Competitive Disclosure of Multiple Product Attributes*

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Abstract

We analyze a duopolistic model of quality disclosure in which product differentiation can be either horizontal or vertical under two vertically differentiated product attributes. Disclosure is fully revealing in the absence of disclosure cost. When disclosure is costly, firms partially disclose. We apply the familiar economic properties of absolute and comparative advantages in a novel way to characterize partial disclosure under the multiple product attributes. The type of product differentiation and intensity of competition interact with absolute and comparative *attribute* advantages in determining the equilibrium non-disclosure sets of the firms. When consumers fall short of forming rational expectation after non-disclosure, firms may partially disclose even without disclosure cost, and the same properties continue to dictate firms' disclosure behavior in the boundedly rational paradigm.

Keywords: Quality Disclosure; Multiple Product Attributes; Product Differentiation; Absolute and Comparative Advantages; Boundedly Rational Consumers

JEL classification: C72; D21; D82; L13; L15

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1 Introduction

Firms engaging in competition are motivated to make known to consumers the superiority of their products relative to their competitors'. When products have multiple attributes, this incentive often manifests as a tendency to underscore the product attributes that represent one's strength and to downplay the attributes that are not as remarkable. In a familiar term, *absolute* and *comparative advantages* of some sorts are at work to influence the marketing efforts of firms in deciding what to promote to, and what to hide from, consumers. This paper analyzes how this and other factors contribute to determine quality disclosure in a competitive setting with multiple product attributes.

To demonstrate the superiority of one's product is to establish the inferiority of the competitors'; the comparative information that firms reveal to consumers is bound to cover not only the qualities of their own products but also those of the rivals. One popular medium that makes available this kind of competitive-product information is the product-category reviews provided by third parties. Examples abound of this kind of reviews, which have been shown to influence consumer choices.[1] There are third-party reviews provided by traditional publishers (*Consumer Reports*, *PC Magazine*, *Car and Driver*, *Runner's World*, *U.S. News & World Report's Best Colleges*), online platforms founded by individuals (*The Points Guy*, *Tom's Guide*), and resellers (*TireRack*).

A classic example of third-party reviews with attribute breakdown is the "Best Cars and Top 10 Lists" provided by the infomediary *Kelley Blue Book*. Their lists contain links to detailed reviews of each selected vehicle, covering its "pros and cons" or the "what we like and don't like," which are essentially reviews of attributes vehicle shoppers care about. The marketing campaign of an automobile manufacturer that promotes its products making the top lists effectively reveals to potential buyers, who otherwise may not be aware of the lists, the attribute qualities of its products as well as the competitors'. What drives a firm's disclosure decision when the quality information being disclosed is bundled, covering one's qualities as well as rivals'? What are the implications of absolute and comparative advantages in product attributes for the informational value of quality disclosure to consumers? How do product differentiation and disclosure cost play a role?

We tackle these questions with a duopoly model with four key features: two products each with two *vertically differentiated attributes*, consumers with heterogeneous preferences, first-stage verifiable attribute-quality disclosure, and second-stage price competition. The two vertically differentiated product attributes bring in a novel disclosure environment in which the products themselves can be either horizontally or vertically differentiated, and

[1] See, e.g., the empirical studies by Pope (2009), Simonsohn (2011), and Luca and Smith (2013).

the key concepts in our characterizations of disclosure behavior, absolute and comparative advantages, are defined in terms of quality differences of these attributes. *Absolute attribute advantages* refer to the quality difference between the two products with respect to a given attribute, and *comparative attribute advantages* are differences (between attributes) in differences (between qualities).

Consider a sports sedan that is *far superior* to a family sedan in terms of driving performance and also offers a *slightly more* comfortable ride. Both performance and comfort are vertically differentiated attributes, and the sports sedan has absolute advantages in both attributes and comparative advantage in performance. Though the family sedan is inferior in both attributes, it has comparative advantage in comfort because it is relatively less inferior in this attribute. In terms of product differentiation, with the sports sedan having two absolute advantages the two vehicles are vertically differentiated. On the other hand, if the performance suspension of the sports sedan were to result in too stiff a ride that would make it less comfortable than the family sedan, then each vehicle would have its own absolute advantage—the two sedans would be horizontally differentiated.

We characterize the equilibrium disclosure outcomes—loosely, what consumers know in equilibrium about the attribute-quality differences or attribute advantages—which reflect the informational value of the disclosure to consumers. In a benchmark case with costless disclosure, full revelation, which arises under the well-known unraveling result of Grossman (1981) and Milgrom (1981) in the canonical model of one firm with one-dimensional quality, also arises in our duopolistic model with two-dimensional quality. The main insights of our modeling exercise, however, lie in the situations where at least some attribute advantages are not disclosed by either firm so that consumers are only partially informed and quality disclosure falls short of bringing the highest informational value to consumers. Such a partially revealing equilibrium arises when disclosure is costly.

We further develop the sedan example to exemplify. The sports sedan is in a sense the leading product of the two. Being in the performance genre, it certainly has advantage—absolute and comparative—in driving performance, and the uncertainty faced by consumers in respect of this attribute is limited to how much more powerful it is than the family sedan. On the other hand, while the sports sedan may or may not offer a more comfortable ride than the family sedan, the difference one way or the other would not be as dramatic as the their difference in performance. Our equilibrium analysis shows that the manufacturers of the two vehicles, with their different prior positions in the market, would have different considerations in their disclosure decisions under costly disclosure.

Selling the *a priori* trailing product, the family-sedan maker's disclosure decision hinges on comparative attribute advantages, which are characterized jointly by the absolute advantages in performance and comfort. Comparative attribute advantages weaken when (a) the sport sedan becomes less powerful (but still more so than the family sedan), in which case the family sedan would be facing the sports sedan priced more competitively, and (b) the sports sedan offers more comfortable (or less uncomfortable) ride relative to the family sedan, in which case the family sedan would be losing consumers looking for comfort to the sports sedan. The effects on both prices and market shares resulting from weaker comparative advantages are unfavorable to the family-sedan maker. Accordingly, when disclosure is costly so that there are some attribute qualities that firms find it not worthwhile to disclose, in equilibrium the firm selling the trailing product would not disclose the attribute qualities at which comparative advantages are sufficiently weak.

Selling the *a priori* leading product, the sports-sedan maker's disclosure decision depends on absolute attribute advantages, which in turn determine the degree of product differentiation. Regardless of whether the two vehicles are horizontally or vertically differentiated, the degree of differentiation decreases when (a′) the sports sedan becomes less powerful, in which case it would be losing consumers looking for performance to the family sedan, and (b′) the difference in comfort between the two vehicles in either direction narrows, in which case the sports sedan would have to be priced more competitively to maintain market share. For these reasons, when disclosure is costly, in equilibrium the firm selling the leading product would not disclose the attribute qualities when absolute attribute advantages are sufficiently weak and thus the products are sufficiently similar.

Our analysis of partially revealing equilibria, which represents the key contribution of this paper, generates three main insights about disclosure behavior and outcomes. The sedan example above illustrates the first insight. In our model with two vertically differentiated product attributes, absolute and comparative advantages defined in terms of these attributes fully characterize the "anchors" of the firms' non-disclosure sets. The leading firm's non-disclosure set anchors at where absolute attribute advantages are weakest, while that of the trailing firm anchors at where comparative attribute advantages are weakest.

The second insight concerns product differentiation and the relationship between the anchors. If consumers know *ex ante* that each firm has an absolute advantage in one attribute (i.e., products are horizontally differentiated), then the anchors of the firms' non-disclosure sets coincide. However, if consumers *ex ante* do not know whether one firm has an absolute advantage in both attributes (i.e., products are vertically differentiated) or each firm has an absolute advantage in one attribute (i.e., products are horizontally differentiated), then the anchors differ. In terms of the sedan example, conditions (a) and (a′) are equivalent, while (b) and (b′) may or may not be the same depending on consumer priors about the type of product differentiation.

The third insight concerns the disclosure outcomes. While the two firms have their own non-disclosure sets, consumers are only uninformed of the attribute qualities that lie in their intersection; outside of the intersection, consumers learn the attribute qualities from at least one firm. We find that even when the anchors of the two non-disclosure sets differ and the disclosure cost is vanishing, there are attribute qualities that neither firm discloses. In equilibrium, firms engage in an optimal tradeoff between, on one side of the equation, letting consumers know precisely the attribute qualities and, on the other side, saving the disclosure cost at the expense of being pooled with the non-disclosed qualities. The partially-revealing disclosure outcomes also arise in the absence of disclosure cost if consumers fail to extract the informational content of non-disclosure. The presence of disclosure cost or boundedly rational consumers limits the informational value that quality disclosure delivers to consumers in our environment with multiple product attributes.

To the best of our knowledge, we are the first to analyze competitive disclosure with two-dimensional vertical quality. Previous studies on quality disclosure have analyzed models with single-dimensional quality (e.g., Board, 2009; Guo and Zhao, 2009; Koessler and Renault, 2012; Celik, 2014; Janssen and Teteryatnikova, 2016; Guo, 2020), multidimensional quality under a monopoly (e.g., Sun, 2011), and duopolies differentiated by one vertical and one horizonal qualities (e.g., Levin et al., 2009; Anderson and Renault, 2009; Ghosh and Galbreth, 2013; Gu and Xie, 2013; Hotz and Xiao, 2013). The apparently slight difference of our environment leads to unique features of the equilibria. The strategic forces that drive non-disclosure in our model are multifaceted and differ from the typical results in the literature, where firms choose not to disclose either when quality is lowest (Board, 2009; Guo and Zhao, 2009; Levin et al., 2009) or when prices are lowest under minimal product differentiation (Board, 2009). Simply put, our difference in this regard is that non-disclosure is characterized by, for one firm, minimal product differentiation but not lowest prices, and, for the other firm, lowest prices but not minimal product differentiation.

Related Literature. We proceed to a more systematic and thorough review of the recent literature on quality disclosure.[2] Studies in this literature can be categorized based on three dichotomies with respect to (a) market structures: monopoly or duopoly; (b) product quality: single-dimensional or two-dimensional; and (c) the unraveling result: whether it holds or fails.[3] Our study belongs to the combination of duopoly, two-dimensional quality, and possible failing of unraveling. Naturally, previous studies in the same category (Levin et al., 2009; Anderson and Renault, 2009; Ghosh and Galbreth, 2013; Gu and Xie, 2013; Hotz and Xiao, 2013) are closest to ours.

[2] Dranove and Jin (2010) provide an excellent survey of the theoretical and empirical studies tracing to the inception of the literature.

[3] This last dichotomy correlates with, and thus largely covers, another important variation: whether disclosure is costly or not.

In a costly-disclosure environment with two-dimensional quality, one horizontal and one vertical, Levin et al. show that a cartel is more likely to disclose than when the firms are in duopoly.[4] Anderson and Renault and Gu and Xie also consider horizontal-vertical dimensions of quality, but the vertical quality in their models is known to consumers. Similar to our disclosure technology, Anderson and Renault allow disclosure of comparative information interpreted as "comparative advertising." The low-quality firm in their model discloses, but neither firm would disclose if comparative advertising was banned. Disclosure in Anderson and Renault is costless; under costly disclosure and without the possibility of disclosing comparative information, Gu and Xie obtain the opposite result, where the high-quality firm discloses and the low-quality firm can possibly free-ride. Free-riding on the other firm's disclosure also can arise in our model.

Ghosh and Galbreth study costly disclosure in the presence of partially informed consumers who can search for information at a cost. They show that firms disclose less information as there are more consumers who are informed about the qualities of one firm but not the other's, or when search costs increase. Hotz and Xiao consider horizontal and vertical qualities that are correlated and unknown to consumers, and the unraveling result fails in their costless-disclosure environment due to the assumed correlation. Despite being in the same broad category, as stated above our model differs from these studies in terms of both the environment and the results.[5]

In environments that intersect with ours in terms of market structures but not quality dimensions, Board (2009) and Guo and Zhao (2009) analyze how competition drives disclosure under a single vertical quality. Board demonstrates that disclosure may intensify price competition to such an extent that it outweighs the benefit of being separated from lower-quality products, upsetting the unraveling even in the absence of disclosure cost. This stands in contrast to our benchmark case with costless disclosure, in which the additional dimension of vertical quality effectively recovers the unraveling. Guo and Zhao examine the effects of timing. They find that the leader in sequential disclosure reveals less information that its counterpart in simultaneous disclosure.

Janssen and Teteryatnikova (2016) analyze quality disclosure in the same category as Board and Guo and Zhao, but the single-dimensional quality is horizontal. As in Anderson and Renault, they examine the role of comparative disclosure. A distinguishing feature of their model is that prices may be announced as part of the product information. They find that unraveling can fail when disclosure is non-comparative or when price information

[4] In a note, Jansen (2017) extends the model in Levin et al. by allowing the degree of differentiation of the horizontal quality to vary and evaluates the resulting impacts on disclosure.

[5] Kuksov and Lin (2010) study strategic information provision in a related environment of multidimensional information and two vertically differentiated firms, but the two dimensions of their information concern product quality and consumers' uncertainty about their own preferences.

is not integral to the disclosure. Sun (2011) analyzes an environment that intersects with ours in terms of the number of quality dimensions, but the dimensions she considers are one horizontal and one vertical. She also considers a different market structure and shows that unraveling can fail in a monopolistic model.[6]

There are other related studies outside of the three dichotomies. Our disclosure technology features comparative information. In a market environment similar to ours with two firms and two product attributes, Shaffer and Zettelmeyer (2009) examine the conditions under which firms prefer comparative to non-comparative advertising. Iyer and Singh (forthcoming) investigate a persuasion contest in which a firm can disclose own positive information or negative information about the rival. They characterize the conditions under which positive or negative communication equilibria prevail.[7]

Finally, our consideration of boundedly rational consumers relates our paper to a line of work that studies how the unraveling result fares when consumers depart from the rational ideals (e.g., Milgrom and Roberts, 1986; Fishman and Hagerty, 2003; Hirshleifer and Teoh, 2003; Hirshleifer et al., 2004; Zhang and Li, 2021). Closest to us is Hirshleifer et al., who find in a single-dimensional environment that the presence of receivers who make naïve inferences about non-disclosure dampens disclosure. Under a different behavioral variation, Zhang and Li find that consumer loss aversion, by contrast, could result in more disclosure.

The rest of the paper is organized as follows. Section 2 describes the oligopolistic disclosure model. Section 3 analyze the simultaneous price competition in the second stage of the game. Section 4 contains the main analysis of the paper, in which we analyze the equilibrium disclosure decisions and outcomes. Section 5 concludes.

2 The Model

Firms and Consumers. Two firms, firm A and firm B, sell product A and product B respectively. The two products belong to the same product class, and each product has two vertically differentiated attributes. The quality of attribute $i \in \{1, 2\}$ of product $K \in \{A, B\}$, q_i^K, is distributed on $\left[\underline{q}_i^K, \overline{q}_i^K\right]$ according to distribution function F_i^K. The

[6]The disclosure environment with no intersection with ours in terms of the number of quality dimension and market structures is monopoly with one single-dimensional quality. Two studies in this category are Koessler and Renault (2012) and Celik (2014), who find conditions under which the unraveling results may or may not hold with horizontal quality and price commitment. In a recent study, Guo (2020) considers single-dimensional disclosure in a bilateral-monopoly model of upstream and downstream firms.

[7]We use non-seller sources of information—third-party product reviews—as motivation for our disclosure technology of comparative information. In relation to this, Chen and Xie (2005) examine how firms adapt their advertising and pricing strategies in the presence of third-party reviews, and Chen and Xie (2008) analyze how firms' marketing communication strategy responds to consumer reviews.

exogenously determined profile of attribute qualities are known by both firms.

The market for the product class is made up of a unit mass of heterogeneous consumers, each interested in purchasing one unit of one of the two products. Consumers are uncertain about the attribute qualities of each product and are commonly known to use the quality distributions as their priors. A consumer who purchases product K receives utility:

$$u_K = r + \theta q_1^K + (1-\theta) q_2^K - p_K,$$

where θ is a parameter measuring the relative weight of consumer's taste toward attribute 1 and attribute 2, p_K is the price of product K, and r is the reservation value of the product class assumed to be large enough so that all consumers purchase one of two products. The heterogeneity of consumers is captured by the distribution of θ, which is uniform on $[0,1]$. While each attribute is vertically differentiated, the heterogeneity of consumer preferences implies that the two products can be either horizontally or vertically differentiated.

Consumers may learn about attribute qualities from the firms. In the first of the two stages of the game, firms A and B simultaneously decide whether to disclose the qualities. A firm incurs $c \geq 0$ when it discloses. This cost parameter is common to both firms. Disclosure is credible and verifiable. All consumers learn all attribute qualities of both firms if at least one firm discloses. In the second stage of the game, the firms simultaneously compete in prices. If neither firm discloses in the first stage, consumers make purchase decisions based on expected attribute qualities $\hat{q}_i^K$, $i \in \{1,2\}$ and $K \in \{A,B\}$.

Attribute Advantages. The unit of analysis of our model is the difference in attribute qualities, which we term *attribute advantage*. We denote by $q_i^{K\text{-}K'} = q_i^K - q_i^{K'}$ the excess in the quality of attribute i of product K over that of product $K' \neq K$. The prior distribution of $q_i^{K\text{-}K'}$, $F_i^{K\text{-}K'}$, is derived accordingly as the convolutions of the primitive distributions, with support $\left[\underline{q}_i^{K\text{-}K'}, \overline{q}_i^{K\text{-}K'}\right] = \left[\underline{q}_i^K - \overline{q}_i^{K'}, \overline{q}_i^K - \underline{q}_i^{K'}\right]$. We assume that $F_i^{K\text{-}K'}$, $i \in \{1,2\}$, is continuously differentiable with everywhere positive density $f_i^{K\text{-}K'}$.

We define two types of attribute advantages, absolute and comparative. Firm K is said to have *absolute advantage* in attribute i if $q_i^{K\text{-}K'} > 0$, i.e., product K has a higher quality than product $K' \neq K$ in respect of attribute i. We refer to a larger difference $q_i^{K\text{-}K'}$ as a greater absolute attribute advantage or the advantage improves.[8] While absolute advantages are differences in qualities, comparative advantages are differences in differences; firm K is said to have *comparative advantage* in attribute i if $q_i^{K\text{-}K'} - q_{i'}^{K\text{-}K'} > 0$, i.e., compared

[8] Since $q_i^{K\text{-}K'} = -q_i^{K'\text{-}K}$, one can also refer to $q_i^{K\text{-}K'} > 0$, which is equivalent to $q_i^{K'\text{-}K} < 0$, as firm K' having an absolute *disadvantage* in attribute i and a larger, i.e., less negative, $q_i^{K'\text{-}K} < 0$ as a smaller absolute disadvantage. Both a greater absolute attribute advantage and a smaller absolute attribute disadvantage would be referred to as an improvement in absolute advantage.

to firm K', firm K has a greater absolute advantage in attribute i than in attribute $i' \neq i$.

As with the term used in the context of trade, comparative advantages in our environment are mutual and reciprocal. When firm K has comparative advantage in attribute i, firm K' must have comparative advantage in attribute i'. We refer to a larger difference in differences in qualities as a stronger comparative attribute advantage or the advantage strengthens. Since $q_i^{K\text{-}K'} - q_{i'}^{K\text{-}K'} = q_{i'}^{K'\text{-}K} - q_i^{K'\text{-}K}$, as a firm's comparative attribute advantage strengthens, so does that of the other firm.

For expositional consistency, our notational default is to express attribute advantages by $q_i^{A\text{-}B}$, supplemented with occasional uses of $q_i^{B\text{-}A}$ to provide intuition. We make the following assumptions about the space of attribute advantages, $\mathbb{Q} = \left[\underline{q}_1^{A\text{-}B}, \overline{q}_1^{A\text{-}B}\right] \times \left[\underline{q}_2^{A\text{-}B}, \overline{q}_2^{A\text{-}B}\right]$, which delineate the absolute and comparative attribute advantages of the two firms:

Assumption 1. *The space of attribute advantages $\mathbb{Q}$ satisfies $\left[\underline{q}_2^{A\text{-}B}, \overline{q}_2^{A\text{-}B}\right] \subseteq \left[-\underline{q}_1^{A\text{-}B}, \underline{q}_1^{A\text{-}B}\right]$.*

Assumption 1 encompasses three sets of restrictions. First, firm A always has absolute advantage in attribute 1. Second, comparative advantage in attribute 1 generically rests with firm A, which implies that comparative advantage in attribute 2 generically resides with firm B. Third, we make no blanket assumption about which firm has absolute advantage in attribute 2, but there is a size restriction: the size of absolute advantage in attribute 2 is never greater than that of firm A's absolute advantage in attribute 1.[9] These three restrictions give a sense that firm A is the firm with the leading product.

Demand and Product Differentiation. As a general formulation, we describe purchase decisions and consumer demand in terms of expected qualities, which would coincide with the actual values if consumers are perfectly informed. Given prices $p = (p_A, p_B)$ and a pair of expected attribute advantages $\hat{q}^{A\text{-}B} = (\hat{q}_1^{A\text{-}B}, \hat{q}_2^{A\text{-}B})$, demand for the two products is delineated by a consumer who is indifferent between purchasing products A and B and thus has a taste threshold:[10]

$$\tilde{\theta} = \frac{p_A - p_B - \hat{q}_2^{A\text{-}B}}{\hat{q}_1^{A\text{-}B} - \hat{q}_2^{A\text{-}B}}. \tag{1}$$

This indifferent consumer divides the unit mass of consumers into two groups, those with $\theta > \tilde{\theta}$ who prefer to purchase Product A and those with $\theta < \tilde{\theta}$ who prefer to purchase

[9]To furnish the details, note that it is implicit in $\left[-\underline{q}_1^{A\text{-}B}, \underline{q}_1^{A\text{-}B}\right]$ that $\underline{q}_1^{A\text{-}B} > 0$ so that firm A always has absolute advantage in attribute 1; $\underline{q}_1^{A\text{-}B} - \overline{q}_2^{A\text{-}B} = \underline{q}_2^{B\text{-}A} - \overline{q}_1^{B\text{-}A} \geq 0$ implies that comparative advantages in attributes 1 and 2 generically rest with firms A and B respectively; finally, given that $\underline{q}_2^{A\text{-}B} \geq -\overline{q}_1^{A\text{-}B}$ is equivalent to $\underline{q}_1^{A\text{-}B} \geq \overline{q}_2^{B\text{-}A}$, the set inclusion $\left[\underline{q}_2^{A\text{-}B}, \overline{q}_2^{A\text{-}B}\right] \subseteq \left[-\underline{q}_1^{A\text{-}B}, \underline{q}_1^{A\text{-}B}\right]$ implies that $\underline{q}_1^{A\text{-}B} \geq \overline{q}_2^{A\text{-}B}$ and $\underline{q}_1^{A\text{-}B} \geq \overline{q}_2^{B\text{-}A}$, i.e., the smallest possible absolute advantage in attribute 1 is no smaller in size than the greatest possible absolute advantage in attribute 2, which may be held by either firm (there is no restriction on the sign of $\overline{q}_2^{A\text{-}B}$). Note also that Assumption 1 places no direct restriction on the upper bound $\overline{q}_1^{A\text{-}B}$.

[10]Throughout the paper, when referring to a pair of variables, we omit the subscript in the single variable.

product B. Accordingly, with $\hat{q}_1^{A\text{-}B} > \hat{q}_2^{A\text{-}B}$ under Assumption 1, consumer demand for the two products, $D(p, \hat{q}^{A\text{-}B}) = (D_A(p, \hat{q}^{A\text{-}B}), D_B(p, \hat{q}^{A\text{-}B}))$, is characterized by:

$$D(p, \hat{q}^{A\text{-}B}) = \begin{cases} (1, 0) & \text{if } p_A - p_B \le \hat{q}_2^{A\text{-}B}, \\ (1 - \tilde{\theta}, \tilde{\theta}) & \text{if } p_A - p_B \in (\hat{q}_2^{A\text{-}B}, \hat{q}_1^{A\text{-}B}), \\ (0, 1) & \text{if } p_A - p_B \ge \hat{q}_1^{A\text{-}B}. \end{cases} \tag{2}$$

The demand function makes clear that under the consumer priors both vertical and horizonal expected product differentiations can arise in our model.It follows from Assumption 1 that firm A has expected absolute advantage in attribute 1 ($\hat{q}_1^{A\text{-}B} > 0$). If expected absolute advantage in attribute 2 also rests with firm A ($\hat{q}_2^{A\text{-}B} > 0$), then there is expected vertical product differentiation: per the first case in (2), all consumers prefer to purchase product A when the two products are priced the same.[11] If instead expected absolute advantage in attribute 2 resides with firm B ($\hat{q}_2^{A\text{-}B} < 0$), then there is expected horizontal product differentiation: the second case in (2) applies, where holding prices the same some consumers prefer product A and some prefer product B.[12]

We assume that firms incur no costs of production, and thus profits equal revenues.[13] The profit of firm $K \in \{A, B\}$ is $\pi_K(p, \hat{q}^{A\text{-}B}) = p_K D_K(p, \hat{q}^{A\text{-}B})$. We analyze how the instrument at firms' disposal to maximize profits—disclosure, which dictates consumer expectations and, in turn, prices and demands—is determined in equilibrium. We begin our analysis in the next section with the pricing decisions in the second stage of the game.

3 Equilibrium Analysis: Pricing

3.1 Equilibrium Prices

The priors and firms' disclosure decisions in the first stage determine the consumer posterior expectations about attribute advantages, which we take as parameters for analysing

[11]Standard definitions of product differentiation (Cremer and Thisse, 1991) compare two products sold at the same price. If each product priced the same has a positive demand, then the products are horizontally differentiated; if consumers demand only one product, then the products are vertically differentiated.

[12]When $\hat{q}_2^{A\text{-}B} = 0$, neither firm has expected absolute advantage in attribute 2. The horizontal differentiation captured in the second case of the demand function reduces to the first case. Note also that with $\hat{q}_1^{A\text{-}B} > 0$, vertical expected product differentiation with product B being the dominant product never arises because in the third case of (2) holding prices the same would require $\hat{q}_1^{A\text{-}B} \le 0$.

[13]Alternatively, we can assume that production costs exist but are sunk, and thus they are not relevant to the decisions being modeled.

equilibrium pricing decisions in the second stage.[14] The following proposition characterizes the equilibrium prices of the products as a function of $\hat{q}^{A\text{-}B}$ under Assumption 1 (all proofs are relegated to Appendix A):

Proposition 1. *Given* $\hat{q}^{A\text{-}B} = (\hat{q}_1^{A\text{-}B}, \hat{q}_2^{A\text{-}B})$ *with* $\hat{q}_1^{A\text{-}B} > 0$ *and* $\hat{q}_2^{A\text{-}B} \in [-\hat{q}_1^{A\text{-}B}, \hat{q}_1^{A\text{-}B}]$*, the unique equilibrium prices set by firms A and B,* $p^*(\hat{q}^{A\text{-}B}) = (p_A^*(\hat{q}^{A\text{-}B}), p_B^*(\hat{q}^{A\text{-}B}))$*, are*

$$p^*(\hat{q}^{A\text{-}B}) = \begin{cases} \left(\frac{2\hat{q}_1^{A\text{-}B} - \hat{q}_2^{A\text{-}B}}{3}, \frac{\hat{q}_1^{A\text{-}B} - 2\hat{q}_2^{A\text{-}B}}{3}\right) & \text{if } \hat{q}_2^{A\text{-}B} \in \left[-\hat{q}_1^{A\text{-}B}, \frac{\hat{q}_1^{A\text{-}B}}{2}\right), \\ (\hat{q}_2^{A\text{-}B}, 0) & \text{if } \hat{q}_2^{A\text{-}B} \in \left[\frac{\hat{q}_1^{A\text{-}B}}{2}, \hat{q}_1^{A\text{-}B}\right]. \end{cases} \tag{3}$$

Proposition 1 captures the interplay of the firms' attribute advantages, market shares, and pricing. Figure 1 illuminates the various scenarios under different values of $\hat{q}_2^{A\text{-}B}$ for a given $\hat{q}_1^{A\text{-}B} > 0$. With firm A being endowed with an absolute advantage in one of the attributes, it always has a positive share of the market. In fact, selling the leading product, firm A never has a smaller market share than the trailing firm B. Within this confine, how firm B shares the market with firm A varies with $\hat{q}_2^{A\text{-}B}$.

The array of scenarios can be compartmentalized using a cutoff at $\hat{q}_2^{A\text{-}B} = 0$, which divides the types of competition into whether the two firms compete under horizontal or vertical product differentiation. At $\hat{q}_2^{A\text{-}B} = 0$, neither firm has absolute advantage in attribute 2, and the products effectively have only one attribute. Except to the boundary consumer who does not care about attribute 1 at all, product A is a better product than product B. The competition environment reduces to the standard duopoly model of vertical product differentiation with one-dimensional quality. Firm B with the inferior product engages in pure price competition to secure a positive demand. In equilibrium, product B is priced sufficiently low for firm B to obtain a market share of one third.[15]

When $\hat{q}_2^{A\text{-}B} > 0$, absolute advantages in both attributes rest with firm A, and there are two quality dimensions with which the products are vertically differentiated. While firm B can continue to rely on lowering price to make up for the inferiority of its product, which is now unequivocally inferior in both attributes, as $\hat{q}_2^{A\text{-}B}$ increases so that the margin of superiority of product A widens, the room for firm B to stay competitive in terms of price narrows. When product B is sufficiently inferior in attribute 2, that $\hat{q}_2^{A\text{-}B} \geq \frac{\hat{q}_1^{A\text{-}B}}{2}$, firm A is in a position where it can set a price high enough to maximize profit but low enough to eliminate any room for firm B to lower price further; any positive price charged by firm B would result in zero market share, and firm A acts as a constrained monopolist.

[14] For brevity, throughout this section we omit "expected" when referring to attribute advantages and product differentiation. Throughout the paper, we denote equilibrium variables or objects with an asterisk.

[15] The equilibrium market share of firm B equals the taste threshold in (1) evaluated at the equilibrium prices, which amounts to $\tilde{\theta}^* = \frac{\hat{q}_1^{A\text{-}B} - 2\hat{q}_2^{A\text{-}B}}{3(\hat{q}_1^{A\text{-}B} - \hat{q}_2^{A\text{-}B})}$.

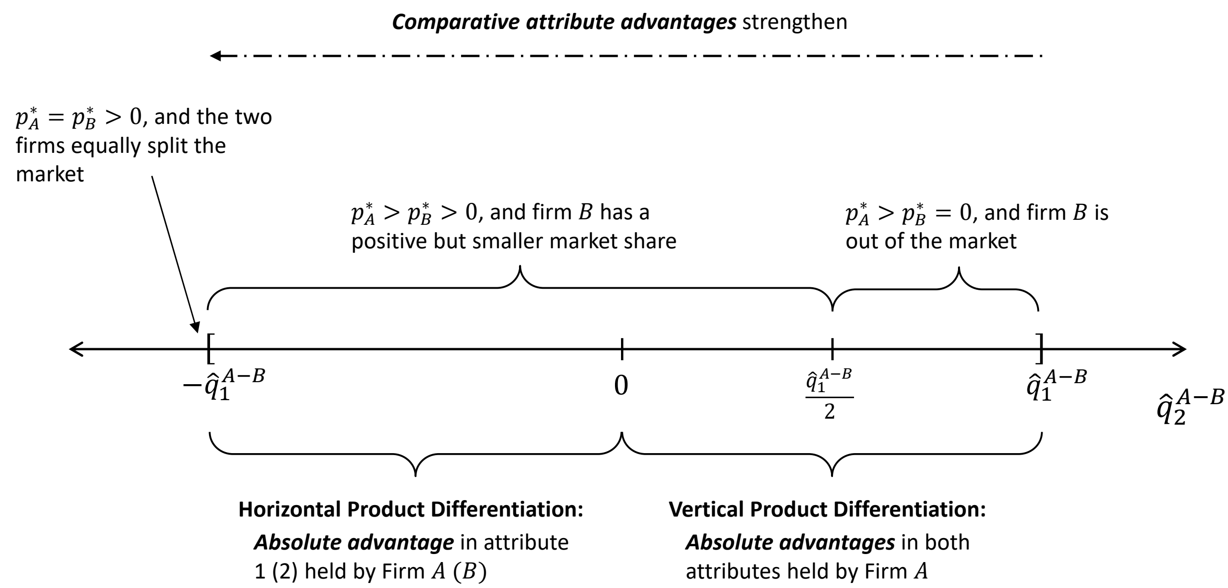


Figure 1: Attribute Advantages, Market Shares, and Pricing

When $\hat{q}_2^{A\text{-}B} < 0$, firm B holds absolute advantage in attribute 2. With each firm having absolute advantage in one attribute, product differentiation becomes horizontal. As the differentiation deepens, i.e., as $\hat{q}_2^{A\text{-}B}$ decreases from zero, each firm enjoys stronger market power over their "clienteles" with less reliance on pure price competition. For $\hat{q}_2^{A\text{-}B} \in (-\hat{q}_1^{A\text{-}B}, 0)$, firm B's absolute advantage in attribute 2 is of a smaller size than its rival's absolute advantage in attribute 1, and firm B still uses a lower price to compensate for the discrepancy. At the boundary where $\hat{q}_2^{A\text{-}B} = -\hat{q}_1^{A\text{-}B}$ so that the two absolute attribute advantages are of equal size, however, the two firms charge the same price and equally split the market, not competing for the same consumer by means of price as the two products are maximally differentiated horizontally.

The cases of the intensity of competition can be organized through the lens of absolute and comparative attribute advantages. The competition faced by firm A from the trailing firm B is most fierce when the absolute advantage in attribute 2, whether it is held by firm A or B, is vanishing, i.e., when $\hat{q}_2^{A\text{-}B}$ approaches 0 from above or below. This is also the point where the two products are least differentiated within the confine of this illustration with a fixed $\hat{q}_1^{A\text{-}B} > 0$. Firm B, on the other hand, faces the most fierce competition from the leading firm A when $\hat{q}_2^{A\text{-}B}$ approaches $\frac{\hat{q}_1^{A\text{-}B}}{2}$. Since the differences in differences that characterize comparative attribute advantages decrease in $\hat{q}_2^{A\text{-}B}$, the competition faced by firm B intensifies as its comparative advantage in attribute 2 weakens.

We conclude this subsection by highlighting two facts of comparative statics about prices stated in terms of absolute and comparative attribute advantages:

Corollary 1. *In the price equilibrium,*

(a) as a firm's absolute advantage in an attribute improves, the firm's price increases if the firm also has comparative advantage in that attribute but decreases if it does not have comparative advantage in that attribute; and

(b) conditional on positive market share, the price of a firm increases as comparative attribute advantages strengthen.

Since a firm's comparative advantage in an attribute strengthens when its absolute advantage in that attribute improves or when its absolute advantage in the other attribute declines, Corollaries 1(a) and 1(b) are two sides of the same coin. Note also that since comparative attribute advantages weaken as $\hat{q}_2^{A\text{-}B}$ increases, Corollary 1(b) makes clear that in our environment lowest prices do not coincide with minimal product differentiation, which occurs at $\hat{q}_2^{A\text{-}B} = 0$.[16]

3.2 Profits in Price Equilibrium

Our main goal is to analyze disclosure decisions. To that end, we further examine the comparative statics of how consumer expectations about attribute advantages, which are influenced by disclosure and taken as parameters in the second stage, affect the firms' profits in the price equilibrium. We consider the meaningful situations where the two firms share the market with competition, restricting attention to the cases where $\hat{q}_2^{A\text{-}B} \in \left[-\hat{q}_1^{A\text{-}B}, \frac{\hat{q}_1^{A\text{-}B}}{2}\right]$.

The reduced-form profit of firm K as a function of $\hat{q}^{A\text{-}B} = (\hat{q}_1^{A\text{-}B}, \hat{q}_2^{A\text{-}B})$ is:

$$\pi_K^*(\hat{q}^{A\text{-}B}) = p_K^*(\hat{q}^{A\text{-}B}) D_K(p^*(\hat{q}^{A\text{-}B}), \hat{q}^{A\text{-}B}), \tag{4}$$

which, using the relevant cases in (1)–(3), can be computed explicitly as $\pi_A^*(\hat{q}^{A\text{-}B}) = \frac{(2\hat{q}_1^{A\text{-}B} - \hat{q}_2^{A\text{-}B})^2}{9(\hat{q}_1^{A\text{-}B} - \hat{q}_2^{A\text{-}B})}$ for firm A and $\pi_B^*(\hat{q}^{A\text{-}B}) = \frac{(\hat{q}_1^{A\text{-}B} - 2\hat{q}_2^{A\text{-}B})^2}{9(\hat{q}_1^{A\text{-}B} - \hat{q}_2^{A\text{-}B})}$ for firm B.

It is constructive for isolating the different effects in the comparative statics by proceeding with the form in (4). We are interested in the signs and magnitudes of the derivatives:

$$\frac{\partial \pi_K^*}{\partial \hat{q}_i^{A\text{-}B}} = p_K^* \left(\frac{\partial D_K}{\partial \hat{q}_i^{A\text{-}B}} + \frac{\partial D_K}{\partial p_{K'}} \frac{\partial p_{K'}^*}{\partial \hat{q}_i^{A\text{-}B}} \right),$$

$K \in \{A, B\}$ and $i \in \{1, 2\}$. The term $\frac{\partial D_K}{\partial \hat{q}_i^{A\text{-}B}}$ is the direct *demand effect* of $\hat{q}_i^{A\text{-}B}$ on firm K's

[16]For expositional ease, Figure 1 and the discussion above have assumed a fixed $\hat{q}_1^{A\text{-}B}$. As we will see below, the properties in terms of absolute and comparative attribute advantages remain valid when $\hat{q}_1^{A\text{-}B}$ varies.

profit, and $\frac{\partial D_K}{\partial p_{K'}}\frac{\partial p^*_{K'}}{\partial \hat{q}_i^{A\text{-}B}}$ is the indirect *strategic effect* through the change in the price of firm $K' \neq K$.[17] Each firm faces two sets of effects, one for each attribute. For firm A, they are:

$$\text{Attribute 1:} \quad \frac{\partial D_A}{\partial \hat{q}_1^{A\text{-}B}} = \frac{\hat{q}_1^{A\text{-}B} - 2\hat{q}_2^{A\text{-}B}}{3(\hat{q}_1^{A\text{-}B} - \hat{q}_2^{A\text{-}B})^2} > 0 \text{ and } \frac{\partial D_A}{\partial p_B}\frac{\partial p^*_B}{\partial \hat{q}_1^{A\text{-}B}} = \frac{1}{3(\hat{q}_1^{A\text{-}B} - \hat{q}_2^{A\text{-}B})} > 0. \tag{5}$$

$$\text{Attribute 2:} \quad \frac{\partial D_A}{\partial \hat{q}_2^{A\text{-}B}} = \frac{2\hat{q}_1^{A\text{-}B} - \hat{q}_2^{A\text{-}B}}{3(\hat{q}_1^{A\text{-}B} - \hat{q}_2^{A\text{-}B})^2} > 0 \text{ and } \frac{\partial D_A}{\partial p_B}\frac{\partial p^*_B}{\partial \hat{q}_2^{A\text{-}B}} = -\frac{2}{3(\hat{q}_1^{A\text{-}B} - \hat{q}_2^{A\text{-}B})} < 0. \tag{6}$$

For firm B, the demand effects are the negatives of those faced by firm A, i.e., $\frac{\partial D_B}{\partial \hat{q}_i^{A\text{-}B}} = -\frac{\partial D_B}{\partial \hat{q}_i^{A\text{-}B}}$; for the strategic effects, the equilibrium price function (3) implies that they are related to firm A's strategic effects by $\frac{\partial D_B}{\partial p_A}\frac{\partial p^*_A}{\partial \hat{q}_1^{A\text{-}B}} = 2\Big(\frac{\partial D_A}{\partial p_B}\frac{\partial p^*_B}{\partial \hat{q}_1^{A\text{-}B}}\Big)$ and $\frac{\partial D_B}{\partial p_A}\frac{\partial p^*_A}{\partial \hat{q}_2^{A\text{-}B}} = \frac{1}{2}\Big(\frac{\partial D_A}{\partial p_B}\frac{\partial p^*_B}{\partial \hat{q}_2^{A\text{-}B}}\Big)$.

The demand effect of $\hat{q}_1^{A\text{-}B}$ on firm A's profit is positive, which is further reinforced by the positive strategic effect, resulting in a higher profit as $\hat{q}_1^{A\text{-}B}$ increases. Product A attracts more consumers when it has a relatively higher quality in attribute 1. As $\hat{q}_1^{A\text{-}B}$ increases, the price of product B also increases as comparative advantages strengthen (Corollary 1), and this further contributes to the relative desirability of product A.

Firm B's profit likewise increases in $\hat{q}_1^{A\text{-}B}$, but it is under a different interaction of the effects. The demand effect, which is now negative, is offset by the positive strategic effect. While the prices of both products increase as comparative advantages strengthen, the price of product A increases more than that of product B as can be seen in the equilibrium price function (1). Even though product B becomes more inferior in respect of attribute 1, it also becomes relatively more price competitive. On balance, the strategic effect dominates.

Firm B's profit decreases in $\hat{q}_2^{A\text{-}B}$. The demand effect is negative, which is reinforced by the negative strategic effect. When $\hat{q}_2^{A\text{-}B}$ increases, either because firm B's absolute advantage in attribute 2 declines under horizontal differentiation or because firm A's improves under vertical differentiation, product B appeals to fewer consumers. Furthermore, as $\hat{q}_2^{A\text{-}B}$ increases, firm A charges a lower price as it does not have comparative advantage in attribute 2 (Corollary 1), and this also makes product B relatively less attractive.

The effect of $\hat{q}_2^{A\text{-}B}$ on firm A's profit is distinct from the other cases, where the relationship is non-monotonic. It is apparent from the derivatives in (6) that, at $\hat{q}_2^{A\text{-}B} = 0$, the negative strategic effect cancels out the positive demand effect. When $\hat{q}_2^{A\text{-}B} > 0$, the demand effect dominates, and vice versa when $\hat{q}_2^{A\text{-}B} < 0$.

We summarize the four cases in the following proposition:

[17] The strategic effect is a cross-price effect, and by the envelope theorem own-price effects do not have a place in the profit derivatives. The interaction of the demand and the strategic effects solely determines how changes in an attribute advantage affect profits.

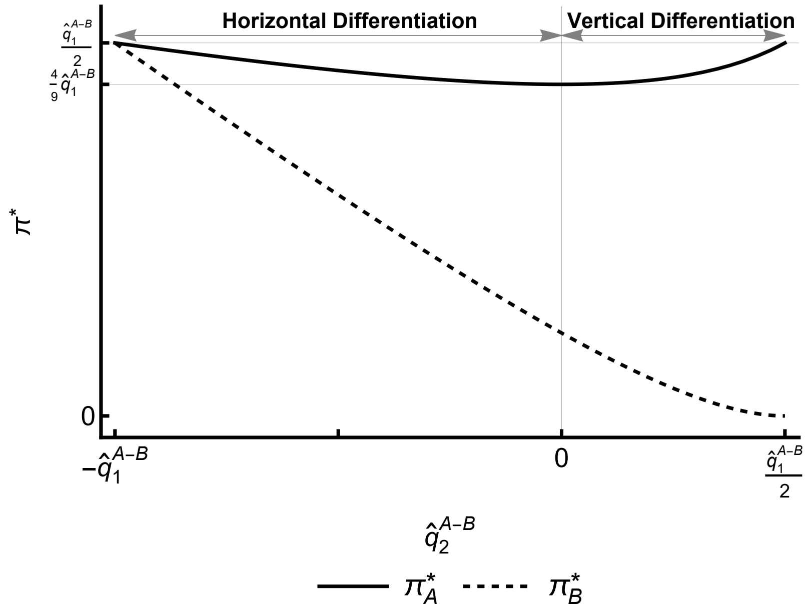


Figure 2: Attribute Advantages and Profits

Proposition 2. *For* $\hat{q}^{A\text{-}B} = (\hat{q}_1^{A\text{-}B}, \hat{q}_2^{A\text{-}B})$ *with* $\hat{q}_1^{A\text{-}B} > 0$ *and* $\hat{q}_2^{A\text{-}B} \in \left[-\hat{q}_1^{A\text{-}B}, \frac{\hat{q}_1^{A\text{-}B}}{2}\right]$*, the reduced-form profits of the firms in* (4) *have the following properties:*

(a) Firm A: $\pi_A^*(\hat{q}^{A\text{-}B})$ *is (i) increasing in* $\hat{q}_1^{A\text{-}B}$*, (ii) increasing in* $\hat{q}_2^{A\text{-}B}$ *when product differentiation is vertical* $(\hat{q}_2^{A\text{-}B} > 0)$*, and (iii) decreasing in* $\hat{q}_2^{A\text{-}B}$ *when product differentiation is horizontal* $(\hat{q}_2^{A\text{-}B} < 0)$*.*

(b) Firm B: $\pi_B^*(\hat{q}^{A\text{-}B})$ *is (i) increasing in* $\hat{q}_1^{A\text{-}B}$ *and (ii) decreasing in* $\hat{q}_2^{A\text{-}B}$*, irrespective of the type of production differentiation.*

The comparative statics further implies the following properties, which, as we will see in the next section, are key to understand firms' disclosure incentives:

Corollary 2. *The firms' profits attain the minimum under the following different scenarios:*

(a) Firm A: $\pi_A^*(\hat{q}^{A\text{-}B})$ *is at the lowest when firm A's absolute advantage in attribute 1 is smallest and when absolute advantage in attribute 2, be it held by firm A or B, is smallest; this is also the point where products A and B are least differentiated.*

(b) Firm B: $\pi_B^*(\hat{q}^{A\text{-}B})$ *is at the lowest when comparative advantages are weakest.*

Corollary 2 resonates with the narrative about intensity of competition in Section 3.1. The point where a firm's profit is lowest is the point where it faces the most fierce competition from the other firm. Figure 2 provides further visualization of the connection, in which we depict the two firms' profit levels for varying values of $\hat{q}_2^{A\text{-}B} \in \left[-\hat{q}_1^{A\text{-}B}, \frac{\hat{q}_1^{A\text{-}B}}{2}\right]$, assuming a fixed $\hat{q}_1^{A\text{-}B}$ as we have done for Figure 1. Firm A earns the lowest profit at $\hat{q}_2^{A\text{-}B} = 0$ and the highest profit at the two end points at which competition is eliminated, in one case under maximal horizontal product differentiation, and in the other under extreme vertical differentiation where the market effectively becomes monopolistic. Firm B's profit, on the other hand, is monotonically decreasing and reaches the lowest level at $\hat{q}_2^{A\text{-}B} = \frac{\hat{q}_1^{A\text{-}B}}{2}$.

4 Equilibrium Analysis: Disclosure

In the first stage of the game, the two firms simultaneously make their disclosure decisions. The interpretation is that the firms disclose a third-party review which reports the attribute qualities of both products. Since profits depend on attribute advantages, we assume without loss of generality that the reported variables are the quality differences instead of directly the four quality variables. The third-party review is subject to a verifiability constraint where it reports $q^{A\text{-}B} = (q_1^{A\text{-}B}, q_2^{A\text{-}B})$ if and only if they are the true attribute advantages. Accordingly, each firm's decision is whether to credibly disclose the realized $q^{A\text{-}B}$ or not. We continue to focus on the cases with competition, analyzing disclosure under the following tightened version of Assumption 1:

Assumption 1′. *The space of attribute advantages* $\mathbb{Q}$ *satisfies* $\left[\underline{q}_2^{A\text{-}B}, \overline{q}_2^{A\text{-}B}\right] \subseteq \left[-\underline{q}_1^{A\text{-}B}, \frac{\underline{q}_1^{A\text{-}B}}{2}\right]$.

Note that Assumption 1′ places no restriction on the sign of the upper bound $\overline{q}_2^{A\text{-}B}$. When $\overline{q}_2^{A\text{-}B} > 0$, from the perspective of an uninformed consumer, product differentiation is *indefinite*: either horizontal or vertical differentiation can arise. On the other hand, when $\overline{q}_2^{A\text{-}B} \leq 0$, product differentiation can only be horizontal.

Let $Q_K \subseteq \mathbb{Q}$ be the set of attribute advantages that firm K discloses. We define a disclosure outcome in terms of $Q = Q_A \cup Q_B$, the set of attribute advantages disclosed by at least one firm. A disclosure outcome is (a) revealing if $Q \neq \varnothing$, (b) *partially revealing* if, in addition to Q being non-empty, $Q \subset \mathbb{Q}$, or equivalently in terms of the complements, $\overline{Q} = \overline{Q}_A \cap \overline{Q}_B \neq \varnothing$, and (c) *fully revealing* if $Q = \mathbb{Q}$, or equivalently, $\overline{Q} = \varnothing$. We analyze the perfect Bayesian equilibria of the game that give rise to these outcomes.[18]

Consumer beliefs on the equilibrium paths are uniquely determined by Bayes' rule. In an equilibrium with outcome $Q^* \neq \varnothing$, all consumers are perfectly informed of the set of attribute advantages disclosed by at least one firm, i.e., for all $q^{A\text{-}B} \in Q^*$, $\hat{q}^{A\text{-}B} = q^{A\text{-}B}$.

[18]Note that a singleton $\overline{Q}$ is not defined because then the singleton would be revealed. An approach to avoid this is to define disclosure outcomes in terms of probability measure. We use the current definition for its intuitive appeal.

If the equilibrium is partially revealing, the event of non-disclosure is on the equilibrium path, and consumers form expectations conditional on the non-disclosed set of attribute advantages so that $\hat{q}^{A\text{-}B} = E\left[q^{A\text{-}B} | q^{A\text{-}B} \in \overline{Q}^*\right]$. Given the verifiability constraint, there are out-of-equilibrium events in which consumer beliefs can still be uniquely pinned down. In a fully revealing equilibrium, however, the absence of disclosure is an out-of-equilibrium event where Bayes' rule cannot be applied; appropriate out-of-equilibrium beliefs will have to be assigned to support a given equilibrium.[19]

Other than the standard analysis described above, we also analyze equilibria under a behavioral variation where firms face boundedly rational consumers who adhere to their priors in an otherwise equilibrium event of non-disclosure.

4.1 Full Disclosure

We begin with a benchmark case where there is no disclosure cost and consumers are fully rational. The well-known unraveling result of disclosure games holds in this baseline environment of our duopolistic disclosure model, as the following proposition shows:

Proposition 3. *Suppose that $\mathbb{Q}$ satisfies Assumption 1′. If the disclosure cost $c = 0$, then the unique equilibrium outcome is fully revealing.*

The fully revealing outcome can be supported by multiple equilibria with different disclosure strategies. In the proof, we construct two types of equilibria: in one type both firms disclose all attribute advantages, and in the other only one firm discloses all attribute advantages, with the other firm disclosing some or none. Both types are fully revealing equilibria in light of the outcome. The multiplicity originates from the fact that consumers learn everything when one firm discloses all attribute advantages—any disclosure strategy by the other firm is then a best response.

While the equilibria are not unique, the outcome is. The non-existence of equilibrium outcome other than fully revealing is a consequence of the fact that the set of attribute advantages that renders a firm a lower profit than does a given $\tilde{q}^{A\text{-}B}$ forms a convex set. Such a set does not exist if $\tilde{q}^{A\text{-}B}$ is the expected value of the attribute advantages in the set. In other words, for any $\overline{Q} \subseteq \mathbb{Q}$, there will be attribute advantages in $\overline{Q}$ where a firm earns a higher profit by disclosing them than leaving consumers to believe that $\hat{q}^{A\text{-}B} = E[q^{A\text{-}B} | q^{A\text{-}B} \in \overline{Q}]$. This result stands in contrast to the monopolistic model of Sun

[19] A full specification of equilibria also requires consumer beliefs to be specified after any out-of-equilibrium prices. We follow, e.g., Board (2009) and adopt the simplest assumption that consumer beliefs are independent of prices. See Janssen and Roy (2014) for an analysis of a duopoly model where in addition to disclosure, prices may signal quality.

(2011) and the oligopolistic model of Hotz and Xiao (2013) with horizontal and vertical attributes. In their models, partially revealing equilibria exist under costless disclosure and the quality variables that are not disclosed form a disjoint set.

4.2 Partial Disclosure

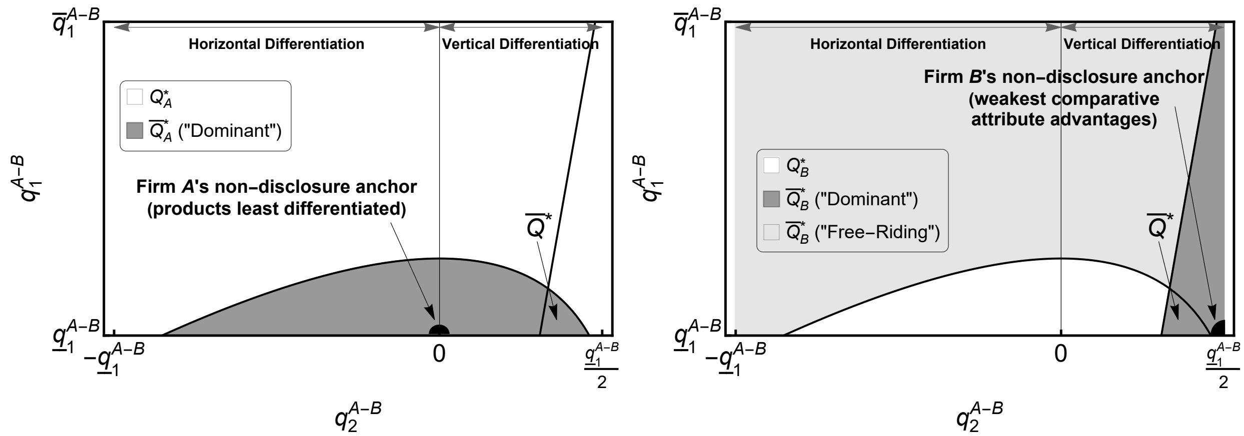
Proposition 3 establishes a benchmark of full disclosure under the assumptions of costless disclosure and fully rational consumers. Relaxing these two assumptions each provides an avenue for partial disclosure to emerge as an equilibrium outcome. The analysis of partial disclosure brings to the fore some of the distinguishing features of our multi-attribute model, in particular how disclosure decisions interact with absolute and comparative attribute advantages as well as product differentiation.

Costly Disclosure. Each firm incurs a common $c > 0$ when choosing to disclose.[20] The presence of disclosure cost renders some attribute advantages not worthwhile to be disclosed. We use the concept of neighborhoods to characterize non-disclosure. For any $\tilde{q}^{A\text{-}B} \in \mathbb{Q}$ and $\epsilon > 0$, we define the ϵ-neighborhood of $\tilde{q}^{A\text{-}B}$, $N_\epsilon[\tilde{q}^{A\text{-}B}] = \{q^{A\text{-}B} \in \mathbb{Q} : d(q^{A\text{-}B}, \tilde{q}^{A\text{-}B}) < \epsilon\}$, where d is the Euclidean distance. It is intuitive to expect, and indeed observed in practice, that when selecting what to disclose and not disclose, firms highlight the favorable information and downplay the unfavorable. The following lemma substantiates this intuition by establishing the properties of the non-disclosure set of *each* firm:

Lemma 1. *Suppose that $\mathbb{Q}$ satisfies Assumption 1′. If the disclosure cost $c > 0$, then there exists $\epsilon^* > 0$ such that for any equilibrium outcome $Q^* = Q_A^* \cup Q_B^*$, $N_{\epsilon^*}\left[\left(\underline{q}_1^{A\text{-}B}, \min\{0, \overline{q}_2^{A\text{-}B}\}\right)\right] \subset \overline{Q}_A^*$ and $N_{\epsilon^*}\left[\left(\underline{q}_1^{A\text{-}B}, \overline{q}_2^{A\text{-}B}\right)\right] \subset \overline{Q}_B^*$.*

The non-disclosure set of each firm anchors at the attribute advantages that, if learned by consumers, would put the firm at the most disadvantageous position facing the most fierce competition and earning the lowest profit. Corollary 2 furnishes what these attribute advantages are for each firm. Firm B is most disadvantaged at the "vertex" $\left(\underline{q}_1^{A\text{-}B}, \overline{q}_2^{A\text{-}B}\right)$, where the trailing firm's comparative advantage in attribute 2 is weakest and any positive price charged for Product B would render a zero sale. Firm A, on the other hand, is most disadvantaged when absolute attribute advantages are smallest, which is also the point where the two products are least differentiated. Since firm A always holds absolute advantage in attribute 1, in respect of this attribute this occurs at $\underline{q}_1^{A\text{-}B}$.

[20] We have interpreted the disclosure decisions as regarding a third-party review that contains quality information on both products. In practice, firms typically publicize third-party reviews on their promotion platforms such as company websites. An example of the disclosure cost could be opportunity cost resulting from the scarcities of promotion spaces or consumer attention.



(a) Firm A (b) Firm B

Figure 3: Disclosure Decisions: Indefinite Product Differentiation

For attribute 2, there are two cases. When product differentiation is indefinite ($\overline{q}_2^{A\text{-}B} > 0$), the least product differentiation occurs at $\left(\underline{q}_1^{A\text{-}B}, 0\right)$, where neither firm has absolute advantage in attribute 2. When the two products are always horizontally differentiated ($\overline{q}_2^{A\text{-}B} \leq 0$), they are least so at $\left(\underline{q}_1^{A\text{-}B}, \overline{q}_2^{A\text{-}B}\right)$, where the smallest absolute advantage in attribute 2 is held by firm B. Note that this coincides with weakest comparative attribute advantages; under definite horizontal differentiation, the two firms' non-disclosure sets anchor at the same point, although with firm A being the leading firm and firm B the trailing the considerations behind their decisions are different.

The purpose of Lemma 1 is to establish the "anchoring property" of $\overline{Q}_A^*$ and $\overline{Q}_B^*$. The ϵ-neighborhood $N_{\epsilon^*}[\cdot]$ in general does not contain all the attribute advantages that a firm does not disclose. In particular, unlike the benchmark case in Section 4.1, when disclosure is costly, in equilibrium the two firms will not disclose the same attribute advantages since a firm can "free ride" on the other firm's disclosure. There may be elements of $\overline{Q}_K^*$ that fall under this category. This is in contrast to the non-disclosure of $N_{\epsilon^*}[\cdot]$, which is a "dominant" decision in that, regardless of what firm K' discloses, firm K does not want to disclose those attribute advantages in the ϵ-neighborhood.

Figure 3 depicts an example with $\mathbb{Q} = \left[\underline{q}_1^{A\text{-}B}, \overline{q}_1^{A\text{-}B}\right] \times \left[-\underline{q}_1^{A\text{-}B}, \frac{\underline{q}_1^{A\text{-}B}}{2}\right]$, where product differentiation is indefinite. The disclosure decisions of firms A and B are separately illustrated in panels (a) and (b). The illustration assumes that firm B is the "free rider" so that firm A discloses all the attribute advantages that are commonly preferred by both firms to be disclosed. For firm A, the white region represents the disclosed Q_A^*, and the shaded region represents the non-disclosed $\overline{Q}_A^*$. For firm B, the white region similarly represents the disclosed Q_B^*, and the shaded region that represents the non-disclosed $\overline{Q}_B^*$ is further divided into the "free-riding" region (lighter shade) and the "dominant" region (darker shade). The free-riding region could be transferred to firm A, which would give rise to an alternative equilibrium with the same disclosure outcome. By contrast, the dominant region cannot be so transferred; even if it was not disclosed by firm A, firm B would still not disclose it. As stated in Lemma 1, $\overline{Q}_A^*$ and $\overline{Q}_B^*$ in the figure contain the respective anchoring points at $\left(\underline{q}_1^{A\text{-}B}, 0\right)$ and $\left(\underline{q}_1^{A\text{-}B}, \overline{q}_2^{A\text{-}B}\right)$.

Lemma 1 presents the disclosure decisions of firms A and B *separately*. Whether a particular set of attribute advantages is disclosed to consumers depends jointly on the decisions. Comparing Figures 3(a) and 3(b) shows that a subset of the non-disclosed $\overline{Q}_B^*$ by firm B is disclosed by firm A, and vice versa for $\overline{Q}_A^*$. The attribute advantages that are not disclosed to consumers at all are those where there is a "double dominance" of non-disclosure—those that lie in the interaction of $\overline{Q}_A^*$ and $\overline{Q}_B^*$ indicated by $\overline{Q}^*$.

As a general matter, the sizes of $\overline{Q}_A^*$ and $\overline{Q}_B^*$ depend on the prior distributions $F^{A\text{-}B} = (F_1^{A\text{-}B}, F_2^{A\text{-}B})$ and the disclosure cost c. The following proposition shows that so long as the disclosure cost is positive, $\overline{Q}_A^*$ intersects $\overline{Q}_B^*$, and fully revealing outcome is out of reach as an equilibrium outcome; as long as disclosure is not too costly given the distributions, a partially revealing equilibrium is guaranteed to exist; and when such an equilibrium exists, product differentiation shapes the form of the partially revealing outcome:

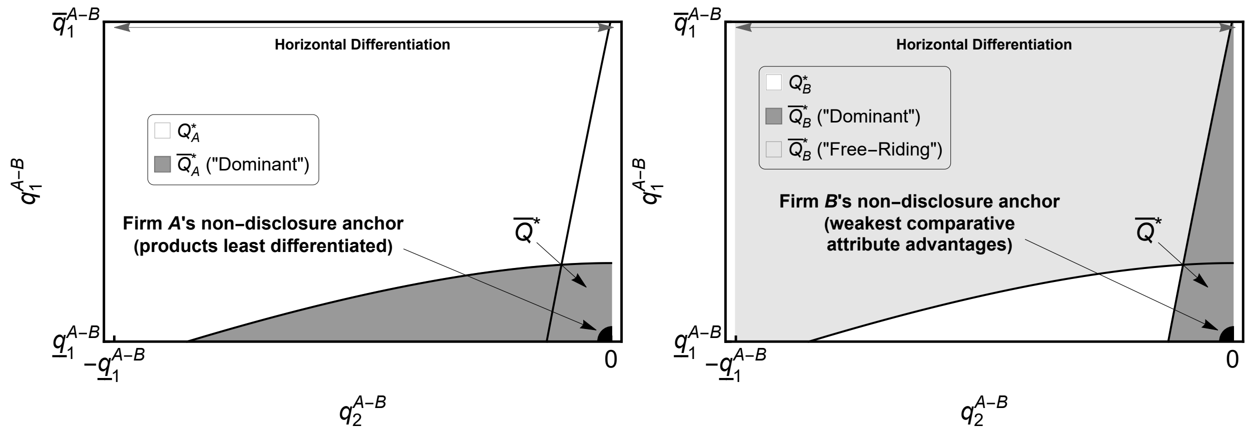
Proposition 4. *Suppose that $\mathbb{Q}$ satisfies Assumption 1′ with distributions $F^{A\text{-}B}$. There exists $\overline{c}_{F^{A\text{-}B}}$ such that for any disclosure cost $c \in (0, \overline{c}_{F^{A\text{-}B}})$, the outcome of any equilibrium Q^* is partially revealing and depends on product differentiation as follows:*

(a) If $\overline{q}_2^{A\text{-}B} > 0$ so that product differentiation is indefinite, then there exists $\epsilon_{hv}^* > 0$ such that $N_{\epsilon_{hv}^*}\left[\left(\underline{q}_1^{A\text{-}B}, q_2^{A\text{-}B}\right)\right] \subset \overline{Q}^*$ for some $q_2^{A\text{-}B} \in \left(0, \overline{q}_2^{A\text{-}B}\right)$.

(b) If $\overline{q}_2^{A\text{-}B} \leq 0$ so that product differentiation is horizontal, then there exists $\epsilon_h^* > 0$ such that $N_{\epsilon_h^*}\left[\left(\underline{q}_1^{A\text{-}B}, \overline{q}_2^{A\text{-}B}\right)\right] \subset \overline{Q}^*$.

The two firms have perfectly aligned interests in not disclosing sufficiently low $q_1^{A\text{-}B}$. But for the case in Proposition 4(a) where either horizontal or vertical differentiation can arise, their disclosure preferences in respect of attribute 2 are not as aligned: firm A prefers not to disclose $q_2^{A\text{-}B}$ in the neighborhood of 0, whereas firm B prefers not to disclose around the upper bound $\overline{q}_2^{A\text{-}B} > 0$. In equilibrium, the set of non-disclosed attribute advantages, $\overline{Q}^*$, the intersection of $\overline{Q}_A^*$ and $\overline{Q}_B^*$, anchors at a point made up of $\underline{q}_1^{A\text{-}B}$ and a value between 0 and $\overline{q}_2^{A\text{-}B}$, as illustrated in Figure 3.

A distinctive difference of the case in Proposition 4(b), in which product differentiation is definitely horizontal, is that the disclosure interests of the two firms are perfectly aligned in both attributes: they both prefer not to disclose $q^{A\text{-}B}$ with sufficiently low $q_1^{A\text{-}B}$ and sufficiently high $q_2^{A\text{-}B}$. Figure 4 displays an example with $\mathbb{Q} = \left[\underline{q}_1^{A\text{-}B}, \overline{q}_1^{A\text{-}B}\right] \times \left[-\underline{q}_1^{A\text{-}B}, 0\right]$.



(a) Firm A (b) Firm B

Figure 4: Disclosure Decisions: Horizontal Product Differentiation

Boundedly Rational Consumers. We further consider disclosure with boundedly rational consumers, who do not update beliefs in the equilibrium event of non-disclosure. Instead, they adhere to the priors and make purchase decisions based on the unconditional expected attribute advantages $E[q^{A\text{-}B}] \in \text{int}(\mathbb{Q})$.[21]

Consumers making purchase decisions under the priors when neither firm discloses renders non-disclosure more attractive to firms, in a way similar to how economization of disclosure cost makes non-disclosure a potentially profitable move. Proposition 3 has established that when disclosure is costless, full revelation is the unique equilibrium outcome with fully rational consumers; the following proposition shows that a partially revealing equilibrium exists even with costless disclosure if consumers are boundedly rational:[22]

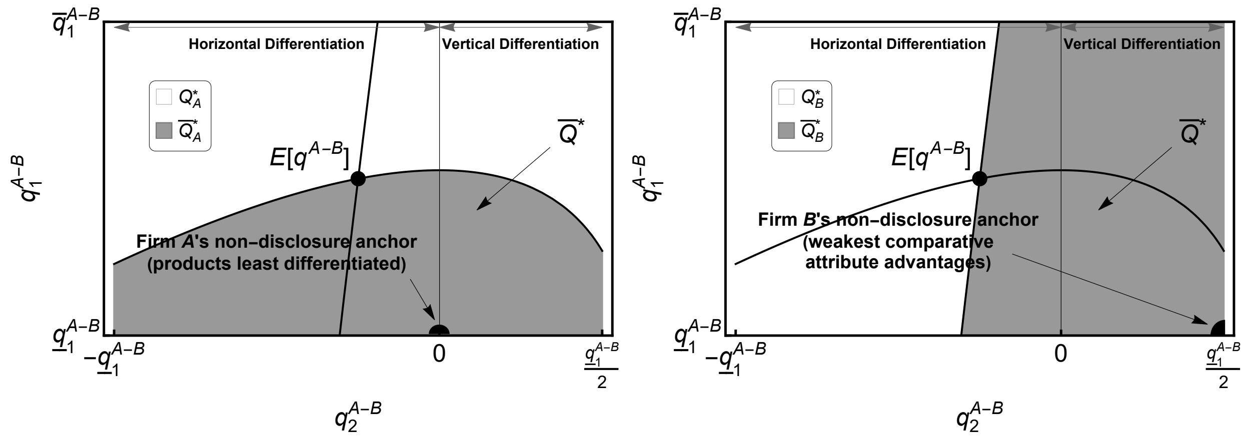
Proposition 5. *Suppose that $\mathbb{Q}$ satisfies Assumption 1′ with distributions $F^{A\text{-}B}$. If consumers adhere to the prior $F^{A\text{-}B}$ when neither firm discloses, then there exists a partially revealing equilibrium for disclosure cost $c = 0$. The outcome of this equilibrium depends on product differentiation as follows:*

(a) If $\overline{q}_2^{A\text{-}B} > 0$ so that product differentiation is indefinite, then there exists $\tilde{\epsilon}^*_{hv} > 0$ such that $N_{\tilde{\epsilon}^*_{hv}}\left[\left(\underline{q}_1^{A\text{-}B}, q_2^{A\text{-}B}\right)\right] \subset \overline{Q}^*$ for some $q_2^{A\text{-}B} \in \left(0, \overline{q}_2^{A\text{-}B}\right)$.

(b) If $\overline{q}_2^{A\text{-}B} \leq 0$ so that product differentiation is horizontal, then there exists $\tilde{\epsilon}^*_h > 0$ such that $N_{\tilde{\epsilon}^*_h}\left[\left(\underline{q}_1^{A\text{-}B}, \overline{q}_2^{A\text{-}B}\right)\right] \subset \overline{Q}^*$.

[21] We would also assume that consumers use the priors in the out-of-equilibrium event of non-disclosure, but this is only for complete specification and is not relevant to the analysis below.

[22] Note that a fully revealing equilibrium in which both firms disclose all attribute advantages also exists with boundedly rational consumers and $c = 0$; partially revealing equilibria are not the unique type of equilibria. Furthermore, a positive disclosure cost operates with boundedly rational consumers in a similar manner as characterized in Proposition 4 for fully rational consumers. The proposition below focuses on costless disclosure to highlight that boundedly rational consumers could supersede positive disclosure cost in yielding a partially revealing equilibrium.



(a) Firm A (b) Firm B

Figure 5: Disclosure Decisions: Boundedly Rational Consumers and Indefinite Product Differentiation

Figure 5 illustrates Proposition 5(a) with $\mathbb{Q} = \left[\underline{q}_1^{A\text{-}B}, \overline{q}_1^{A\text{-}B}\right] \times \left[-\underline{q}_1^{A\text{-}B}, \frac{\underline{q}_1^{A\text{-}B}}{2}\right]$, where product differentiation is indefinite. The boundaries of the firms' non-disclosure sets, $\overline{Q}_A^*$ and $\overline{Q}_B^*$, pass through the unconditional $E(q^{A\text{-}B})$ that the boundedly rational consumers rely on after non-disclosure. Comparing and contrasting Figure 5 and Figure 3 of costly disclosure exemplify the similarities and differences between the disclosure decisions in the two environments. For similarities, firm B's non-disclosure set $\overline{Q}_B^*$ must contain, in both cases, $(\underline{q}_1^{A\text{-}B}, \overline{q}_2^{A\text{-}B})$, the point where comparative attribute advantages are weakest. In a partially revealing equilibrium, this set intersects with firm A's non-disclosure set $\overline{Q}_A^*$ to give rise to $\overline{Q}^*$, the set of attribute advantages that are not disclosed by either firm.

The differences are more subtle. The partially revealing outcome illustrated in Figure 5 is made up of (a) disclosed region in which both firms disclose (region that is white in both panels), (b) disclosed regions in which only one firm discloses (regions that are shaded in only one panel), and (c) the non-disclosed region (region that is shaded in both panels, indicated by $\overline{Q}^*$). Given that disclosure is costless, there is no longer dominant non-disclosure; similar to the benchmark in Section 4.1, there are multiple equilibria supporting the same disclosure outcome that stem from a firm's indifference when the other firm discloses. One could construct other equilibria with the non-disclosed region unchanged but each of the other three regions being disclosed by one or both firms.

Despite these differences, the behavioral environment with costless disclosure complements the costly-disclosure environment in that partial revealing equilibria arise in both cases; when product differentiation must be horizontal ($\overline{q}_2^{A\text{-}B} \leq 0$), the insights from the two environments further align in which the partial revealing equilibria feature non-disclosure of attribute advantages where comparative advantages are weakest. Consumers falling short of applying Bayes' rule and carrying out the iterative reasoning assumed by equilibrium are perhaps more descriptive of the consumers in the real world.[23] Our behavioral analysis suggests that the major insights from the costly-disclosure analysis regarding the interplay of partial disclosure, absolute and comparative attribute advantages, and product differentiation remain relevant when one departs from the fully rational paradigm.

5 Concluding Remarks

This paper analyzes a novel variation of otherwise well-studied quality disclosure. In a duopolistic environment with disclosure of comparative quality information, we show that modeling two vertical product attributes instead of one horizontal and one vertical as is done in previous studies introduces unexplored considerations for analyzing competitive disclosure. These strategically rich considerations, though new in the context of disclosure, surround the familiar notion of absolute and comparative advantages.

In our model, one of the two firms sells a product that is *ex-ante* leading in terms of absolute attribute advantages (Assumption 1). When disclosure is costly, firms strategically select what to disclose and what to withhold. The firm selling the leading product withholds disclosure when it faces sufficiently intense competition from the firm selling the trailing product, which occurs when absolute attribute advantages are smallest and the two products are least differentiated. The trailing firm survives in the market on the comparative attribute advantage of its product. Naturally, this firm has incentives not to disclose when comparative advantages are sufficiently weak; at the point where comparative advantages are the weakest, its sales drop to zero under any positive price.

Least product differentiation coincides with weakest comparative attribute advantages when under the priors consumers consider the two products to be horizontally differentiated. When instead the two products can be either vertically or horizontally differentiated from the *ex-ante* perspective, least product differentiation differs from weakest comparative advantages. In this case, the two firms' interests in what to withhold are not aligned, giving the hope that consumers may still be fully informed by combining the information disclosed by each firm. However, we find that even for an infinitesimal amount of disclosure cost, the non-disclosure sets of the two firms, though possibly anchoring at different points, intersect. Consequently, in equilibrium consumers are uninformed of at least some attribute qualities regardless of the type of product differentiation. The partially revealing disclosure outcome may also arise under costless disclosure when consumers fall short of

[23] While relevant field data are hard to come by, Jin et al. (2021), e.g., obtain laboratory evidence that receivers in a disclosure game fail to form correct beliefs about non-disclosed states.

forming rational expectation upon non-disclosure.

An important debate in the theory and practice of quality disclosure concerns the role of mandatory disclosure. The unraveling result provides a basis to argue that mandatory disclosure has no place under the profit incentives of firms: voluntary disclosure completely resolves the problem of asymmetric information in the marketplace. Our paper contributes to the debate by providing and analyzing a market environment under which mandatory disclosure may be favored. In a duopolistic setting with two vertically differentiated product attributes, in which revelation of comparative quality information is practiced, relying on the marketing incentives of firms to promote the attributes of their products may not be enough to render consumers fully informed. Our analysis suggests that in such an environment mandatory disclosure has a role to play when, as is often the case in practice, disclosure is costly or consumers are not as intelligent as economic models would postulate.

One direction that future research can extend our analysis involves consumer search. The presence of consumers who access third-party product reviews on their own (e.g., Ghosh and Galbreth, 2013) may possibly expand the set of product attributes that firms choose not to disclose to consumers directly by themselves. If so, then active search by some consumers might create a negative spillover on other consumers who do not search by paradoxically reducing the amount of product information available to them. Another related direction involves the disclosure of a subset of product attributes. If third-party reviews evaluate products only on certain attributes but not others, then depending on absolute and comparative attribute advantages firms may have an incentive to expand the set of attributes that is made known to consumers. Formal analysis to substantiate or refine this intuition would further enhance our understanding of quality disclosure.

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Appendix A Proofs

Proof of Proposition 1. Consider first the case where $\hat{q}_2^{A\text{-}B} < p_A - p_B < \hat{q}_1^{A\text{-}B}$ so that there is a positive demand for each product. Maximizing the profit of firm $K \in \{A, B\}$, $\pi_K(p, \hat{q}^{A\text{-}B}) = p_K D_K(p, \hat{q}^{A\text{-}B})$, with respect to p_K yields best-response functions $p_A(p_B) = \frac{p_B + \hat{q}_1^{A\text{-}B}}{2}$ and $p_B(p_A) = \frac{p_A - \hat{q}_2^{A\text{-}B}}{2}$. Given that the functions are linear, there exist unique mutual best responses $p_A^* = \frac{2\hat{q}_1^{A\text{-}B} - \hat{q}_2^{A\text{-}B}}{3}$ and $p_B^* = \frac{\hat{q}_1^{A\text{-}B} - 2\hat{q}_2^{A\text{-}B}}{3}$. It can be verified that the equilibrium prices satisfy $\hat{q}_2^{A\text{-}B} < p_A^* - p_B^* < \hat{q}_1^{A\text{-}B}$ if and only if $\hat{q}_2^{A\text{-}B} < \frac{\hat{q}_1^{A\text{-}B}}{2}$, which is the first case in (3).

Consider next the case where $p_A - p_B \leq \hat{q}_2^{A\text{-}B}$ so that there is a positive demand only for product A. In any equilibrium under this case, we must have that $p_A - p_B = \hat{q}_2^{A\text{-}B}$. Suppose instead that $p_A - p_B < \hat{q}_2^{A\text{-}B}$. Then by raising its price to $p_A + \varepsilon$, with $\varepsilon > 0$ sufficiently small so that the demand for product A remains to be $D_A(p, \hat{q}^{A\text{-}B}) = 1$, firm A can increase its profit. We next argue that in any equilibrium with $p_A - p_B = \hat{q}_2^{A\text{-}B}$, it must be the case that $p_B = 0$. Suppose instead that $p_B > 0$. Then by charging $p_B' = p_B - \delta$ so that $p_A - p_B' = \hat{q}_2^{A\text{-}B} + \delta > \hat{q}_2^{A\text{-}B}$, firm B secures a positive demand and earns positive profit rather than the zero profit from charging p_B. We further identify the condition under which firm A has no incentive to deviate to charge higher than $p_A = \hat{q}_2^{A\text{-}B}$. For any such deviation, where product A is no longer the only product with positive demand, the highest profit firm A can obtain is by setting $p_A' = \frac{\hat{q}_1^{A\text{-}B}}{2}$ with resulting $\pi(p_A', \hat{q}^{A\text{-}B}) = \left(\frac{\hat{q}_1^{A\text{-}B}}{2}\right)\left(1 - \frac{\frac{\hat{q}_1^{A\text{-}B}}{2} - \hat{q}_2^{A\text{-}B}}{\hat{q}_1^{A\text{-}B} - \hat{q}_2^{A\text{-}B}}\right)$, while the profit from charging $p_A = \hat{q}_2^{A\text{-}B}$ is $\pi(p_A, \hat{q}^{A\text{-}B}) = \hat{q}_2^{A\text{-}B}$. It can be verified that $\pi(p_A, \hat{q}^{A\text{-}B}) \geq \pi(p_A', \hat{q}^{A\text{-}B})$ if and only if $\frac{\hat{q}_1^{A\text{-}B}}{2} \leq \hat{q}_2^{A\text{-}B}$, and this is the second case in (3) with $p_A^* = \hat{q}_2^{A\text{-}B}$ and $p_B^* = 0$.

Finally, we argue that there is no equilibrium under the case where $p_A - p_B \geq \hat{q}_1^{A\text{-}B}$ with a positive demand only for product B. Given that $\hat{q}_1^{A\text{-}B} > 0$, we have that $p_A > p_B$. But then firm B always has a profitable deviation by charging a slightly higher price.

□

Proof of Corollary 1. The corollary follows immediately from the derivatives of the prices in (3) and the definition of absolute and comparative advantages.

□

Proof of Proposition 2. The explicit expressions of the firms' reduced-form profits are

$$\pi_A^*(\hat{q}^{A\text{-}B}) = \frac{(2\hat{q}_1^{A\text{-}B} - \hat{q}_2^{A\text{-}B})^2}{9(\hat{q}_1^{A\text{-}B} - \hat{q}_2^{A\text{-}B})}, \text{ and} \tag{A.1}$$

$$\pi_B^*(\hat{q}^{A\text{-}B}) = \frac{(\hat{q}_1^{A\text{-}B} - 2\hat{q}_2^{A\text{-}B})^2}{9(\hat{q}_1^{A\text{-}B} - \hat{q}_2^{A\text{-}B})}. \tag{A.2}$$

Evaluating the signs of the following four derivatives, $\frac{\partial \pi_A^*}{\partial \hat{q}_1^{A\text{-}B}} = \frac{(2\hat{q}_1^{A\text{-}B} - 3\hat{q}_2^{A\text{-}B})(2\hat{q}_1^{A\text{-}B} - \hat{q}_2^{A\text{-}B})}{9(\hat{q}_1^{A\text{-}B} - \hat{q}_2^{A\text{-}B})^2}$, $\frac{\partial \pi_A^*}{\partial \hat{q}_2^{A\text{-}B}} = \frac{\hat{q}_2^{A\text{-}B}(2\hat{q}_1^{A\text{-}B} - \hat{q}_2^{A\text{-}B})}{9(\hat{q}_1^{A\text{-}B} - \hat{q}_2^{A\text{-}B})^2}$, $\frac{\partial \pi_B^*}{\partial \hat{q}_1^{A\text{-}B}} = \frac{\hat{q}_1^{A\text{-}B}(\hat{q}_1^{A\text{-}B} - 2\hat{q}_2^{A\text{-}B})}{9(\hat{q}_1^{A\text{-}B} - \hat{q}_2^{A\text{-}B})^2}$, and $\frac{\partial \pi_B^*}{\partial \hat{q}_2^{A\text{-}B}} = \frac{(2\hat{q}_2^{A\text{-}B} - 3\hat{q}_1^{A\text{-}B})(\hat{q}_1^{A\text{-}B} - 2\hat{q}_2^{A\text{-}B})}{9(\hat{q}_1^{A\text{-}B} - \hat{q}_2^{A\text{-}B})^2}$, under the parameter restrictions that $\hat{q}_1^{A\text{-}B} > 0$ and $\hat{q}_2^{A\text{-}B} \in \left[-\hat{q}_1^{A\text{-}B}, \frac{\hat{q}_1^{A\text{-}B}}{2}\right]$ gives us the properties.

□

Proof of Corollary 2. The corollary follows immediately from Proposition 2 and the definition of absolute and comparative advantages.

□

Proof of Proposition 3. We begin by constructing two types of equilibria that support the outcome $Q^* = \mathbb{Q}$. Suppose that in a putative equilibrium only firm A discloses all $q^{A\text{-}B} \in \mathbb{Q}$, while firm B may disclose some or none of the attribute advantages. Any event of non-disclosure must be resulting from a deviation by firm A, and we assign consumer beliefs so that in any of these out-of-equilibrium events the expected attribute advantages are $(\underline{q}_1^{A\text{-}B}, 0)$. Since $\pi_A^*(q^{A\text{-}B}) \geq \pi_A^*(\underline{q}_1^{A\text{-}B}, 0)$ for all $q^{A\text{-}B}$, firm A has no strict incentive to deviate. Given that all $q^{A\text{-}B}$ are disclosed by firm A, any disclosure strategy by firm B is a best response. It follows that this is an equilibrium. By a similar argument, only firm B disclosing all $q^{A\text{-}B}$ constitutes an equilibrium under out-of-equilibrium expected attribute advantages $(\underline{q}_1^{A\text{-}B}, \overline{q}_2^{A\text{-}B})$ in the event of non-disclosure. These give us one type of fully revealing equilibria where $Q_{K'}^* \subset Q_K^* = Q^* = \mathbb{Q}$, $K, K' \in \{A, B\}$ and $K \neq K'$. Another type of equilibrium has both firms disclose all $q^{A\text{-}B}$ so that $Q_A^* = Q_B^* = Q^* = \mathbb{Q}$. To see that this is an equilibrium, note that under the verifiability constraint consumers believe that the attribute advantages are $q^{A\text{-}B}$ when one firm deviates and does not discloses $q^{A\text{-}B}$.

We proceed to show that there exists no equilibrium in which $Q^* \subset \mathbb{Q}$. For a given $\tilde{q}^{A\text{-}B} \in \mathbb{Q}$, define $LP_K(\tilde{q}^{A\text{-}B}) = \{q^{A\text{-}B} \in \mathbb{Q} : \pi_K^*(q^{A\text{-}B}) \leq \pi_K^*(\tilde{q}^{A\text{-}B})\}$ to be the set of attribute advantages that would give firm $K \in \{A, B\}$ a weakly lower profit than would $\tilde{q}^{A\text{-}B}$. We first establish that $LP_K(\tilde{q}^{A\text{-}B})$ is a convex set. From the explicit expressions of the reduced-form profits in (A.1) and (A.2), we obtain the Hessian matrix of firm K's profit:

$$H_{\pi_K^*}(\tilde{q}^{A\text{-}B}) = \begin{bmatrix} \frac{2(\tilde{q}_2^{A\text{-}B})^2}{9(\tilde{q}_1^{A\text{-}B} - \tilde{q}_2^{A\text{-}B})^3} & -\frac{2\tilde{q}_1^{A\text{-}B}\tilde{q}_2^{A\text{-}B}}{9(\tilde{q}_1^{A\text{-}B} - \tilde{q}_2^{A\text{-}B})^3} \\ -\frac{2\tilde{q}_1^{A\text{-}B}\tilde{q}_2^{A\text{-}B}}{9(\tilde{q}_1^{A\text{-}B} - \tilde{q}_2^{A\text{-}B})^3} & \frac{2(\tilde{q}_1^{A\text{-}B})^2}{9(\tilde{q}_1^{A\text{-}B} - \tilde{q}_2^{A\text{-}B})^3} \end{bmatrix},$$

which is positive semidefinite for all $\tilde{q}^{A\text{-}B} \in \mathbb{Q}$ under Assumption 1′. It follows that $\pi_K^*(\cdot)$ is a convex function. Note that $LP_K(\tilde{q}^{A\text{-}B})$ is a lower contour set for $\pi_K^*(\cdot)$. The fact that $\pi_K^*(\cdot)$ is convex implies that $LP_K(\tilde{q}^{A\text{-}B})$ is a convex set for all $\tilde{q}^{A\text{-}B} \in \mathbb{Q}$.

We utilize this convex property to show that in any equilibrium, if there exists a non-empty $\overline{Q}^*_{K'}$, then all $q^{A\text{-}B} \in \overline{Q}^*_{K'}$ must be disclosed by firm $K \neq K'$. Suppose not so that $\overline{Q}^*_{K'} = \overline{Q}^* \subseteq \overline{Q}^*_K$ and let $\hat{q}^{A\text{-}B}(\overline{Q}^*) = E[q^{A\text{-}B} | q^{A\text{-}B} \in \overline{Q}^*]$. For this to constitute an equilibrium, it must be that $\overline{Q}^* \subseteq LP_K(\hat{q}^{A\text{-}B}(\overline{Q}^*))$. The fact that $LP_K(\hat{q}^{A\text{-}B}(\overline{Q}^*))$ is a convex set then implies that $\hat{q}^{A\text{-}B}(\overline{Q}^*) \in LP_K(\hat{q}^{A\text{-}B}(\overline{Q}^*))$. A contradiction follows immediately if $\hat{q}^{A\text{-}B}(\overline{Q}^*) \in \text{int}(LP_K(\hat{q}^{A\text{-}B}(\overline{Q}^*)))$ because then $\pi^*_K(\hat{q}^{A\text{-}B}(\overline{Q}^*)) > \pi^*_K(\hat{q}^{A\text{-}B}(\overline{Q}^*))$. Given that the density $f_i^{A\text{-}B}$, $i \in \{1, 2\}$, is everywhere positive, if $\hat{q}^{A\text{-}B}(\overline{Q}^*) \in \partial(LP_K(\hat{q}^{A\text{-}B}(\overline{Q}^*)))$, then there must exist a $q'^{A\text{-}B} \in \overline{Q}^*$ such that $\pi^*_K(q'^{A\text{-}B}) > \pi^*_K(\hat{q}^{A\text{-}B}(\overline{Q}^*))$ and thus $q'^{A\text{-}B} \notin LP_K(\hat{q}^{A\text{-}B}(\overline{Q}^*))$, again a contradiction. It follows that $Q^* = \mathbb{Q}$ is the unique equilibrium outcome.

□

Proof of Lemma 1. We begin with the case where $\mathbb{Q}$ satisfying Assumption 1′ is such that $\overline{q}_2^{A\text{-}B} > 0$. Consider any equilibrium disclosure outcome $Q^* \subseteq \mathbb{Q}$. For any $q^{A\text{-}B} \in \mathbb{Q}$, there are three possibilities for firm K's profit, $K \in \{A, B\}$: (a) if $q^{A\text{-}B}$ is revealed by firm K, then its profit is $\pi^*_K(q^{A\text{-}B}) - c$, (b) if $q^{A\text{-}B}$ is revealed only by firm $K' \neq K$, then its profit is $\pi^*_K(q^{A\text{-}B})$, and (c) if $q^{A\text{-}B}$ is not revealed at all, then its profit is either $\pi^*_K(\hat{q}^{A\text{-}B}(\overline{Q}^*))$ where $\hat{q}^{A\text{-}B}(\overline{Q}^*) = E[q^{A\text{-}B} | q^{A\text{-}B} \in \overline{Q}^*]$ or, if this is an out-of-equilibrium event, $\pi^*_K(\acute{q}^{A\text{-}B})$ where $\acute{q}^{A\text{-}B}$ is some expected attribute advantages based on consumer out-of-equilibrium beliefs. The reduced-form profits, $\pi^*_A(q^{A\text{-}B}) = \frac{(2q_1^{A\text{-}B} - q_2^{A\text{-}B})^2}{9(q_1^{A\text{-}B} - q_2^{A\text{-}B})}$ and $\pi^*_B(q^{A\text{-}B}) = \frac{(q_1^{A\text{-}B} - 2q_2^{A\text{-}B})^2}{9(q_1^{A\text{-}B} - q_2^{A\text{-}B})}$, are minimized at $\left(\underline{q}_1^{A\text{-}B}, 0\right)$ and $\left(\underline{q}_1^{A\text{-}B}, \overline{q}_2^{A\text{-}B}\right)$ respectively. Since $\pi^*_A\left(\underline{q}_1^{A\text{-}B}, 0\right) - c < \pi^*_A(q^{A\text{-}B})$ for any $q^{A\text{-}B} \in \mathbb{Q}$, firm A does not reveal $\left(\underline{q}_1^{A\text{-}B}, 0\right)$ regardless of whether firm B reveals $\left(\underline{q}_1^{A\text{-}B}, 0\right)$ or not. It follows that we must have that $\left(\underline{q}_1^{A\text{-}B}, 0\right) \in \overline{Q}^*_A$. The continuity of $\pi^*_A(\cdot)$ implies that there exists $\epsilon^*_A > 0$ such that $N_{\epsilon^*_A}\left[\left(\underline{q}_1^{A\text{-}B}, 0\right)\right] \subset \overline{Q}^*_A$. By a similar argument, for firm B, there exists $\epsilon^*_B > 0$ such that $N_{\epsilon^*_B}\left[\left(\underline{q}_1^{A\text{-}B}, \overline{q}_2^{A\text{-}B}\right)\right] \subset \overline{Q}^*_B$. Let $\epsilon^* = \min\{\epsilon^*_A, \epsilon^*_B\}$, and the lemma follows. For the case where $\overline{q}_2^{A\text{-}B} \leq 0$, $\pi^*_A(q^{A\text{-}B})$ and $\pi^*_B(q^{A\text{-}B})$ are minimized at the same point $\left(\underline{q}_1^{A\text{-}B}, \overline{q}_2^{A\text{-}B}\right)$. Thus, for firm A, the condition $\left(\underline{q}_1^{A\text{-}B}, 0\right) \in \overline{Q}^*_A$ is replaced by $\left(q_1^{A\text{-}B}, \overline{q}_2^{A\text{-}B}\right) \in \overline{Q}^*_A$. The lemma then follows by a similar argument.

□

Proof of Proposition 4. We prove the proposition by verifying five claims. Claims 1 and 2 establish two properties of equilibrium outcomes.

Claim 1. *If Q^* is an equilibrium outcome for $c > 0$, then $\overline{Q}^* \neq \varnothing$.*

Proof of Claim 1. We prove by contradiction. Suppose that $c > 0$ and there exists an equilibrium in which $\overline{Q}^* = \varnothing$ or equivalently $Q^* = \mathbb{Q}$. Lemma 1 and the fact that the two firms will not disclose the same $q^{A\text{-}B}$ in equilibrium when $c > 0$ implies that the equilibrium must not involve only one firm disclosing all $q^{A\text{-}B} \in \mathbb{Q}$ or both firms disclosing all $q^{A\text{-}B} \in \mathbb{Q}$, the types of fully revealing equilibria constructed in the proof of Proposition 3 for costless disclosure. We further rule out the remaining possibilities for a fully revealing outcome where $Q^*_A \subset \mathbb{Q}$, $Q^*_B \subset \mathbb{Q}$, $Q^*_A \cup Q^*_B = \mathbb{Q}$, and $Q^*_A \cap Q^*_B = \varnothing$. For a putative equilibrium with these properties, the absence of disclosure is an out-of-equilibrium event in which consumer beliefs are neither pinned down by Bayes' rule nor restricted by the verifiability constraint. Denote by $\acute{q}^{A\text{-}B}$ an expected attribute advantages based on consumer out-of-equilibrium beliefs. With $c > 0$, firms A and B strictly prefer not to disclose any $q^{A\text{-}B} \in LP_A(\acute{q}^{A\text{-}B}) \cap LP_B(\acute{q}^{A\text{-}B})$, and computation using the profit functions $\pi^*_A(\cdot)$ and $\pi^*_B(\cdot)$ shows that $LP_A(\acute{q}^{A\text{-}B}) \cap LP_B(\acute{q}^{A\text{-}B}) \neq \varnothing$ for any $\acute{q}^{A\text{-}B} \in \mathbb{Q}$. Hence, for any putative equilibrium with the above properties supported by any $\acute{q}^{A\text{-}B}$, there exists a profitable deviation.

□

We introduce some notations and definition for Claim 2. Denote the unconditional expected attribute advantages under distributions $F^{A\text{-}B}$ by $\hat{q}^{A\text{-}B}(F^{A\text{-}B})$ and define $\overline{c}_{F^{A\text{-}B}} = \min\{\overline{\pi}_A - \pi^*_A(\hat{q}^{A\text{-}B}(F^{A\text{-}B})), \overline{\pi}_B - \pi^*_B(\hat{q}^{A\text{-}B}(F^{A\text{-}B}))\}$, where $\overline{\pi}_K = \max_{q^{A\text{-}B} \in \mathbb{Q}} \pi^*_K(q^{A\text{-}B})$, $K \in \{A, B\}$.

Claim 2. *Given $F^{A\text{-}B}$, if Q^* is an equilibrium outcome for $c \in (0, \overline{c}_{F^{A\text{-}B}})$, then $Q^* \neq \varnothing$.*

Proof of Claim 2. We prove by contradiction. Suppose that $c < \overline{c}_{F^{A\text{-}B}}$ and there exists an equilibrium in which $Q^* = \varnothing$ or equivalently $\overline{Q}^* = \mathbb{Q}$. We show that firm $K \in \{A, B\}$ has a profitable deviation. Note that the unconditional $\hat{q}^{A\text{-}B}(F^{A\text{-}B})$ is the expected attribute advantages conditional on non-disclosure given that $\overline{Q}^* = \mathbb{Q}$. By construction, $c < \overline{c}_{F^{A\text{-}B}} \leq \overline{\pi}_K - \pi^*_K(\hat{q}^{A\text{-}B}(F^{A\text{-}B}))$, which implies that $\overline{\pi}_K - c > \pi^*_K(\hat{q}^{A\text{-}B}(F^{A\text{-}B}))$. Thus, firm K strictly prefers to disclose $\operatorname{argmax}_{q^{A\text{-}B} \in \mathbb{Q}} \pi^*_K(q^{A\text{-}B})$.

□

Claims 1 and 2 have established that for any $c \in (0, \overline{c}_{F^{A\text{-}B}})$, if an equilibrium exists, then it is partially revealing. Claims 3 and 4 are about existence. We further introduce additional notations and definitions. Let $S(q^{A\text{-}B})$ be the set of disclosure signals available to a firm when the realized attribute advantages are $q^{A\text{-}B} \in \mathbb{Q}$. Under the accurate verifiability constraint, $S(q^{A\text{-}B}) = \{\{q^{A\text{-}B}\}, \mathbb{Q}\}$; a firm decides whether to send $s = \{q^{A\text{-}B}\}$, disclosing the realized attribute advantages, or to send $s = \mathbb{Q}$, which amounts to not disclosing. Define $S = \bigcup_{q^{A\text{-}B} \in \mathbb{Q}} S(q^{A\text{-}B})$. A pure disclosure strategy of firm $K \in \{A, B\}$ is a mapping $\delta_K : \mathbb{Q} \to S$ with the restriction that for all $q^{A\text{-}B} \in \mathbb{Q}$, $\delta_K(q^{A\text{-}B}) \in S(q^{A\text{-}B})$.

We specify a conditional best response of firm K, CBR_K, relative to an arbitrary profit threshold $\pi_K \geq 0$. Given π_K, CBR_K prescribes firm K the following response to $\delta_{K'}$,

$K' \neq K$: for every $q^{A\text{-}B} \in \mathbb{Q}$,

$$CBR_K[q^{A\text{-}B}, \delta_{K'}(q^{A\text{-}B})|\pi_K] = \begin{cases} q^{A\text{-}B} & \text{if } \delta_{K'}(q^{A\text{-}B}) = \mathbb{Q} \text{ and } \pi_K^*(q^{A\text{-}B}) - c \geq \pi_K, \\ \mathbb{Q} & \text{if } \delta_{K'}(q^{A\text{-}B}) = q^{A\text{-}B} \text{ or } \pi_K^*(q^{A\text{-}B}) - c < \pi_K. \end{cases}$$

Claim 3. *In any equilibrium, δ_K^* satisfies: for every $q^{A\text{-}B} \in \mathbb{Q}$,*

$$\delta_K^*(q^{A\text{-}B}) = CBR_K[q^{A\text{-}B}, \delta_{K'}^*(q^{A\text{-}B})|\pi_K^*(\hat{q}^{A\text{-}B}(\overline{Q}^*))],$$

where $K, K' \in \{A, B\}$, $K \neq K'$, and $\hat{q}^{A\text{-}B}(\overline{Q}^*) = E[q^{A\text{-}B}|q^{A\text{-}B} \in \overline{Q}^*]$.

Proof of Claim 3. By Claim 1, we can focus on putative equilibria in which $\overline{Q}^* \neq \varnothing$. Suppose that firm K' does not disclose $q^{A\text{-}B}$. If firm K discloses $q^{A\text{-}B}$, then its profit is $\pi_K^*(q^{A\text{-}B}) - c$; if firm K does not disclose $q^{A\text{-}B}$, then its profit is $\pi_K^*(\hat{q}^{A\text{-}B}(\overline{Q}^*))$. It follows that if in a putative equilibrium $\delta_{K'}^*(q^{A\text{-}B}) = \mathbb{Q}$, then $\delta_K^*(q^{A\text{-}B}) = q^{A\text{-}B}$ if and only if $\pi_K^*(q^{A\text{-}B}) - c \geq \pi_K^*(\hat{q}^{A\text{-}B}(\overline{Q}^*))$. Note that specifying either disclosure or non-disclosure by firm K for $\pi_K^*(q^{A\text{-}B}) - c = \pi_K^*(\hat{q}^{A\text{-}B}(\overline{Q}^*))$ does not affect the measure of disclosed attribute advantages. Finally, the condition that $\delta_{K'}^*(q^{A\text{-}B}) = q^{A\text{-}B}$ implies that $\delta_K^*(q^{A\text{-}B}) = \mathbb{Q}$ follows from the fact that in equilibrium the two firms will not disclose the same $q^{A\text{-}B}$ when $c > 0$.

□

Claim 4. *An equilibrium exists.*

Proof of Claim 4. The set of non-disclosed attribute advantages by firm K can be characterized as $\overline{Q}_K = \{q^{A\text{-}B} \in \mathbb{Q} : CBR_K[q^{A\text{-}B}, \delta_{K'}(q^{A\text{-}B})|\pi_K] = \mathbb{Q}\}$. It can be seen from this characterization that a pair of profit thresholds (π_A, π_B) delineates the set of attribute advantages not disclosed by either firm, $\overline{Q} = \overline{Q}_A \cap \overline{Q}_B$. The conditional expectation function, $\hat{q}^{A\text{-}B}(\cdot) = E[q^{A\text{-}B}|\cdot]$, and then firm K's profit function, $\pi_K^*(\cdot)$, further map $\overline{Q}$ to $\pi_K^*(\hat{q}^{A\text{-}B}(\overline{Q}))$. Let $g_K : [\underline{\pi}_A, \overline{\pi}_A] \times [\underline{\pi}_B, \overline{\pi}_B] \to [\underline{\pi}_K, \overline{\pi}_K]$ be this composite mapping for firm $K \in \{A, B\}$, where $\underline{\pi}_K = \min_{q^{A\text{-}B} \in \mathbb{Q}} \pi_K^*(q^{A\text{-}B})$ and $\overline{\pi}_K = \max_{q^{A\text{-}B} \in \mathbb{Q}} \pi_K^*(q^{A\text{-}B})$. We further define a function $g : [\underline{\pi}_A, \overline{\pi}_A] \times [\underline{\pi}_B, \overline{\pi}_B] \to [\underline{\pi}_A, \overline{\pi}_A] \times [\underline{\pi}_B, \overline{\pi}_B]$ by $g(\pi_A, \pi_B) = g_A(\pi_A, \pi_B) \times g_B(\pi_A, \pi_B)$.

Claim 3 has established that if an equilibrium exists, then each firm's disclosure strategy must constitute a conditional best response. It follows that a fixed point of g, $(\tilde{\pi}_A, \tilde{\pi}_B) = g(\tilde{\pi}_A, \tilde{\pi}_B)$, characterizes an equilibrium that induces the non-disclosure set $\overline{Q}^*$. If a fixed point exists, then an equilibrium exists. In the domain of g, the upper and lower bounds of the sets, $\underline{\pi}_K$ and $\overline{\pi}_K$, $K \in \{A, B\}$, represent firm K's lowest and highest profit in the space $\mathbb{Q} = \left[\underline{q}_1^{A\text{-}B}, \overline{q}_1^{A\text{-}B}\right] \times \left[\underline{q}_2^{A\text{-}B}, \overline{q}_2^{A\text{-}B}\right]$. Therefore, the domain of g is non-empty, compact, and convex. The continuity of the distribution functions $F^{A\text{-}B}$ and of the profit functions

$\pi_A^*(\cdot)$ and $\pi_B^*(\cdot)$ imply that g is continuous. Thus, with all the conditions of the Brouwer's fixed point theorem satisfied, there exists a fixed point for g.

□

Claim 5 concerns the characterizations of $\overline{Q}^*$ in parts (a) and (b) of the proposition.

Claim 5. *For any partially revealing equilibrium outcome Q^*, if $\overline{q}_2^{A\text{-}B} > 0$, then there exists $\epsilon_{hv}^* > 0$ such that $N_{\epsilon_{hv}^*}\left[\left(\underline{q}_1^{A\text{-}B}, q_2^{A\text{-}B}\right)\right] \subset \overline{Q}^*$ for some $q_2^{A\text{-}B} \in \left(0, \overline{q}_2^{A\text{-}B}\right)$; if $\overline{q}_2^{A\text{-}B} \leq 0$, then there exists $\epsilon_h^* > 0$ such that $N_{\epsilon_h^*}\left[\left(\underline{q}_1^{A\text{-}B}, \overline{q}_2^{A\text{-}B}\right)\right] \subset \overline{Q}^*$.*

Proof of Claim 5. With $c > 0$, the existence of a partially revealing equilibrium implies that $LP_A(\hat{q}^{A\text{-}B}(\overline{Q}^*)) \cap LP_B(\hat{q}^{A\text{-}B}(\overline{Q}^*)) \subset \overline{Q}^* \neq \varnothing$. Also recall that $LP_A(q^{A\text{-}B}) \cap LP_B(q^{A\text{-}B}) \neq \varnothing$ for any $q^{A\text{-}B} \in \mathbb{Q}$. It follows that for the case where $\overline{q}_2^{A\text{-}B} > 0$, since $\pi_A^*(q^{A\text{-}B})$ and $\pi_B^*(q^{A\text{-}B})$ are minimized at $\left(\underline{q}_1^{A\text{-}B}, 0\right)$ and $\left(\underline{q}_1^{A\text{-}B}, \overline{q}_2^{A\text{-}B}\right)$ respectively, we must have that $\left(\underline{q}_1^{A\text{-}B}, q_2^{A\text{-}B}\right) \in LP_A(\hat{q}^{A\text{-}B}(\overline{Q}^*)) \cap LP_B(\hat{q}^{A\text{-}B}(\overline{Q}^*))$ for some $q_2^{A\text{-}B} \in \left(0, \overline{q}_2^{A\text{-}B}\right)$. The continuity of the profit functions and the set inclusion $LP_A(\hat{q}^{A\text{-}B}(\overline{Q}^*)) \cap LP_B(\hat{q}^{A\text{-}B}(\overline{Q}^*)) \subset \overline{Q}^*$ then imply that there exists $\epsilon_{hv}^* > 0$ such that $N_{\epsilon_{hv}^*}\left[\left(\underline{q}_1^{A\text{-}B}, q_2^{A\text{-}B}\right)\right] \subset \overline{Q}^*$ for some $q_2^{A\text{-}B} \in \left(0, \overline{q}_2^{A\text{-}B}\right)$. For the case where $\overline{q}_2^{A\text{-}B} \leq 0$, since $\pi_A^*(q^{A\text{-}B})$ and $\pi_B^*(q^{A\text{-}B})$ are both minimized at $\left(\underline{q}_1^{A\text{-}B}, \overline{q}_2^{A\text{-}B}\right)$, we must have that $\left(\underline{q}_1^{A\text{-}B}, \overline{q}_2^{A\text{-}B}\right) \in LP_A(\hat{q}^{A\text{-}B}(\overline{Q}^*)) \cap LP_B(\hat{q}^{A\text{-}B}(\overline{Q}^*))$. It follows that there exists $\epsilon_h^* > 0$ such that $N_{\epsilon_h^*}\left[\left(\underline{q}_1^{A\text{-}B}, \overline{q}_2^{A\text{-}B}\right)\right] \subset \overline{Q}^*$.

□

The proofs of Claims 1–5 combined to prove Proposition 4.

□

Proof of Proposition 5. We construct a partially revealing equilibrium where $Q^* \neq \varnothing$ and $\overline{Q}^* \neq \varnothing$. We continue to use the notation for Claim 2 for the proof of Proposition 4, denoting the unconditional expected attribute advantages for distributions $F^{A\text{-}B}$ by $\hat{q}^{A\text{-}B}(F^{A\text{-}B})$ (instead of $E[q^{A\text{-}B}]$ in the main text). Upon non-disclosure, the expected attribute advantages of the boundedly rational consumers are $\hat{q}^{A\text{-}B}(F^{A\text{-}B})$. We assign $\overline{Q}^* = LP_A(\hat{q}^{A\text{-}B}(F^{A\text{-}B})) \cap LP_B(\hat{q}^{A\text{-}B}(F^{A\text{-}B}))$. Recall that $LP_A(q^{A\text{-}B}) \cap LP_B(q^{A\text{-}B}) \neq \varnothing$ for any $q^{A\text{-}B} \in \mathbb{Q}$, and it follows from the proof of Claim 5 for Proposition 4 that, if $\overline{q}_2^{A\text{-}B} > 0$, then $\left(\underline{q}_1^{A\text{-}B}, q_2^{A\text{-}B}\right) \in LP_A(\hat{q}^{A\text{-}B}(F^{A\text{-}B})) \cap LP_B(\hat{q}^{A\text{-}B}(F^{A\text{-}B}))$ for some $q_2^{A\text{-}B} \in \left(0, \overline{q}_2^{A\text{-}B}\right)$, and, if $\overline{q}_2^{A\text{-}B} \leq 0$, then $\left(\underline{q}_1^{A\text{-}B}, \overline{q}_2^{A\text{-}B}\right) \in LP_A(\hat{q}^{A\text{-}B}(F^{A\text{-}B})) \cap LP_B(\hat{q}^{A\text{-}B}(F^{A\text{-}B}))$. Furthermore, the proof of Proposition 3 has established that $LP_K(q^{A\text{-}B})$, $K \in \{A, B\}$, is a convex set for any $q^{A\text{-}B} \in \mathbb{Q}$. These properties imply that $\text{int}(LP_A(\hat{q}^{A\text{-}B}(F^{A\text{-}B})) \cap LP_B(\hat{q}^{A\text{-}B}(F^{A\text{-}B}))) \neq \varnothing$. Thus, firms A and B prefer not to disclose any $q^{A\text{-}B} \in LP_A(\hat{q}^{A\text{-}B}(F^{A\text{-}B})) \cap LP_B(\hat{q}^{A\text{-}B}(F^{A\text{-}B}))$ and strictly so for any $q^{A\text{-}B} \in \text{int}(LP_A(\hat{q}^{A\text{-}B}(F^{A\text{-}B})) \cap LP_B(\hat{q}^{A\text{-}B}(F^{A\text{-}B})))$, as we desired.

We next argue that for $\overline{Q}^* = LP_A(\hat{q}^{A\text{-}B}(F^{A\text{-}B})) \cap LP_B(\hat{q}^{A\text{-}B}(F^{A\text{-}B}))$, $Q^* \neq \varnothing$ or equivalently $\overline{Q}^* \subset \mathbb{Q}$. Suppose instead that $\overline{Q}^* = \mathbb{Q}$. Since the density $f_i^{A\text{-}B}$, $i \in \{1,2\}$, is everywhere positive, $\overline{\pi}_K > \pi_K^*(\hat{q}^{A\text{-}B}(F^{A\text{-}B}))$, and thus firm K strictly prefers to disclose $\operatorname{argmax}_{q^{A\text{-}B} \in \mathbb{Q}} \pi_K^*(q^{A\text{-}B})$, yielding a contradiction. The proof for the characterizations of $\overline{Q}^*$ in parts (a) and (b) of the proposition is similar to the proof of Claim 5 for Proposition 4 and is therefore skipped.

□