)^2+4)} = \frac{2(s+1)}{(s+1)^2+1} - \frac{2(s+1)}{(s+1)^2+4}.$$

Problem 8. (15 pts.) Solve the initial value problem

$$y'' + 4y = \begin{cases} t & \text{for } 0 \leq t < \pi/2, \\ \pi/2 & \text{for } \pi/2 \leq t < \infty, \end{cases} \quad y(0) = y'(0) = 0.$$

Problem 9. (15 pts.) Solve the initial value problem

$$y'' + 2y' + 2y = \delta(t - \pi), \quad y(0) = y'(0) = 0,$$

where δ is the Dirac impulse function.