PHYSICS 262

GEOMETRIC OPTICS

Part I – Position and Size of Image: Cardinal Points

If the indices of refraction of all elements are known, together with the positions and radii of curvature of all surfaces, it is always possible to calculate the approximate position and size of the image of a given object by considering successively the effect of each surface. But information on refractive indices and radii of curvature is usually not available and these parameters are difficult to measure.

Fortunately, details of an optical system are unimportant for most purposes. One can locate the cardinal points and determine the focal lengths of any system or any component of a system and use these to calculate image size and position. (These calculations are based upon the paraxial approximation in which it is assumed that all rays make small angles with the axis of the system). The cardinal points, which occur in pairs with one point in object space and the other in image space, are defined in the following way.

Focal Points

Rays entering a system parallel to the axis converge to a focal point in image space, while rays emanating from the focal point in object space leave the system parallel to the axis. Like all cardinal points, these are fixed relative to the system, and interchange roles when the system is turned around.

Principal Points

The image of an object at one principal point appears at the other principal point, is erect, and has the same lateral size as the object. These are sometimes called points of unit magnification.

Nodal Points

A ray entering the system through the nodal point in object space will leave as a ray from the nodal point in image space parallel to the incident ray.

Cardinal Points

All cardinal points are on the axis of the system. Planes through these points and perpendicular to the axis are called the focal planes, the principal planes and the nodal planes. Focal planes are important because, for an ideally perfect paraxial system, they are the loci of points to which all entering bundles of mutually parallel rays converge. A bundle of mutually parallel rays converges to a focal point only if the entering rays are parallel to the axis of the system. Principal planes have the property that a ray entering the object principle plane at a given distance from the axis leaves the image principal plane at the same distance from the axis. Nodal planes have no important properties; it is the nodal points which are useful.

Focal Lengths

The focal lengths of a system are the distances from the principal points to the corresponding focal points. If the media on the two sides of the system have different indices of refraction, n and n', then the corresponding focal lengths, f and f', are related by:

$$n/f = n'/f' \qquad (1)$$

Unprimed quantities refer to the object space and primed quantities to the image space. In nearly all practical cases, however, with the notable exceptions of the oil or water inversion microscope and the eye, the medium is the same (usually air) on both sides of the system so the two focal lengths almost always have the same magnitude.

Although it is not immediately obvious from the definitions, the locations of the focal and principal points of a system are all that need be known to be able to calculate the approximate position and size of any image, given the position and size of the object. The positions of the nodal points are easy to find experimentally and once found can be used to locate the principal points.

There are two useful systems of equations relating object and image. In the Gaussian system, object and image distances are measured from the corresponding principal points, while in the Newtonian system they are measured from the focal points. The form of the equations in both systems is the same as for a thin lens.

The important quantities are pictured in Fig. 1 where, as in Eq. (1), unprimed quantities refer to object space and primed quantities to image space. The equations which follow Fig. 1 are given in general form and can usually be simplified, since, as previously pointed out, $n = n'$ and $f = f'$ in nearly all practical cases. The sign convention used here is that distances are positive in object space and negative in image space if they are measured opposite to the direction in which the light is traveling; a distance measured along a direction perpendicular to the axis is positive if the direction is up and negative if it is down. All quantities in Fig. 1 are positive except the image height, y'.

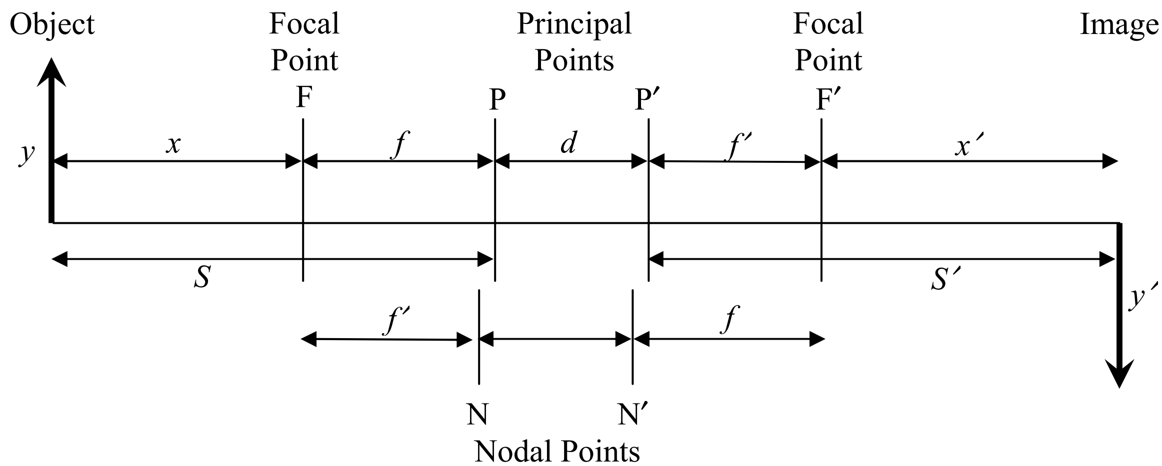


Figure 1: General optical system

Gaussian Equations

$$\frac{f'}{S'} + \frac{f}{S} = 1 \qquad (2)$$

$$M = \frac{y'}{y} = -\frac{n}{n'}\frac{S'}{S} \qquad (4)$$

Newtonian Equations

$$xx' = ff' \qquad (3)$$

$$M = \frac{y'}{y} = -\frac{x'}{f'} = -\frac{f}{x} \qquad (5)$$

where S and S' are object and image distances measured from principal points P and P′, x and x' are object and image distances measured from focal points F and F′, f and f' are focal lengths, y and y' are object and image heights, M is the lateral magnification, and d is the distance between principal points P and P′, and also between nodal points N and N′ (if any two points are in the opposite order from that shown in the diagram above, one merely takes the corresponding distance, for example f, to be negative, e.g. a 'negative', diverging lens).

Location of Cardinal Points and Determination of Focal Lengths

Refracting surfaces are not pictured in Fig. 1 since there is usually no need to know their positions. As noted previously, image position and size can be calculated for any object given only the positions of the focal and principal points, and these can be located with respect to any reference mark fixed to the system.

The focal points are easy to locate directly. Principal points are difficult to locate directly by means of their own properties, but in cases where the media on the two sides of the system are the same they coincide with the nodal points, which can be found with the use of an instrument known as a nodal slide. Even when the principal and nodal points do not coincide, their positions are simply correlated: N′ is a distance f from F′ and, similarly, N is a distance f' from F, as shown in Fig. 1. When the nodal slide is unsuitable, or when an independent method is needed to provide a check, measured quantities can be used in the object-image equations (1) – (5) to calculate the focal lengths. The principal points can then be located by displacing each focal point an amount equal to the corresponding focal length. Again the situation if simplest when the media on the two sides of the system are the same, because the focal lengths are then equal. Only this case will be considered further, although some of the techniques discussed might be adapted to more general cases.

Focal Points

A focal point can be found experimentally by locating the image of a distant object.

The object may actually be a long distance away or it may be placed artificially at infinity with the use of an auxiliary lens. In order to produce a virtual object at infinity the position of the auxiliary lens must be adjusted to that the real object is at the focal point. This adjustment can be easily and accurately made by autocollimation.

The generally useful **method of autocollimation** is illustrated in Fig. 2. A plane mirror is used to direct the light back through the lens, which is moved until the image falls in the plane of the object. Under these conditions the object and image both lie in

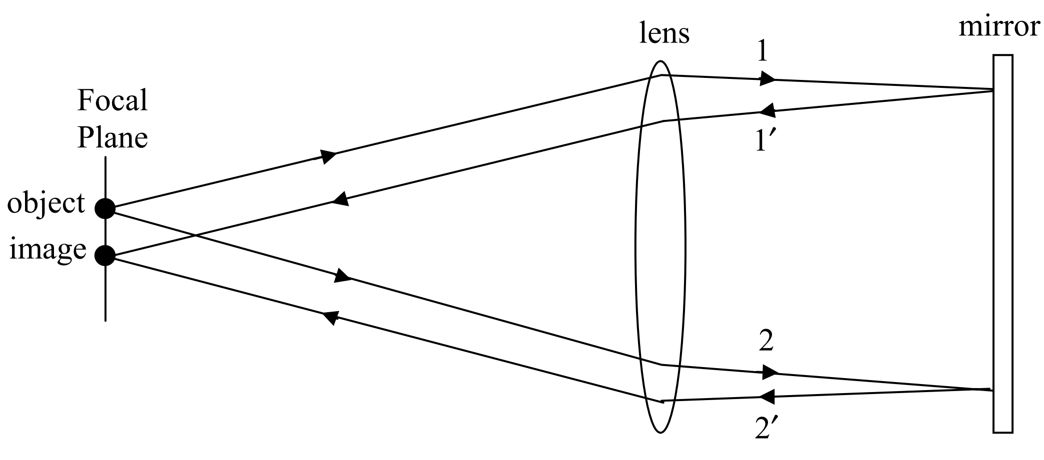


Figure 2: Autocollimation for finding the focal point of a lens

the focal plane and the lens produces a bundle of mutually parallel rays (1 and 2) for each point in the object. The word collimation means to form a parallel beam or column of light. (Since the object is necessarily finite in size, will all rays leaving the lens be parallel to each other?)

Returning to the system with the object located at infinity, the image can be found if it is real by focusing it on a screen. This is easy to do but usually not very accurate. A better method in the case of a real image is to view it with a magnifier focused on a set of cross-hairs. The magnifier and cross-hairs are moved until there is no parallax (i.e. no relative movement when the eye is moved slightly from side to side) between the image and the cross-hairs. The image and the cross-hairs are then coincident. Neither of these methods can be used if the focal points are inside the lens system because a distant object then forms a virtual image. In this case the image can be located with a microscope having a short depth of focus. The microscope is focused first on the image and then on the back lens surface, which can usually be identified by adhering dust particles. Clearly, the distance between these two settings is the distance of the image, and therefore the focal point, inside the back surface.

An alternative to employing an auxiliary lens is to use the method of autocollimation in which case the system provides its own distant object.

In any case, when one focal point has been found, the other can be located by turning the system around.

Nodal Points

A nodal slide is an arrangement for moving an optical system relative to a fixed axis of rotation, R, as shown schematically in Fig. 3.

The position of the system is adjusted, by shifting it relative to R, until a small rotation produces no movement of the image, I, of a distant object. The nodal point in image space, N′, is then coincident with the axis of rotation, R. As the system is shifted to arrive at this condition the image must be kept in sharp focus on a set of cross-hairs or viewing screen.

As pictured in Fig. 3, the nodal slide is being used with a distant object or an auxiliary lens to produce the parallel rays. It can also be used with a plane mirror to produce the parallel rays by autocollimation.

In either case the nodal point being located is the one in image space (remember in autocollimation that the parallel rays enter the system from the mirror). To find the other nodal point the system can be reversed.

As pointed out previously, the principal points coincide with the nodal points in practical cases where the optical system has the same medium (air) on both sides. In this case, with the system in adjustment, the principal point is at the axis of rotation, the focal point is at the image, and the distance between these positions is the focal length.

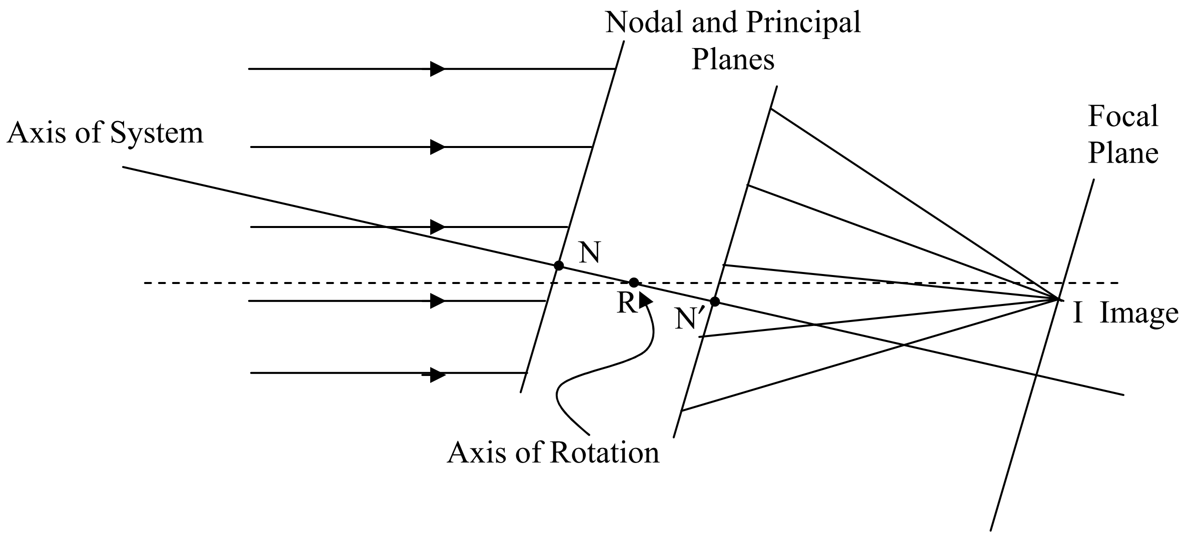


Figure 3: Nodal slide

Focal Length: Object-Image Distances

If the focal points of a system have been located, the Newtonian object and image distances, x and x', can be measured for a particular setting and the focal length calculated by means of equation (3), which reduces to $xx' = ff' = ff$ for practical cases in which the system is immersed in air.

A convenient way of determining x and x' when the focal points are outside the front and back surfaces of the system, but have not been located previously, is the three step process pictured in Fig. 4.

In the first step, autocollimation is used to locate the system when the primary focal point is at the object-image screen on the left. In the second step, the position of the system is determined when it forms an image on the screen at the right. In the third step autocollimation is again used, this time to locate the system when the secondary focal point is at the object-image screen on the right. If the object-image screens are in the same positions throughout, the values of x and x' are the distances the system has been moved between the first and second step, and between the second and third step, respectively.

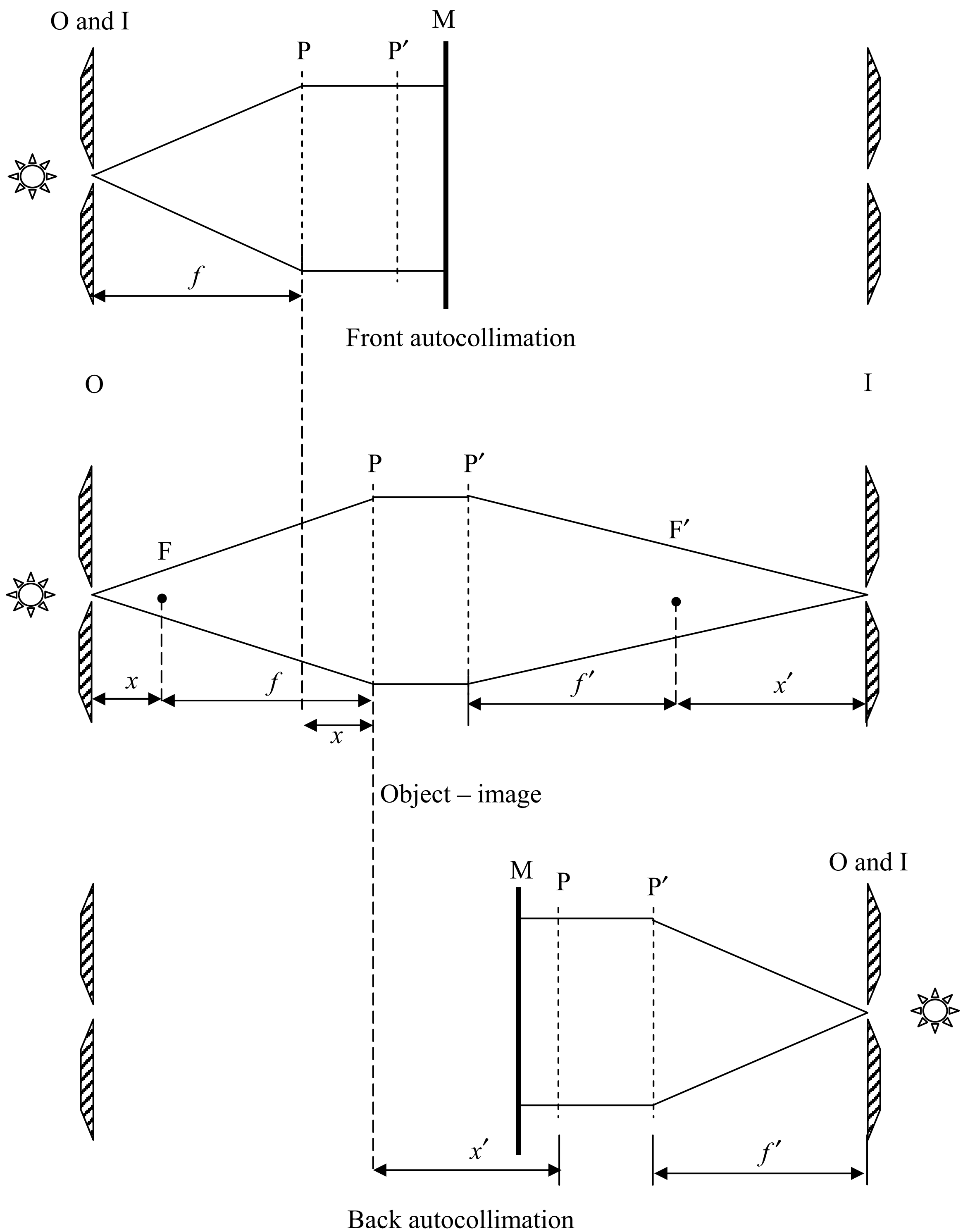


Figure 4: Three step method to find x and x'

Focal Length: Magnification

With systems having very small focal lengths, such as microscope objectives, it is difficult to measure object-image distances accurately. In this case, the focal length can be found by using an object of known dimensions and measuring the size of the image formed at two distances from the system. The object can be a fine scale (0.05mm/division) ruled on glass and the size of the image can be measured with a micrometer eyepiece or with a traveling microscope.

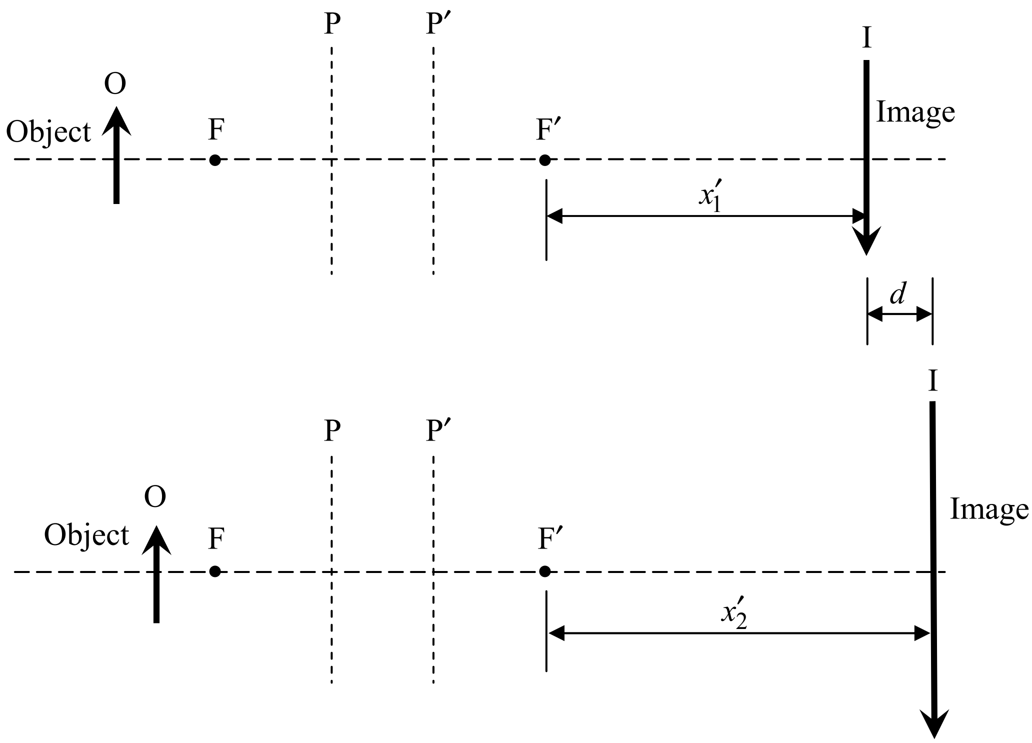


Figure 5: Focal Length by Magnification

For each position of the image, the magnification, M_1 or M_2, is determined by the ratio of the size of the image to the size of the object.

In addition, the distance *d*, which is the distance between the two images measured relative to the system, must be determined. The focal length is then given by

$$f = -d/(M_2 - M_1)\,. \tag{6}$$

Procedure for the Laboratory

1) Long Focal Length Lens (telescope objective). Locate the focal and principal points, find the separation between nodal points, and determine the focal length.

2) Try to develop equations (2) through (6) from the basic definitions.

3) Short Focal Length Lens (eyepiece). Locate the focal and principal planes with respect to the body of the eyepiece.

Part II – Magnifying Power of Optical Systems

As previously pointed out, the cardinal points and focal length can be specified for any optical system no matter how complicated it is. Often, however, cardinal points for the whole system are inconvenient to use, as in the case of the telescope where the focal and principal points are both normally located large distances from the actual components. Also, it is often useful to have somewhat more detailed information about the system, particularly when the positions of the cardinal points can be changed by varying the spacing between the components.

Telescopes and microscopes are usually described, not by giving the positions of the cardinal points and focal length for the whole system, but by giving them for each of two groups of elements. (There may be additional components, such as an erecting lens). One group, or component, is called the objective and the other is called the eyepiece.

A usual arrangement of a long focal length objective and a short focal length eyepiece to provide a telescope for visual observation of distant objects is shown in Fig. 6.

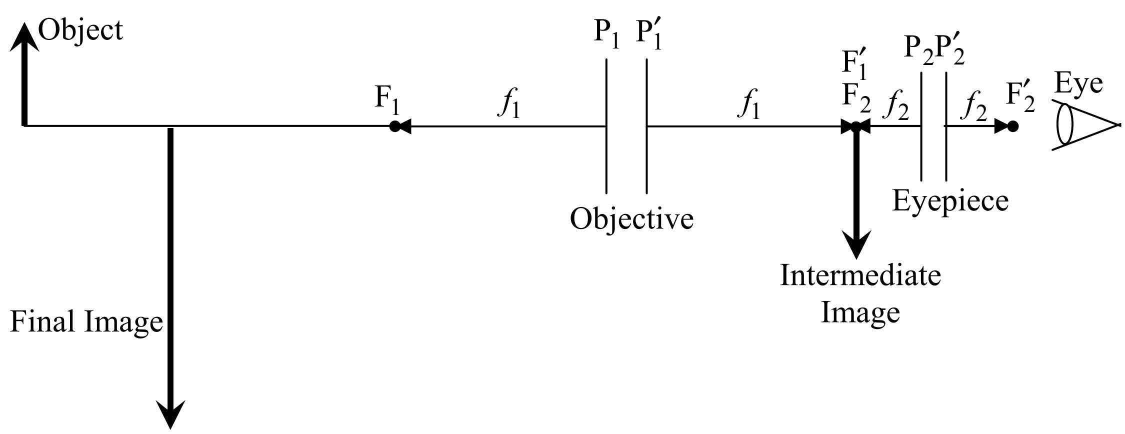


Figure 6: Telescope for Visual Observation

The object distance is out of scale since normally it will be many times the focal length of the objective. Here the final image is shown at an arbitrary position which, depending on the adjustment of the eyepiece, might be anywhere beyond the near point of clear vision for the eye to infinity. The eye becomes less fatigued during long periods of observation when the image is at infinity, but it might be placed at the usual reading distance which for the normal eye is about 25cm.

In a microscope, which is designed for viewing close objects, the objective normally has a shorter focal length than the eyepiece. A usual arrangement is shown in Fig. 7. As in the case for the telescope, the final image might be anywhere beyond the near point of clear vision for the eye to infinity.

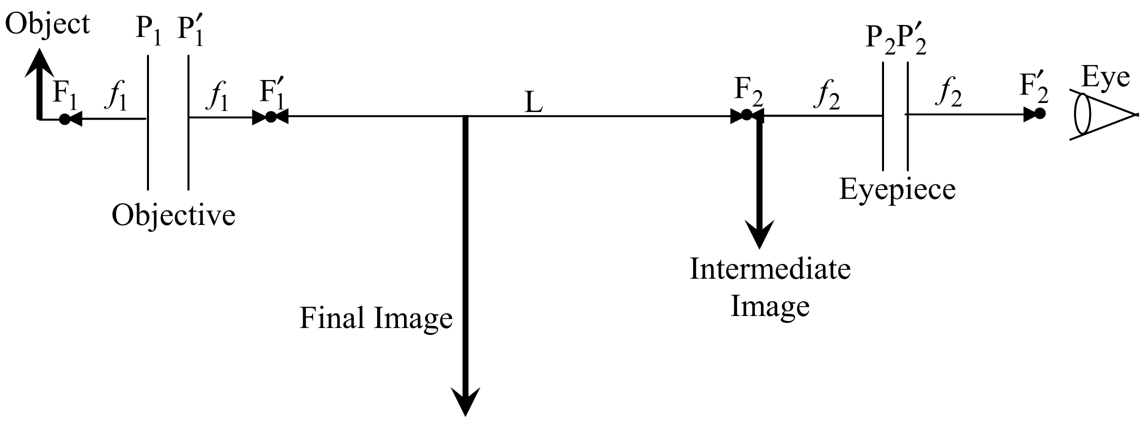


Figure 7: Microscope for Visual Observation

Magnifying Power

In visual observation the apparent size of an object is determined by the size of the image on the retina of the eye. This in turn is determined by the angle which is subtended at the eye by the object, or by the image formed by an optical instrument.

The **magnifying power** of an instrument is defined in terms of this angle. Explicitly, it is the angle subtended at the eye by the image formed with the instrument divided by the angle subtended by the object without the instrument.

$$\text{Magnifying Power} = \frac{\theta'}{\theta} = \frac{y'}{y}\frac{D}{D'} \tag{8}$$

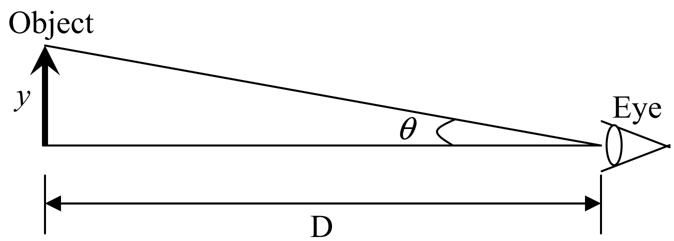
See Fig. 8 for the meaning of the symbols. It will be seen from Eq. (8) that the **magnification**, which is defined as the ratio of the size of the image to the size of the object and can be calculated from equation (4) or (5) of the introductory section on geometric optics, is not equal to the **magnifying power** unless the viewing distance is the same with as without the instrument.

Standard formulas are often used to specify the magnifying power of an instrument, and it is well to remember that these are developed for certain assumed conditions. The actual magnifying power is not necessarily the standard value in any particular situation.

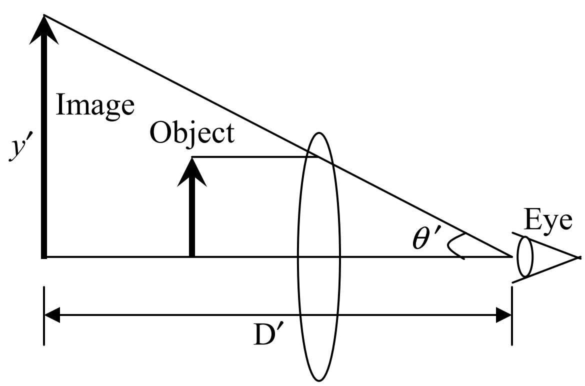
Simple Magnifier

$$\text{Magnifying Power} = (25\text{cm})/f \tag{9}$$

Magnifiers are usually designated as 10x, 5x, etc., which means that this is the ratio of 25cm, the standard viewing distance assumed for a normal eye, and the focal length of the lens used as a magnifier. It will indeed be the magnifying power if the eye is placed at the emergent focal point, and, without the magnifier, the object is viewed 25cm from the eye.



Viewing distance without instrument



Viewing distance with instrument

Figure 8: Apparent Size of Object

Telescope

$$\text{Magnifying Power} = f_1/f_2 \tag{10}$$

where f_1 and f_2 are the focal lengths of the objective and eyepiece, respectively. It is assumed that the object is a great distance from the telescope (actually, an infinite distance) and that the eye is at the emergent focal point of the eyepiece. Also

$$\text{Magnifying Power} = r/r' \tag{11}$$

where r is the radius of the objective lens and r' is the radius of the image of the objective lens formed by the eyepiece. (This is useful knowledge for determining the minimum apertures of telescope components without unnecessary loss of light).

Microscope

$$\text{Magnifying Power} = (18 \text{ cm}/f_1)(25 \text{ cm}/f_2) \qquad (12)$$

where f_1 and f_2 are the focal lengths of the objective and eyepiece, respectively. It is assumed that the focal points of the objective and eyepiece are spaced a standard distance of 18 cm (see L in Fig. 7), that the eye is placed at the emergent focal point of the eyepiece, and that without the microscope the object is viewed at a distance of 25 cm.

Measurement of Magnification

The lateral magnification of an image can be determined directly from the definition, y'/y, by measuring a distance, y', between two image points corresponding to object points separated by a known distance, y.

One way of measuring y' is to focus on the image with an auxiliary instrument, such as a traveling microscope or telescope, whose lateral displacement is read from a calibrated scale. As an auxiliary instrument of this type if moved, it measures the distance between image points which appear successively at the cross-hairs. Such an instrument can be focused on an image even when the image is virtual (i.e., the rays from an image point are diverging as they leave the last optical component, and the image can therefore not be formed on a screen.)

Another way of measuring y' is to focus the image on a scale with divisions of known size. In this method, the image and the scale can be viewed with the same eye (or simultaneously with both eyes) only if the image is real. If the image is virtual, as is the case with most instruments used for visual observation, one eye (looking through the instrument) must be used to view the image, while the other eye (looking around the instrument) views the measuring scale directly. To insure that the image is superimposed on the measuring scale, the position of the image (or the scale) must be adjusted until there is no parallax between the image and the scale (i.e., when the head is moved slightly, there should be no **relative** movement of the image and the scale).

Procedures for Studying Magnifying Power

The purpose of the studies proposed here is different from the purpose of most laboratory experiments in which one attempts either to measure precisely some objectively defined physical quantity or to disprove a theoretical prediction based on certain stated assumptions. Instead, the purpose if primarily to develop an understanding of the distinction between magnification and magnifying power. In considering this distinction, it may be helpful to remember that measurements are made only of magnification. Magnifying power is computed from the magnification and appropriate values of D and D′ using Eq. (8) or its equivalent.

**** Suggestions of what you might do are made in conjunction with questions related to typical viewing situations . . .

1) Use of a Magnifier to View a Nearby Object: In order to distinguish detail in a nearby object, the distance between the object and the eye is normally decreased until the eye is no longer able to form a distinct image on the retina. The distance between the eye and the object when the object is at the closest point to which the eye can focus is called the distance of most distinct vision, *d*. To observe greater detail a magnifying lens is used.

Eq. (9), When and Why?

The practice of specifying a simple lens system as 5x, 10x, etc., which is the magnifying power obtained from Eq. (9), provides no more information about the system than would be given by specifying its focal length. However, this standard value of magnifying power is a rough measure of the usefulness of the lens system as a magnifier. To see why this is so, use Eq. (5) to show that Eq. (8) reduces to Eq. (9) when a value of 25 cm (standard viewing distance) is chosen for D, and it is assumed that the eye is located at the focal point of the magnifier.

What is the standard value of magnifying power for the eyepiece you are using (Instructor will provide value or you can measure this)? Note that a short focal length is needed to provide a large magnifying power.

Does Eq. (9) provide the magnifying power of a lens in actual use?

In other words, how does the magnifying power depend on the distance of most distinct vision of the observer, *d*, which may differ from the standard value of 25 cm?

Would a given magnifying lens be of more benefit to a near-sighted or far-sighted person? Why does a near-sighted person take off his glasses to view a small object?

When using a magnifier, is there any advantage to positioning the image so that it is at the distance of most distinct vision?

By moving the magnifier relative to the object, the position of the image can be changed at will, and this is accomplished by a change in magnification. How does such a change affect the magnifying power?

Why do books often recommend that the eyepiece of telescopes and microscopes be focused so that the image is at infinity when long periods of observation are anticipated?

How does the magnifying power depend on the position of the eye relative to the magnifying lens?

This question can be answered with the use of Eq. (8) by noting that both D and the magnification, y'/y, do not depend on the position of the eye relative to the magnifier, which determines D':

$$\text{Magnifying Power} = \frac{y'}{y}\frac{D}{D'} \tag{8}$$

For a given position of the image, the magnifying power is greatest when D′ is smallest, or, in other words, when the eye is placed as close to the lens as possible. Despite this, it has previously been assumed that the eye will be held near the secondary focal point rather than at the surface of the lens. The reason for this is that usually (for example, when the magnifier is used as an eyepiece in a microscope or telescope) the image will appear brightest when the eye is near the focal point and very little increase in magnifying power results from moving the eye closer to the lens.

Show, by derivation, that:

$$MP(L) = MP(F)(1 + \frac{f}{D'}) \tag{13}$$

where MP(F) is the magnifying power when the eye is placed at the focal point and MP(L) is the magnifying power when the eye is placed at the lens. Note that the greatest magnifying power possible is obtained when the eye is at the lens and the image is at the position of most distinct vision ($D' = d$).

How much greater is MP(L) than MP(F) for the eyepiece you have been using?

2) Experimental Use of a Telescope to View Distant Objects: When the object is far away, D is necessarily much greater than the distance of most distinct vision. A long focal length lens is then needed to form a nearby real image, which can be viewed through a short focal length lens operating as a simple magnifier. As noted earlier, this is the usual arrangement for a visual telescope.

Arrange the long and short focal length lenses, whose focal lengths have been provided by the instructor, as a telescope and determine its magnifying power when used to view a distant object by direct observation. This you can do by looking through the telescope at a distant scale (something like a meter stick) and adjusting the focus so that the final image formed by the telescope is superimposed on the scale viewed directly. It is important that the object and image be focused by both eyes simultaneously, one eye looking at the object directly and the other eye at the image looking through the telescope. For this condition, D and D′ in Eq. (8) are equal so that the magnification is equal to the magnifying power. The magnification is found in the usual way by taking the ratio of the size of the image to the actual size of the scale.

Would the magnification of the final image be different if it were not located at the scale? Would the magnifying power of the telescope be different in this case?

Beginning with Eq. (8), derive Eq. (10). Compare the value predicted by Eq. (10) with your measured magnifying power.

3) Experimental Use of a Telescope with Intermediately Distant Objects: Eqs. (10) and (11) hold only if the distance of the object from the telescope is large compared to the focal length of the objective lens. Develop a general expression for the magnifying power of a telescope which is not based on the assumption that the object is far away. The expression should involve only x, the distance of the object from the primary focal point of the objective, f_1, the focal length of the objective, f_2, the focal length of the eyepiece, and D, the distance between the object and the eye. Make the usual assumption that the eye is at the secondary focal point of the eyepiece. It is interesting that the general expression does not contain the image distance, indicating that the magnifying power does not depend on the position of the final image.

Focus your telescope on an objective placed only three or four focal lengths in front of the objective, measure the magnification of the image and determine the magnifying power by substitution into Eq. (8). Compare the value obtained for MP with the value predicted by the general expression you have just developed. Also compare these values with the prediction of Eq. (10).