Should Corporate Pension Funds Invest in Risky Assets?

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Abstract

We address an ongoing debate on pension investment policy: should defined-benefit corporate pension plans invest aggressively in risky securities or completely de-risk their assets? In our model, firms maximize shareholder value subject to the participation constraint of employees, who are wealth-constrained and are partially exposed to pension investment risk via a corporate bankruptcy channel and a pension surplus sharing channel. For a reasonable set of parameter values, the model-suggested optimal pension allocation to risky assets exceeds 50%. The level of pension risk-taking predicted by the model, and its relation with a firm's bankruptcy probability and pension funding ratio, match with empirical observations. We show that due to limited sharing of the investment risk by employees, defined-benefit pensions may take on even more risk than what employees choose in the defined contribution plans. Further, firms may substantially reduce their overall pension funding costs under an alternative arrangement in which employees bear all the systematic pension investment risk. This is consistent with the secular trend of firms switching from defined benefit plans to defined contribution plans.

I. Introduction

Defined-benefit corporate pension plans (hereafter simply referred to as pensions or DB plans) are legal entities set up by companies to provide a stable stream of incomes to employees upon their retirement. Despite a long history of pension evolution, how pension plans should invest remains an unsettled issue. At one extreme, many corporate pensions consider themselves as patient, long-term investors, and are major holders of illiquid, risky assets such as real estate, hedge funds, and private equities. At the other extreme, some pensions plans have subscribed to a complete "de-risking" strategy, holding only safe fixed-income securities or annuities. Collectively, U.S. corporate pensions' allocation to stocks and alternative risky assets hovers above 50% in recent years. Aggressive pension risk-taking, coupled with prevalent pension underfunding, has often been the subject of concern in news media and by policymakers.

Researchers have identified at least two reasons that defined-benefit pension plans should avoid risky securities and invest safely. The first, as pointed out by Black (1980) and Tepper (1981), is a corporate tax advantage for pensions to invest in fixed income securities. The second, and an even stronger reason, can be understood in the context of the corporate risk management model of Froot and Stein (1998). Any risky corporate investment without a positive alpha – including pension investment – does not create value for shareholders; meanwhile, risky investment may reduce firm value by forcing firms to raise costly financing. Therefore, to maximize shareholder value, pensions should only invest in riskfree assets that match the horizon of pension obligations.[1] From this perspective, the risky asset allocation by corporate pensions at the aggregate level poses a puzzle.[2]

[1]The intuitive version of this argument is suggested in the early pension literature; e.g., Bodie (1988). The risk-management based argument serves as a rationale for the pension de-risking strategy proposed by practitioners (e.g., Cooper and Bianco, 2003).

[2]Possibly, some firms invest pension money in risky assets with a hope to generate positive alphas. But evidence provided by a long stream of academic studies, from Lakonishok, Shleifer, and Vishny (1992) to Busse, Goyal, and Wahal (2010), suggests that actively equity portfolios managed on behalf of pension funds on average fail to generate positive alphas. Further, the alpha-seeking motivation does not explain why pensions invest substantially in index portfolios. According to French (2008), during the period of 2000-2006, about 30% of DB plans' equity investments are passively managed.

Several studies (e.g., Sharpe 1976; Treynor 1977) link the risky investments by corporate pensions to a moral hazard problem, in a way similar to the risk-shifting problem for firms with high financial leverage. But moral hazard is unlikely the sole driver of corporate pensions' risky asset allocation decisions. Studies such as Bodie, Light, Morck, Taggart (1985), Ruah (2009), and An, Huang, and Zhang (2013) find that pensions sponsored by firms with higher bankruptcy risk – thus stronger risk-shifting incentives – take on lower investment risk. In addition, Lucas and Zeldes (2006) and Sundaresan and Zapatero (1997) point out that pensions may invest in stocks to hedge against the future growth of pension obligations. However, Lucas and Zeldes (2006) note that equity allocation by pension plans with low future wage growth remains quite high and cannot be explained by the hedging demand. It suffices to say that the search is still on for a better understanding of the risky investment policies adopted by most corporate pensions.

In this study, we take a stakeholder approach to model key pension decisions by firms, including the investment decision as well as the choices of pension benefit level and pension funding. In our model, firms maximize shareholder value, but are additionally subject to the participation of wealth-constrained employees. As a consequence, firms must balance the risk management concern of shareholders with employees' preference for systematic risk exposure. The model predicts substantial risky assets in optimal pension portfolios, and thus offers a perspective for understanding the observed pension risk-taking behavior. For a reasonable set of parameters, the model-suggested optimal level of pension investment risk and its relation with the bankruptcy risk and pension funding ratio are consistent with those reported by empirical studies. We also show that the typical defined benefit pensions offer inefficient risk sharing between shareholders and employees, which exacerbates pension risk taking.

A key element of our model is the difference between a firm's shareholders and its employees in their capacity for bearing pension investment risk. Notably, the investment risk ensuing from a diversified pension portfolio is systematic. And as it turns out, employees may have a stronger capacity or appetite than shareholders for such systematic risk. To understand this, think about a standard assumption in the existing literature (e.g., Froot and Stein 1998) that shareholders hold an optimally diversified portfolio to maximize their utility. By already holding an optimal portfolio, shareholders are indifferent to a small change in the level of systematic risk brought about by an incremental investment, as long as the investment is fairly valued (i.e., having a zero alpha). Employees, on the other hand, have a substantial part of their wealth tied up in safe wages. Because of this wealth constraint, employees may be under-exposed to systematic risk. As a consequence, they may desire systematic risk exposure in their pension payoffs, even when such exposure does not deliver a positive alpha.[3] In sum, the different capacities for systematic risk are not because shareholders and employees have different utility functions, but rather because of the different levels of systematic risk they are already exposed to.

Another important element of our model is that despite the fixed pension benefits promised by firms, employees are at least partially exposed to pension investment risk. Employees' share of pension investment risk intuitively comes from two channels. The first is the bankruptcy channel – when a firm goes bankrupt, employees' pension payoff depends on pension assets value relative to pension liabilities, and the payoff becomes risky when the pension is underfunded. The second is a pension surplus sharing channel – when the value of pension assets exceeds the value of promised benefits, employees are entitled to at least a fraction of the excess. This pension surplus sharing mechanism, in short, can be understood as a direct outcome of the current tax code. Pension assets are held in a separate legal entity from the sponsoring firm, and there is a 50% punitive exercise tax when a firm converts pension surplus back into firm assets. However, the tax rate is reduced to 20% if employees receive at least 20% of the surplus, e.g., in the form of increased pension benefits.

[3]This is certainly different from the case of bearing firm-specific risk – unconstrained shareholders can more effectively diversify away firm-specific risk than wealth constrained employees do.

This creates an incentive for firms to share pension surplus with employees.[4] Note that the bankruptcy channel mainly exposes employees to the downside of pension investment risk, while the surplus sharing channel mainly exposes employees to the upside of pension investment risk.

The difference in their risk-bearing capacities and the sharing of pension investment risk between shareholders and employees jointly drive pension investment decisions. Suppose a pension chooses between a fairly-valued stock index fund and a riskfree asset. Although shareholders are indifferent to a small systematic risk increase in terms of their utility, an increase in risk reduces their value by increasing the firm's expected financing cost. Therefore, if shareholders are to make pension decisions solely on their own, as predicted in the standard corporate risk management setting, they would strictly prefer the riskfree investment. However, pension investment risk is attractive to employees. Under reasonable assumptions for the financing cost function and employees' utility function, there is an optimal level of pension investment risk, born by both shareholders and employees. Further, we show that the decisions on the level of pension benefits and on pension contribution are also affected by pension risk sharing. For example, for firms with low probabilities of bankruptcy, employee's exposure to pension investment risk is mainly through the pension surplus sharing channel. For such firms, it is optimal to maintain a high funding ratio (i.e., high pension contribution relative to promised pension benefits) and at the same time invest aggressively, to increase the chance of pension surplus. By contrast, for firms with high bankruptcy probabilities, the model predicts a low funding ratio.

We show that with reasonable parameter values, the model outcome matches several

[4] The Employee Retirement Income Security Act (ERISA) of 1974 requires that pensions assets are managed to the exclusive benefit of beneficiaries, which can be construed as that beneficiaries have claims on pension surplus. However, the laws in this aspect are incomplete, and firms' effective control rights on pensions matter. Firms have various means to reclaim part of pension surplus, from reducing pension contributions, terminating overfunded pension plans, merging overfunded pensions with underfunded ones, to diverting pension assets to cover operating costs and restructuring costs. Meanwhile, various restrictions are in place to prevent firms from recapturing all the pension surplus and encourage firms to share pension surplus with employees – for example, the punitive exercise tax for pension surplus reversion. Section II provides further details on pension surplus sharing. The fact that employees enjoy a fraction of pension surplus has been noted in existing literature; see, e.g., Miller and Scholes (1981) and Bulow and Scholes (1983).

observed patterns on pension investments. For example, in a baseline calibration analysis, we consider a 30-year retirement horizon and an annual bankruptcy probability of 0.5% for the firm (typical for BBB-rated firms), with employees' share of pension surplus at a modest 20% and with other parameters such as market risk premium, market volatility, and financing costs either calibrated to historical data or taken from estimates of existing studies. The optimal weight on the risky assets in this case is around 55%, matching well with the observed level of risky allocation by corporate pensions. By varying model parameters, we also find that the optimal portfolio weight on risky assets tends to decrease with the bankruptcy probability, and increase with the pension funding ratio. These two patterns are consistent with empirical findings reported in existing studies (e.g., Bodie, Light, Morck, Taggart 1985; Ruah 2009; An, Huang, and Zhang 2013).

Across various parameter choices, the relation between pension investment risk and employees' share of pension risk – through either the bankruptcy channel or the surplus sharing channel – tends to be substitutive in our model. That is, when employees' share of pension risk is lower, the pension invests more aggressively.[5] This relation, although surprising at first, can be intuitively understood as an effect to maintain a desirable level of systematic risk exposure in employees' pension payoffs. To further understand this relation, we explore a hypothetical pension arrangement where employees bear all the pension investment risk, which is similar to a defined-contribution plan (referred to as a "variable benefit" plan). We find that the optimal level of investment risk is lower under the "variable benefit" plan than under the typical defined-benefit arrangement. That is, investment decisions by defined benefit pension plans are more aggressive than what employees would choose for themselves in defined contribution plans. This gives rise to an interesting policy implication – if pensions' risk-taking behavior is a cause for concern, then designing a better pension risk-sharing mechanism for employees could be part of the redress.

Ultimately, our analysis highlights that risk sharing between shareholders and employees in typical defined benefit plans is inefficient. Given employees' preference for systematic

[5] An exception is that when employees' share of pension surplus is very low (close to zero), a small increase in the surplus sharing leads to an increase in pension investment risk.

risk and shareholders' concern for risk management, letting employees bear all the pension investment risk could be a better arrangement. To showcase the potential welfare improvement, we compare firms' total pension funding cost under the typical defined benefit plan with that under the "variable benefit" plan. For most parameter choices, firms' pension funding cost is substantially lower under the latter arrangement. It is worthwhile noting that in the recent decades, driven by the desire to reduce pension funding cost, many firms have switched away from defined benefit plans to defined contribution plans. DB plans' inefficiency in investment risk sharing might be one of the reasons for this secular trend.

Overall, our paper provides a new perspective to understand the risky asset allocation policies pursued by corporate pensions. The insight of our model is that pension risk taking could be driven by employees' preference for systematic risk exposure. In this regard, our paper is related to Love, Smith, and Wilcox (2011), yet with different objectives and implications. The main purpose of their paper is to study the effect of PBGC insurance on pension risk taking. In their model, employees do not face wealth constraints and they price pension investment risk the same way as shareholders. As a result, their model has a strong prediction that pensions should invest riskfree as long as PBGC insurance is fairly priced.

The rest of the paper is organized as follows. Section II provides a discussion on the evolution of defined benefit pension plans and pension investments, and several legal aspects relevant for understanding pension investment risk sharing. In Section III, we introduce a one-period model of pension investments and provide analytical results under certain model specifications. Section IV reports the numeric solutions of the one-period model under more general model specifications, and examine various extensions of the model. Section V examines a dynamic model of pension investments and pension funding. Finally, Section VI provides concluding remarks.

II. Background on Corporate Pensions and Pension Investments

II.1 Evolution of Corporate Pensions and Pension Investments

The origin of U.S. corporate-sponsored pensions could be traced to the 1880s, when companies in the booming railroad industry used pension benefits to recruit workers. In 1875, American Express – a railway company at the time – established the first pension plan with defined benefit features. By 1929, there were about four hundred corporate pension plans in operation, sponsored by many large corporations of the time (Munnell, 1982). Corporate pensions took a hit during the Great Depression, but recovered afterwards and grew rapidly after World War II, covering 25%, 41%, 45%, 46%, and 43% of all private-sector workers in 1950, 1960, 1970, 1980, and 1990, respectively (McDonnell, 1998). In the recent decades, however, with the rise of defined contribution plans and individual retirement accounts (since late 1970s and 1980s), the importance of defined benefit pensions in overall retirement savings has declined. According to recent statistics, in 2014, DB plan assets stand at $3.96 trillion, compared with $5.32 trillion for defined contribution plans and $6.23 trillion for individual retirement accounts (IRAs).[6] The waning popularity of corporate DB plans relative to DC plans and IRAs has been well noted and discussed in existing studies; see, e.g., Munell and Soto (2008), and Rauh and Stefanescu (2009).

Prior to the stock market boom of the 1950s, corporate pensions invested only in safe assets such as bank deposits, government bonds and corporate bonds. In 1950, a DB plan of General Motors became the first to invest in the stock market (McDonnell, 1998). Over time, pensions have shifted toward substantial risky investments. By mid-1960s, the aggregate corporate pension allocation to stocks exceeded allocation to fixed income assets.[7] According

[6] Statistics for DB and DC plans are from Private Pension Plan Bulletin, Abstract of 2014 Form 5500 Annual Reports, by Employee Benefits Security Administration, Depart of Labor. Available at: https://www.dol.gov/agencies/ebsa/researchers/statistics/retirement-bulletins/private-pension-plan. Statistics for IRAs are from The IRA Investor Profile: Traditional IRA Investor's Activity 2007-2015. Available at: https://www.ici.org/pdf/rpt_17_ira_traditional.pdf.

[7] Based on statistics provided by Federal Reserve Board Flow of Funds data (Z.1 Statistical Release).

to various recent statistics and surveys, corporate pensions allocate between 50% to 60% of the investments to risky assets such as stocks, hedge funds, private equities, and real estates (e.g., Stockton, 2012; Andonov, Bauer, and Cremers 2012; Panis and Brien 2015; and Willis Towers Watson 2016). For example, based on corporate filings (Department of Labor Form 5500) in 2013, Panis and Brien (2015) find that on average 50.6% of pension assets are invested in stocks, with another 12% in real estate and alternative assets. They also note that pension allocation to risky assets is trending down in recent years.

Corporate pensions' wage into the stock market in the 1950s was motivated by the market boom during that period. In the 1980s, corporate pensions were among the early investors (together with public pensions and insurers) in junk bonds. Due to their long-term nature of pension obligations, DB plans are considered long-term investors who can afford to invest in illiquid and risky assets (e.g., Campbell and Viceira 2005). Starting from the 1990s, corporate pensions also become pioneering investors, and remain a major force today, in alternative assets such as hedge funds and private equities. Meanwhile, their relative importance in the public equity market sees a peak in the mid 1990s and has since been on a decline (French 2008).

The notion of liability-driven investing (LDI) by pensions was developed in the 1980s (Liebowitz, 1986; Ang 2014). LDI takes into account pension liabilities when making investment decisions. An extreme version of this approach is to completely de-risk pension assets, i.e., investing in annuities and avoiding any form of market risk.[8] A more flexible version of LDI is to set the pension investment objective to be a concave function of pension surplus ratio (i.e., the ratio of pension assets to pension liabilities), thus introducing a hedging component in the optimal portfolio against the interest rate risk of pension liabilities. LDI strategies have gained traction among pension managers in recent years (e.g., Cooper and Bianco, 2003; Leibowitz and Ilmanen, 2016).

[8]Pension de-risking may also involve removal of uncertainty in future pension obligations.

II.2 Ownership and Control of Pension Plans: Legal Aspects

Corporate DB pension plans are typically set up as trusts, with trust beneficiaries being qualified (current and retired) employees. The trustees are in charge of pension administration, supposedly in the interest of the beneficiaries. In practice, trustees are appointed by firms sponsoring the pension plans. This means that plan sponsors have effective control of pension decisions.

Prior to the Employee Retirement Income Security Act (ERISA) of 1974, corporate pension plans are governed by the general trust laws. ERISA sets a comprehensive list of standards for pension vesting, funding, termination, and disclosure. It subjects pension trustees and pension asset managers to an explicit set of fiduciary duties, and establishes the Pension Benefit Guaranty Corporation (PBGC) to provide insurance to beneficiaries against pension failures. ERISA is amended by subsequent legislations, such as the Pension Protection Act (PPA) of 2006 (which tightens pension funding and reporting requirements); but its major framework remains intact.

Several aspects of pension laws are of particular relevance to this study. First, Section 403(a) of ERISA explicitly requires pension assets to be held in a trust. This separates pension assets from the rest of a plan sponsor's assets. The same section further sets forth that "a fiduciary shall discharge his duties with respect to a plan solely in the interest of the participants and beneficiaries...". And Section 403(c) of ERISA requires that "the assets of a plan shall never inure to the benefit of any employer and shall be held for the exclusive purposes of providing benefits to participants in the plan ...". Combined, they suggest that pension decisions, including investment decisions, should be made in the interest of plan beneficiaries instead of plan sponsors. However, ERISA falls short of making specific requirements on how plan trustees should be appointed or how pension assets should be invested. With the exception of multi-employer plans (where labor unions have some controls over pension plans), corporate plan sponsors have effective control rights on pensions via the ability to appoint trustees.

Second, when a plan sponsor is in bankruptcy (either Chapter 7 or Chapter 11), because pension assets are held in a separate trust, they are protected from the claims of the plan sponsor's creditors. However, the plan sponsor in bankruptcy may choose to terminate an underfunded pension plan, leaving insufficient pension assets to cover pension liabilities. This distress termination (Section 4041(c) of ERISA) triggers PBGC to take over the pension plan and provide insurance to beneficiaries. Section 4062 of ERISA sets forth that in distress termination, PBGC holds a claim against a plan sponsor for unfunded pension liabilities, which is treated as general unsecured debt in bankruptcy proceedings. Also note that PBGC's pension benefit coverage has limits. In 2017, the maximum insured benefit for a 65-year old retiree in a single-employer plan is $64,000 annually. Further, certain types of vested benefits are not covered by PBGC.[9]

Third, the most uncertain legal aspect about pension plans is perhaps who have ownership of pension surplus, i.e., the part of pension assets in excess of pension liabilities. Although ERISA requires pension assets to be managed in the exclusive benefit of plan participants and beneficiaries, firms have several ways to claw back pension surplus into corporate assets, which is known as "surplus reversion". Section 4041(b) of ERISA provides a standard termination procedure, allowing a plan sponsor to terminate a pension plan and claim pension assets under the condition that the sponsor makes alternative arrangements to meet all plan liabilities (e.g., paying a lump sum of cash to beneficiaries or replacing pensions with annuities). Outright terminations of overfunded pensions for surplus reversion were quite popular in the 1980s (e.g., VanDerhei 1987; Cather, Cooperman, and Wolfe 1991), and generally survived court challenges. But in the Omnibus Budget Reconciliation Act of 1990, the congress imposed a 50% exercise tax on surplus reversion. This effectively stopped the practice of surplus reversion through standard pension termination.[10] The exercise tax rate is reduced to 20% if at least 25% of the surplus reversion is transferred into a qualified replacement plan (e.g., a defined contribution plan) or at least 20% of the surplus is used to

[9] According to a PBGC (2008) analysis of 125 (still healthy) pension plans with 525,000 participants, 16% of the participants would see their benefits reduced if PBGC takes over, and their benefits on average would be reduced by 28%.

[10] The Tax Reform Act of 1986 initially introduced an exercise tax rate of 10% on pension surplus reversion, which was increased to 15% by the Technical and Miscellaneous Revenue Act of 1988, before the 50% rate imposed in 1990.

increase the pension benefits of qualified participants. This provides an incentive for pension surplus sharing between plan sponsors and plan beneficiaries.

Finally, because plan sponsors have effective control over pension decisions, there are alternative ways for firms to recapture pension surplus. For example, a firm experiencing growth in pension liabilities can reduce surplus by simply reducing ongoing pension contributions. The exercise tax can be avoided by merging an over-funded plan with an under-funded plan, either within a firm or through a merger of two firms. Finally, pension regulations have some vague parts in treating health benefits and severance benefits, allowing firms to dip into pension surplus to pay for what are otherwise considered normal operating costs or restructuring costs.

III. One-period Model

III.1 Model Setup

Our baseline one-period model has two dates: time 0 and time T. At time 0, a firm hires a representative employee, contributes to a pension plan, and makes an investment decision for the pension. At time T, the employee retires, the pension investment return is realized, and the employee receives a one-time pension payment.[11]

At time 0, the firm has a certain amount of discretionary cash, denoted by H0, to be contributed to the pension. The firm can make incremental (non-negative) contributions to the pension fund at time 0 and T, denoted by h_0 and h_T respectively. We assume that the firm additionally incurs a financing cost (or opportunity cost) for pension contribution, denoted as $C_0(h_0)$ and $C_T(h_T)$, respectively. The cost functions $C_0(.)$ and $C_T(.)$ are increasing and convex.

At time 0, the pension can invest in a risk free asset and a risky asset (e.g., a stock). The gross riskfree rate for the period is R_f, and the gross return on the risky asset for the

[11]DB plans cover the longevity risk of retired employees, in that retirees receive annuities as pension payment until death. For simplicity we abstract away from the longevity risk coverage and assume a one-time pension payment.

period is R_m. The pension asset at time T (before time-T pension contribution) is then

$$W_T = W_0(wR_m + (1-w)R_f) \tag{1}$$

where w is the portfolio weight of the pension fund on the risky asset, and W_0 and W_T are the values of pension assets at time 0 and T respectively.

The firm's total pension related cash flow at time 0 is the contribution plus the financing cost, i.e., $CF_0 = -W_0 - C_0(h_0)$, where $h_0 = Max(W_0 - H_0, 0)$. Let S denote the pension payment to the employee at time T. Then the firm's time-T total cash flow is $CF_T = W_T - S - C_T(h_T)$ where $h_T = \max(S - W_T, 0)$. S depends on the pension assets W_T and other conditions, e.g., whether the firm is in bankruptcy and how pension surplus is shared. We will provide further details below.

The firm's objective is to maximize shareholder value subject to the employee's participation constraint. We assume that shareholders do not have wealth constraints or portfolio constraints. Thus they value the firm's pension-related cash flows CF_0 and CF_T via the risk-neutral probability approach. Further, let D be the random variable indicating firm bankruptcy. We assume that the firm's bankruptcy event is idiosyncratic, and thus shareholders do not demand a risk premium for the bankruptcy risk. That is, $E(D) = E^Q(D) = p$, where Q denote the risk neutral probability, and the objective probability of bankruptcy is the same as the risk neutral probability of bankruptcy, p.

The employee is wealth constrained. For simplicity, we assume that at time T, the employee does not have any wealth other than the pension payment S she receives. They value the pension payment using an increasing and concave utility function $U(S)$.

The firm's objective is to maximize shareholder value subject to the employee's participation constraint:

$$\max_{w,W_0,S} CF_0 + \frac{1}{R_f} E^Q(CF_T) \tag{2}$$

$$\text{subject to:} \quad E(U(S)) = \mathbb{U} \tag{3}$$

where Q denotes the risk neutral probability. $\mathbb{U}$ is the employee's reservation utility. We omit from the above specification the pension asset process (Equation (1)) and several additional constraints imposed by the pension contract, which will become explicit once we specify the pension payment schedule.

III.2 Analytical Results for Some Payment Specifications

In the basic model setup above we leave out the details for the pension payment S, which depends on the contractual benefit of the pension plan, pension funding status, firm bankruptcy, and pension surplus sharing arrangements. Here we consider a few simple specifications of S that are susceptible to analytical examination. They serve as illustrations of the basic intuition and highlight some common underlying themes. We later examine more general specifications.

First, consider the following case. Let F denote the promised pension payment at time T. When the firm is not in bankruptcy, we assume that the employee receives the promised benefit F regardless of the pension funding status, while the firm gets the pension surplus $W_T - F$ and bears the pension deficit. In the case of bankruptcy, since the pension assets serve as collateral for pension obligations, we assume that the employee receives the entire pension assets. That is, the pension payment S can be specified as:

$$S_B = F(1 - D) + W_T D. \tag{4}$$

We use the subscript B to indicate this base case. As introduced earlier, D is a random variable indicating the firm's bankruptcy event, with $E(D) = E^Q(D) = p$.

We have the following results:

Proposition 1. *Consider the case of the pension payment specified in Equation (4)*

(1) In the case that the firm's default probability is zero (i.e., $p = 0$), the pension fund will invest only in risk free asset (i.e., $w = 0$)

(2) In the case of a positive default probability (i.e., $p > 0$), the pension fund's risky asset allocation is strictly positive (i.e., $w > 0$), and is lower than the optimal allocation of

a stand-alone investor who has the same utility as the employee, with the sole exception that the pension is fully funded and the cost function's slope at 0 is higher than some threshold (i.e., $C_T'(0) > C$, where C is specified in the appendix), in which case, the pension fund's stock allocation may be zero.[12]

Proof. See Appendix.

From the firm's stand point, investing in the risky asset generates zero net present value to begin with, and further, the convexity in the financing cost function makes the firm averse to the investment risk. When there is zero probability of bankruptcy, the pension payment is fixed at F with no uncertainty. Therefore, the employee's utility is not affected by the pension's investment decision. The firm's aversion to risk drives to the pension to invest completely in the risk free asset.

When a firm defaults, the employee's payoff is the value of pension assets. From the employee's perspective, her utility, conditional on the event of the firm's default, will be maximized if the pension asset is allocated between the risk free asset and the risky asset in line with her risk aversion to achieve a desirable risk and return trade-off. Thus, with a positive probability of default, the employee's preference for the risk-return trade-off is balanced with the firm's aversion to risk, leading to a compromised solution: the allocation on the risky asset will be strictly positive but lower than what is optimal for a stand-alone investor.[13]

Proposition 1 can be extended to cover additional cases. We report results regarding two cases in the following corollary.

[12]We refer to the pension fund as over, under, or fully funded at time zero by comparing the fund asset W_0 with the PBO (i.e., the present value of promised payment F discounted at the risk free rate).

[13]The sole exception in the proposition is when the pension fund is fully funded and the financing cost function has a high slope at zero. In this case, investing in the risk free will result in the time T fund value from the investment being equal to the committed payment. Consider increasing the risky asset allocation by an infinitesimal amount from 0. On the positive side, the expected return for the pension fund investment portfolio will increase proportional to w. On the negative side, the firm faces the chance of fund deficit, which incurs financing cost. Whether it is desirable to move away from the pure risk free investment depends on which of the above two effects dominates, the precise condition of which is given in the proposition.

Corollary 1. *(1) In the case the payment is specified as*

$$S = F(1 - D) + \min(W_T, F)D. \tag{5}$$

and the default probability is positive, if the pension is underfunded at time 0, the optimal stock market allocation is strictly positive (i.e. $w > 0$).

(2) In the case that there is zero probability of default and the payment is specified as

$$S = F + (1 - \alpha)\max(W_T - F, 0), \tag{6}$$

where $0 \leq \alpha < 1$, if the pension is over-funded at time 0, the optimal stock market allocation is strictly positive.

Proof. See Appendix.

The corollary examines very different cases. It highlight the fact that our basic intuition can apply in a straightforward way to very different scenarios. In (1) of the corollary, the employee receives F when the firm does not default, the same as our baseline case. Different from the baseline case, when the firm defaults, the employee gets paid by the pension asset up to the level of the promised payment. In (2) of the corollary, we take out the consideration of firm default, and assume that, in the case of pension surplus, the firm keeps a fraction α of the surplus, and the employee gets the remaining fraction $1 - \alpha$. It is worth noting in the first case of the corollary, the employee never receives a payment from the pension higher than the promised payment F. Yet, when the pension investment return is low, she receives less payment in the case of firm default. In effect, she bears the downside risk of the pension investment in the case of firm default. In the second case, there is no default possibility. The pension payment is never below the promised payment F. Yet, the employee shares the upside of pension investment outcome. Despite their apparent differences, in both cases, the optimal investment strategy involves positive allocation to the risky asset. So, as long as the employee is exposed to the investment risk, being it through upside surplus sharing or downside risk bearing, it can be optimal for the pension fund to invest some portion of its asset in the stock market.

IV. Numerical Analysis

In this section, we numerically calibrate the one-period model. We consider a variety of specifications of the pension payment schedule. We focus on the optimal risk allocation for the pension asset, and examine how different parameters affect this solution.

IV.1 Calibration

We calibrate the model parameters in the following way. First, we set the risk premium on the risky asset, the riskfree rate, and the volatility of the risky asset to the historical averages of market risk premium, riskfree rate, and market return volatility from the Fama-French data on the market portfolio from 1926 to 2016. Specifically, the annual risk premium for the risky asset is 6.1%, the annual risk free rate is 3.3%, and the annual volatility of the risky asset return is 18.5%.

We assume that the employee retires in 30 years (i.e., T=30). We assume that the firm has a BBB credit rating, with an annual bankruptcy probability of 0.5% (from Berk and DeMarzo, 2016). This means that over a 30-year horizon, the probability of firm bankruptcy is 13.93%. We assume a CRRA utility function for the employee with the relative risk averse coefficient $\gamma = 6$, based on Constantinides (1990).

To calibrate the employee's reservation utility, we take the following approach. We consider a hypothetical pension commitment F and use the risk free rate to discount F to estimate the Projected Benefit Obligation (PBO), i.e., the present value of promised pension payment. Note that all the variables calibrated so far are scale invariant. So, it is without lose of generality to unify this hypothetical PBO to 1. Then we assume that the investor is handed this amount (PBO) to invest for herself for the 30-year period. We use the resulting utility at year 30 as the reservation utility. According to Chen et al (2014), the average pension PBO is $1,016.25 million. Therefore, 1 unit of PBO in our model represents approximately \$1 billion.

Following Hennessy and Whited (2007), we assume a quadratic form for the financing cost function. Specifically, $C_t(h) = c_a + c_b h + c_c h^2$, where $t = 0, T$. We take their estimated values with the following adjustment. In Hennessy and Whited (2007), the financing amount h is in millions. Given that our unit is in billions, we make the scale adjustment accordingly. We set the cost function parameters as $c_a = 5.98 \times 10^{-5}$, $c_b = 0.091$, $c_c = 0.4$. We assume that the firm's initial discretionary cash H_0 is zero.

We consider a more general form of the pension payment schedule S:

$$S = F(1 - D) + \min(W_T, F)D + (1 - \alpha) \times \max(W_T - F, 0). \tag{7}$$

In this specification, the employee gets the promised payment F when the firm does not default, represented by the term $F(1 - D)$. In the case of a default and if the pension asset falls below the promised payment, the employee only gets the pension asset, as represented by the term $\min(W_T, F)D$. Finally, if there is a surplus, the firm keeps a fraction α of the surplus and the employee gets the remainder fraction $(1 - \alpha)$, as represented by the term $(1 - \alpha) \times \max(W_T - F, 0)$. For the baseline setup, we assume that the firm keeps 80% of the surplus while the employee keeps the remaining 20%. That is, $\alpha = 0.8$. The baseline parameter choices are summarized in the following table.

Table 1: Benchmark Parameters

μ	r_f	σ	c_a	c_b	c_c	p	γ	H_0	α
0.094	0.033	0.185	5.98×10^{-5}	0.091	0.4	0.005	6	0	0.8

IV.2 Comparative Results

Figure 1 plots the portfolio's risk allocation (w), the promised pension payment (F) and the initial contribution (W_0), when we vary the default probability while keeping other parameters constant. Panel (1) of the figure shows that for the baseline firm with a credit rating of BBB, the portfolio weight on the risky asset is about 55%, similar to the observed level of allocation to risky assets by corporate pensions as a whole. Further, in Panel (1), we see that the portfolio's risk allocation decreases as the default probability increases. This is consistent with the empirical findings in existing studies; e.g., Bodie, Light, Morck, Taggart (1985), Ruah (2009), and An, Huang, and Zhang (2013).

To see the intuition, it is helpful to pay attention to the sensitivity of the employee's pension payment to the pension asset value. We refer to this sensitivity as the delta of S to W_T, or briefly, delta. Formally,

$$\delta = (\partial S)/(\partial W_T) = (W_T < F)D + (1-\alpha)(W_T > F).^{14} \tag{8}$$

Consider the case of default probability being 0. The pension payment takes an option-like form: $S = F + (1-\alpha) \times \max(W_T - F, 0)$. That is, the employee gets the fixed payment F in the case that the pension's investment asset falls below F. In the case the investment asset is higher, the employee gets $1-\alpha$ share of the surplus. Thus, $\delta = 0$ in the region $W_T < F$ and $\delta = 1-\alpha$ in the region $W_T > F$. The average delta is thus somewhere in between 0 and $1-\alpha$. With a relative low delta, it takes a high allocation of the pension fund to the risky asset for the employee to achieve desirable risk exposure from the pension payment. The firm invests thus aggressively in order to get closer to the employee's desired level. Furthermore, as shown in Panel (2) of the figure, the firm chooses a relatively low promised payment, which helps enlarge the relatively high delta region (i.e. the region of $\delta = 1-\alpha$ or equivalently $W_T > F$) and reduce the 0 delta region (i.e, the region of $W_T < F$), with the effect of a higher average delta. The higher average delta is desirable because it improves the effectiveness of passing the risk exposure of the pension fund investment to the pension payment that the employee gets. Furthermore, as part of the optimal strategy that involves aggressive risk allocation, the firm chooses to fund aggressively at time 0 which helps reduce the financing concern at time 1 and thus reduce the potentially high impact of the convex financing cost from the aggressively risky investment.

[14] We adopt de Finetti's notation convention and do not distinguish a random event with its indicate variable. Further, the partial derivative, conditional on D, is to be understood in the general sense, with a few indifferentiable points being ignored.

On the other extreme, consider the case that the firm defaults for sure. The pension payment is $S = \min(W_T, F) + (1-\alpha) \times \max(W_T - F, 0)$. The pension payment's delta to the pension investment asset is 1 in the region of $W_T < F$, and the delta is $1-\alpha$ in the region of $W_T > F$. The average delta is thus somewhere in between $1-\alpha$ and 1. The overall delta in the case is in a relative high comparing with the first case discussed above. The riskiness of the pension asset investment can be effectively transferred to the employee through the highly variable pension payment. Therefore, the firm needs only to allocate a modest amount of the pension asset in the risky asset to achieve optimal. The firm would choose a high promised payment F in order to enlarge the average delta by enlarging the region of delta being 1 (i.e., the region of $W_T < F$). There will be severe underfunding at time zero, in order to balance the effect of the high level of the promised payment F, so as to keep the employee's utility at the reservation level. Furthermore, doing so is optimal because, the future financing cost is a not a concern as it incurs only in the nondefault case. Once we understand the two extreme cases of the default probability, the firm's decisions for intermediary cases fall naturally somewhere in between the two extremes.

In Figure 2, we examine the effects of various factors on the optimal asset allocation and the initial funding. In Panel A, we look at the effect of the initial free cash flow. The intuition for this analysis is straightforward. With high free cash flow at time 0, naturally the firm contributes more at time 0, as shown on the right side chart of Panel A. Such contribution reduces the concern the potential deficit at time T. As a consequence, as shown on the left chart of the panel, the firm makes more aggressive risk allocation for the pension fund to be in line with the employee's preference on the risk-return trade-off. Consequently, the firm can promise a lower payment F while still keeping the employee at the reservation utility, as shown on the right chart of the panel.

In Panel B of Figure 2, we examine the effect of varying the ratio of surplus that the firm keeps. As shown in the left chart of Panel B, as the share of surplus the firm keeps (i.e., α)increases, the risk allocation initially increases and then decreases. As the firm keeps some surplus, the pension payment's delta in the region of $W_T > F$ decreases. Consequently, the overall average delta decreases as well. Naturally, it would take a more aggressive risk allocation in the pension fund to offset the decrease in delta, in order to keep the riskiness of the pension payment leveled. Indeed, the firm acts accordingly, as shown in the chart. As the firm's share of surplus further increases, the delta in the no-default case starts to approach zero. As a result, any reasonable level of risk allocation in the pension fund leads to only meager stock market exposure of the pension payment to the employee. That is, with extreme low values of delta, the channel of passing the risk exposure of the pension investment to the employee through the varying pension payment is by and large shut down, making it ineffective for the pension fund to provide the risk exposure to the stock market for the employee. Overall, when the firm takes large enough share of the surplus, further increasing in the firm's share of the surplus and thus reducing the employee's share of the surplus, the optimal risk allocation decreases.

As the firm's share of the surplus increases, the delta of the pension payment to the pension fund investment value decreases in the surplus region (i.e., the region of $W_T > F$). As shown in Plot B.2 , the firm chooses a high level of the PBO and accordingly the low level of the the initial funding ratio so as to reduce the probability of the surplus region, the region of the reduced delta. Overall, the firm chooses the funding ratio in an attempt to maintain a relatively high average delta.

The cases discussed above help establish the basic intuition of our model. We further examine other comparative statistics, including the effect of varying financing cost (Panels C and D of Figure 2), the effect of varying stock market condition (Panels E, F, and G). The results are largely consistent with our model's basic intuition: the optimal risky allocation decision is reached by negotiating between the demand of risky exposure from the employee and the convex financing cost.

IV.3 Cross-section of Pension Investments

In our model, both the initial funding ratio and the asset allocation are endogenous. Thus, the empirically observed cross-sectional relation between the funding ratio and the risk allocation should be interpreted with care. In this subsection, we examine various cross sectional relations that can result from the model. The results of the analysis are presented in Figure 3.

In the model, a simple way to vary the initial funding ratio is to vary the initial free cash. If a firm has less free cash at time 0, it is naturally costly for the firm to fund the pension generously. The pension is thus likely to be underfunded, ceteris paribus. How the initial free cash affects the firm's initial funding, PBO, and the asset allocation is examined in Panel A of Figure 2, in Subsection IV.2. Here, we present the cross-sectional relation of the initial funding ratio and the risky allocation in Plot 1 of Figure 3 (the dot-line). Basically, as the initial funding ratio due to higher initial free cash flow, the fund's risky allocation is also higher.

In the plot, we also examine the cross-section by letting the bankruptcy probability vary. Some properties of this case are examined in Figure 1, in Subsection IV.2. As the bankruptcy probability increases, the promised obligation increases and the initial funding decreases. At the same time, the risky asset allocation decreases. So, if we simply relate the funding ratio and the asset allocation, we again see a positive relation.

Finally, in the plot, we present the cross-section by varying the surplus sharing. Some properties of the case are examined in Figure 2, Panel B, in Subsection IV.2. As the firm's share in the upside increases, the employee's share in the upside decreases. The funding ratio decreases and the asset allocation increases initially and decreases at the end. The intuition is discussed in detail in Subsection IV.2. Overall, there is a negative relation between the funding ratio and the risky allocation in some range of the two variables when the firm's share of the surplus is low, but in other range of the two variables when the firm's share of the surplus is high.

In summary, we see that, with different choice of the underlying force that causes the cross-sectional variation, the relation between the funding ratio and the risky allocation is mostly positive, but can be negative.

IV.4 A "Variable Benefit" Plan

Our analysis so far suggests a general result that it is optimal for the employee to bear a large fraction of pension investment risk, i.e., having a high delta. The defined benefit feature, however, severely limits employee's exposure to investment risk by having a low value of delta. This leads naturally to the question we explore in this part: what if we let the employee bear the full risk of the investment? That is, let $S = W_T$. This makes the employee's payoff similar to that in a defined contribution plan, although we still assume that the investment asset is pooled among all the employees and is managed uniformly, and the firm insures the longevity risk (thus, strictly speaking, a hybrid of the defined benefit plan and the defined contribution plan). We call it the variable benefit plan (the VB plan). Such a plan eliminates the firm's concern of financing cost at time T. More important, such a plan has the highest possible delta in our model, namely 1.

We compare the outcome of the defined benefit plan with that of the variable benefit plan. In particular, we examine how asset allocation differs across the two plans, and how much more the defined benefit plan costs the firm than the variable benefit plan while keeping the employee's reservation utility fixed. In Panel A of Figure 4, we study the comparison for firms with different bankruptcy probabilities. For this part of the analysis we keep the funding ratio exogenous. One can also interpret the result as if there is an exogenous shock to funding ratio. Alternatively, we can generate essentially the same chart by varying the time 0 free cash flow. The plot shows that as the funding ratio increases, similar to the results in Panel A of Figure 2, the risky allocation increases.

Interestingly, the DB plan in general invests much more aggressively than the VB plan, as shown in Panel A(1). In Panel A(2), we see that the total funding cost of the DB plan to the firm is much higher than that of a VB plan (about 40% higher for an investment-grade firm). The total funding cost is calculated as $CF_0 + \frac{1}{R_f} E^Q(CF_T)$. It may come as a surprise that the funding cost of the DB plan is the lowest for a low credit-rating firm. Thus, the results highlight that the importance of risk sharing in pension design. To minimize pension funding cost, it is optimal for the employee to bear all the (systematic) investment risk.

In recent decades, there is a trend of firms shifting away from defined benefit plans into defined contribution plans. One reason often cited by firms making this shift is that defined benefit plans have much higher funding cost (Munnell and Soto 2008). The analysis performed here supports this notion.

IV.5 The Effect of PBGC Insurance

In this section, we analyze the effect of PBGC pension insurance. PBGC provides insurance coverage on employees' promised pension benefits up to a ceiling — in 2017, \$64,000 a year for a retiree at age of 65. For highly paid employees such as pilots, PBGC insurance provides only a partial coverage. To capture the partial coverage feature of PBGC insurance, we assume that the employee receives θF, with $\theta \in (0,1)$, from PBGC when the firm is bankrupt and when the pension is underfunded. We initially set $\theta = 0.7$ and investigate variations of θ in later analysis. The employee's expected pension payment becomes:

$$S = F(1-D) + \min(W_T, 0.7 \times F)D + 0.2 \times \max(W_T - F, 0). \tag{9}$$

We also assume that PBGC has access to the optimal risky portfolio, but is thinly capitalized and faces a convexly shaped financing cost function (with lower parameter values relative to the firm's). We set $c_{ins,a} = 0$, $c_{ins,b} = 0.0353$, and $c_{ins,c} = 0.1333$. The insurance premium by PBGC is considered fairly priced if PBGC breaks even on its insurance. Too low a premium (undervalued premium) will result in PBGC subsidizing the firm, and too high a premium will result in PBGC gains at the expense of the firm. To model the over-/under-pricing of PBGC insurance, we follow the insurance literature (i.e., Doherty and Schlesinger 1990) and set the PBGC insurance premium as the product of the risk-neutral expectation of cash flow to PBGC in case of bankruptcy and a loading factor. The insurance is overpriced if the loading factor m is greater than 1, underpriced if m is smaller than 1, and fairly priced if m equals to 1. Thus PBGC insurance premium can expressed as

$$I = mD\frac{1}{R_f}E^Q\left[\max(0.7 \times F - W_T, 0) + C_B(\max(0.7 \times F - W_T, 0))\right] \tag{10}$$

where C_B is the financing cost function of PBGC. Firms need to purchase PBGC insurance when forming the pension portfolio, which changes firm's cash flow at period 0 to $CF_0 = H_0 + h_0 - W_0 - I$. Again, initial contribution h_0 needs to be chosen such that CF_0 is non-negative.

Figure 5 displays the pension plan's risky allocation and funding decision in the presence of PBGC. In Panel A, we look at the effect of bankruptcy risk. When the bankruptcy risk is relatively low (less than 0.5), the relation between risky allocation and bankruptcy risk is similar to the case without PBGC insurance – pension investment becomes more aggressive when the bankruptcy risk is higher. However, as the bankruptcy probability increases above 0.5, the relationship is reversed. Two effects are driving the results here. First, insurance coverage reduces the employee's downside risk exposure in bankruptcy, therefore the employee desires more risky allocation as the perceived bankruptcy risk increases. Second, higher bankruptcy risk increases insurance premium, which discourages the firm from investing in the risky asset. The pattern shown in Panel A (1) results from the trade-off of these two effects. Finally, the relation between pension funding and bankruptcy risk in Panel A (2) is similar to that without PBGC – the pension becomes more underfunded when bankruptcy risk increases.

In Panel B, we investigate the effect of initial free cash. When firm has little initial free cash to fund pension plan, it takes advantage of pension insurance and the lower financing cost of PBGC by investing aggressively in the risky asset and contributing as little as possible. Essentially, the firm bets on its bankruptcy and PBGC insurance to meet the employee's reservation utility. As the firm's initial free cash increases, the above strategy becomes less attractive. The firm starts to contribute more, promise less, and allocate more to the risky asset.

The effect of surplus sharing, as shown in Panel C, is similar to the case without PBGC.

Further, Panel D shows that the effect of PBGC's financing cost basically replicates the pattern of firm's own financing cost at time T.

As shown in Panel E, pension insurance encourages pension risk taking particularly when the insurance premium is under valued. Even with fair valued insurance, the risky allocation under PBGC insurance is still higher than the case without PBGC. This is because of the lower financing cost of PBGC relative to that of the firm. Meanwhile, overpriced PBGC insurance makes the firm promise more to compensate for the reduction in risk exposure. Pension plan becomes less underfunded as a result of increased insurance premium.

Finally, we look at the effect of partial PBGC coverage in Panel F. Higher PBGC coverage reduces the employee's downside risk exposure, which requires firm to make more risky investment. If the PBGC coverage is too low, the employee bears almost the same downside risk exposure as in the case without PBGC. And as a result, the risky investment in the presence of PBGC gets close to that without PBGC.

For robustness, we further examine the cross-sectional relations among key variables in a setting where the pension is fully funded at the initial stage. We thus focus only on the the firm's asset allocation decision, while the fund contribution decision is fixed by assumption. We repeat the above numerical exercises for both the cases of with and without the PBGC. Figure 6 shows that all the results obtained earlier in this section remain qualitatively robust to this variation. It is noticeable that even without PBGC insurance, firms with overly high bankruptcy probability when required to fully fund the pension plan will have a very aggressive investment policy. This is because for firms with high bankruptcy probability, it is optimal to underfund the pension plan, which, however, is not allowed in this setting. Thus firms lower the promised pension payment and compensate employees with higher risky allocation.

IV.6 The Effect of Surplus Reversion Tax

As discussed earlier, to prevent firms clawing back all the pension surplus, in 1990 the congress imposed a punitive excise tax on pension surplus being clawed back to the firm.

We include the surplus reversion tax feature to the model in this section.

According to the Omnibus Budget Reconciliation Act of 1990, the excise tax rate on pension surplus reversion is 50%, which is reduced to 20% if the plan sponsor is in Chapter 7 bankruptcy liquidation, or at least 20% of pension surplus is used to increase plan participants' benefits, or the firm transfers 25% or more of the surplus to a qualified replacement plan.[15] In the case of no bankruptcy, the firm is motivated to share 20% pension surplus with its employee and pay 20% as reversion tax instead of suffering the 50% as deadweight loss. In the case of bankruptcy, the firm is no longer motivated to share pension surplus because it has to pay 20% reversion tax regardless of sharing or not. Therefore, the employee's pension payment becomes:

$$S = F(1-D) + \min(W_T, F)D + (1-\alpha) \times \max(W_T - F, 0)(1-D). \tag{11}$$

Parameterizing the reversion tax rate as τ which equals 20% in current regulation and is subject to change in later analysis, the firm's cash flow at time T turns to

$$CF_T = W_T + h_T - S - \max(W_T - F, 0) \times \tau(D + \alpha(1-D)). \tag{12}$$

The numerical analysis focuses on how the surplus reversion tax affects the two risk sharing channels - bankruptcy and surplus sharing, and how the reversion tax affects the pension's risky allocation and pension funding. In general, as shown in Figure 7, taxing the pension surplus reversion reduces the pension's risky allocation. It is because the reversion tax lowers the firm's cash flow at time T, thus making it less attractive for the firm at time 0 to plan on bearing the future financing cost. Figure 7 also shows that the firm is more likely to underfund the pension plan under higher reversion tax. The lowered funding ratio is driven by the employee's demand of more promised pension payment, which stems from the reduced risk exposure in the pension portfolio. Besides the aforementioned endogenous reduction of risk exposure, the employee also experiences a loss of upside risk sharing in the

[15] In the static model, firms are not allowed to adjust the sponsored plan hence we do not consider the case of 25% reversion tax on plan replacement here.

case of bankruptcy, because the firm is not willing to share the surplus if it has to pay the reversion tax either way.

It is worthy to mention that, as illustrated in Panel A.1 of Figure 7, under surplus reversion tax, firms with very high bankruptcy risk will choose to implement a more aggressive investment policy than firms with moderate bankruptcy risk. This leads to a U-shape relationship of the bankruptcy risk and the risky allocation. This effect is because the employee loses too much upside risk exposure in the case of bankruptcy, which in turn drives up the desire for risky allocation in general. This model prediction is consistent with the empirical finding by An, Huang, and Zhang (2013) that for firms already in financial distress, the relation between bankruptcy probability and investment risk becomes positive.

V. Dynamic Model

While our one-period model delivers a rich set of results, it assumes that firms cannot adjust pension investment allocations or adjust pension contribution in the interim. This might be restrictive. In this section, we consider a dynamic model that relaxes these restrictive assumptions.

V.1 Model Setup

The entire time period in the dynamic model is from time 0 to T. At time 0, the firm hires the employee. At time T, the employee retires and receives the pension payment. The investment opportunity again consists of two assets, the risk free asset with the instantaneous rate r_f, and the risky asset its value process following a geometrical Brownian motion:

$$\frac{dP_m}{P_m} = (\mu + r_f)dt + \sigma dB \tag{13}$$

where B is a standard Brownian motion, μ the risk premium, and σ the asset volatility. We also assume that the market is dynamically complete, and there is the unique the risk neutral probability Q.

$$\frac{dP_m}{P_m} = r_f dt + \sigma dB^Q, \tag{14}$$

where B^Q is a standard Brownian motion under the risk neutral probability Q.

Let w_t be the portfolio weight on the market portfolio. We denote the cumulative pension contribution by the firm up to time t by H_t, which is a non-decreasing stochastic process. Together, the pension value process is governed by

$$dW_t = W_t[(w_t\mu + r_f)dt + w_t\sigma dB] + dH_t. \tag{15}$$

To facilitate numerical solutions, we assume that the firm only adjusts its portfolio weight and makes pension contribution at T+1 discrete time points: at times $n = 0, 1, \ldots, T$. Specifically, the portfolio weight is kept constant between the two discrete decision time points, i.e., $w_t = w_n$ if $t \in [n, n+1)$ for integer n. Furthermore, the cumulative contribution process H is a stochastic process that increases only by steps at the discrete time points. That is, $H_t = H_n$ if $t \in [n, n+1)$, and $dH_n = h_n$ for some discrete process h_n. At the time when $dH_n > 0$, a financing cost is incurred. We denote the cumulative cost process by C_t. We use $C(\cdot)$ to denote the cost function. Formally, we can write $dC_t = C(dH_t)(dH_t > 0)$ for $t \in [0, T]$. Of course, dC_t defined this way can be non-zero only at the discrete time points n and only when there is a positive pension contribution. Specifically, $dC_n = C(h_n)(h_n > 0)$.

The firm's default indicator, D_t, is a dynamic process governed by the first arrival of a Poisson process with constant intensity δ. We denote the firm default time by τ, a stopping time defined by $\tau = \min(t|D_t = 1)$. The pension will be liquidated and payment is made to the employee either when the firm defaults at time τ ($\tau \leq T$) or at the terminal time T. The payment schedule is in principle the same as in Equation (7) with some notation adjustments to accommodate the dynamic model structure. Specifically,

$$S_\tau = F(1 - D_\tau) + \min(W_{\tau-}, F)D_\tau + (1 - \alpha) \times \max(W_{\tau-} - F, 0) \tag{16}$$

The firm keeps the difference between the pension assets and pension payment. That is,

$CF_\tau = W_\tau - S_\tau$. At the payment time, we further adopt the convention that $W_\tau \geq S_\tau$. That is, if the pension's asset falls short of the payment S_τ (i.e., $W_{\tau-} < S_\tau$), the firm's time-τ contribution has to make up the shortfall. This ensures that CF_τ is non-negative.

The employee has a CRRA utility function on time-T wealth, $U(S) = \frac{S^{1-\gamma}}{1-\gamma}$. If at time τ, the employee receives the pension payment as the consequence of firm bankruptcy, she will invest optimally with the weight on the market portfolio $w = \frac{\mu}{\gamma\sigma^2}$ for the remainder time interval $[\tau, T]$. Thus,

$$S_T = S_\tau e^{(r_f + w\mu - \frac{1}{2}w^2\sigma^2)(T-\tau) + w\sigma(B_T - B_\tau)} \tag{17}$$

The firm values future cash flows (including the non-negative cash flows, the contributions, and the financing cost) using the risk neutral probability and the risk free discount rate. The firm's optimization problem is:

$$\max E^Q \left[e^{-r_f \tau} CF_\tau - \int_0^\tau e^{-r_f t}(dH_t + dC_t) \right]. \tag{18}$$

$$\text{subject to:} \quad E(U(S_T)) = \mathbb{U}. \tag{19}$$

There are additional constraints, including CF_τ being non-negative, pension asset value process being governed by Equation (15), and the final pension payment to the employee being specified by Equations (16) and (17).

We numerically solve the dynamic model. Most of the economic parameters, including the stock market information, and parameters in the financing cost function, the employee's risk aversion, the number of years till retirement, and the annual default rate are the same as those in the one-period model. The only difference is that we assume the fixed cost of external financing as zero in the dynamic model to smooth the optimization solutions.

V.2 Initial Pension Decisions

Within the dynamic model setup, we revisit some of the cross-sectional relations observed for the one-period model. In particular, we examine how the firm's initial (time-0) pension funding decision and the initial (time-0) investment decision vary when we change model parameters. The results are reported in Figure 8. We have three sets of results regarding the cross-section of the firm's initial investment allocation (w_0), cash contribution (h_0) and PBO (F), as we vary the probability of bankruptcy (δ), the firm's initial free cash (H_0), and the percentage of the pension surplus the firm keeps (α).

The cross-sectional patterns observed in Figure 8 is qualitatively similar to those seen in Figures 1 and 2.

In Panel A.1 of the figure, similar to what we find in Figure 1 for the one-period model, we see that a firm with higher default probability tends to choose less risky allocation, everything else the same. For an AAA-rated firm, the risky asset weight of the portfolio is above 80%. In comparison, for a BBB-rated firm, the weight drops to 62%. Also note that the in general the risky allocation in the dynamic model here is higher that that in the one-period model. Thus, having the ability to adjust asset allocation and make pension contribution at multiple time points does not reduce a pension's investment risk-taking.

In Panels B and C, we vary the initial free cash (H_0) and the parameter of the surplus sharing (α), respectively. The results are qualitatively similar as those in Figure 2. Specifically, when the firm has more free cash at time 0, it contributes more to the pension fund. This further leads to more aggressive asset allocation because more initial contribution mitigates the concern of financing cost due to future funding shortfalls. The relation between investment risk-taking and the fraction of pension surplus kept by the firm is non-monotonic, with investment risk being the highest when the firm's share of the surplus is moderate. The investment risk is lower when the firm's share of surplus is either very high or very low. The intuition is the same as discussed in Section III.2. It is worth noting that the portfolio weight on the risky asset stays above 55% throughout the entire range of the surplus sharing parameter.

Overall, we find that in the dynamic model, corporate pension may continue to take substantial investment risk, and the relation of pension risk taking with some of the key parameters remain similar to what we find in the one-period model. Thus, the ability of the firm to dynamically adjust pension contribution over time does not significantly reduces pension risk taking.

V.3 The Dynamics of Pension Investments and Contributions

We further investigate how pension investments and pension contributions vary over time. Figure 9 displays the portfolio weight on the risky asset over time under different sets of parameters. In Panel A, we start with the benchmark case where the annual default probability is 0.05% (for BBB-rated firms) and the firm's fraction of pension surplus is 80%. We show the portfolio weights in the left plot and pension contributions in the right plot. In Panel B, we change the default probability to 0.01% (for AAA-rated firms). In Panel C, we change the surplus sharing parameter: the firm keeps 60% of the surplus and the employee keeps 40%.

To varying degrees, all the plots share some common features. In the early years, that is, when the retirement time (T) is far into the future, the risky allocation is relatively high. As the retirement time draws closer, for most range of the funding ratio, the risky allocation drops. The intuition in Proposition 1 can help us understand this pattern, but there is also important differences due to the added richness of the dynamic model, as we explain below.

Unlike the one-period model where the asset allocation decision is made only once, in the dynamic model, asset allocation varies over time in response to funding level. The two main counterbalancing factors identified in Proposition 1 for optimal asset allocation are: (1) the employee's desire to have investment risk exposure; and (2) the firm's financing cost. In the dynamic model, the firm chooses to have high risk exposure at times and states that such investment risk exposure can be effectively passed on to the employee and that the cost of such risk exposure (due to the convex financing cost) is relatively low.

Along the time dimension, the firm would choose to have high risk exposure when the retirement time is far away. A long investment horizon means that the optimal risk exposure has a bigger impact on the employee's utility, ceteris paribus. Furthermore, when the time horizon is long, the firm has the time to make the periodic contributions, and this would substantially reduces the impact of the convexity in the financing cost. Consider this simple thought experiment: there is a shortfall of \$10. If the firm has to make a one time contribution, the convex component of the financing cost is proportion to $10^2 = 100$. In contrast, if the firm breaks down the contribution into 10 equal pieces. Then the convex component of the financing cost adds up to an amount proportion to only $10 \times 1^2 = 10$, which is only one tenth of the one time contribution cost. In addition, the firm has the option to choose whether and when to make the contributions, as well as to adjustment portfolio allocations when needed. Such option has a higher value when the uncertainty is higher, that is, when the pension's risk allocation is high. All this point to the direction that the pension plan should have a high risk allocation early on, ceteris paribus.

In addition to the time dimension, when consider the state dimension of the time-state space, the firm should also allocation the pension fund's risk exposure across the state in an optimal fashion. A number of factors influence the decision. One can enhance the overall risk exposure, while keeping the average risk exposure across the state space fixed, by take up more risky positions in the up and down sides of the funding status. This idea is simple, as we make both ends of the state-space more volatile, the total volatility increases. Therefore, high risk allocations in the both the highly over-funding and the highly under-funding states, which make the pension asset more risky, help to expose the employee to the stock market risk.

Second, from a cost-effective stand point, when the under-funding is severe, the ex ante average delta of the pension payment to the pension asset approaches 1, the upper bound of the delta in the model. Therefore, it is the most effective in this range to expose the employee to the investment risk. In the middle range, when the pension plan is close to the fully funded or slightly over-funded, given the surplus sharing, the investment risk can still be passed to the employee, though at a lower delta. Yet, the convex cost is still a concern. The combination result is that the pension fund's risk allocation is relatively low in this range. Yet on the other end, when the pension plan is substantially over-funded, the decrease in the average delta reaches a plateau, and yet the financing cost become a lesser concern because the likelihood of future fund shortfall is low. The joint effect is that the risk allocation is high.

The patterns we observe from asset allocations across firms with different default probabilities help us gain further intuition on the risk allocation problem in the dynamic setting. When we reduce the default probability from that of a BBB firm in our benchmark case in Panel A to a AAA firm in Panel B, we see that overall the risk allocation is higher. This is consistent with what we find in Panel A of Figure 1. The pension payment's delta to the pension investment asset is higher in the default case than in the non-default case, because the employee is exposed to the downside risk in the default case and is protected by the promised obligation in the non-default case. Therefore, it takes a higher risk allocation in the pension investment to achieve the same level of risk exposure in the pension payment. That is, an AAA firm has a low pension payment to pension asset delta than a BBB firm, and therefore, the AAA firm should choose a higher risk allocation for the pension investment to make up the reduced delta. On surface, this statement seems to contradict what we stated above, that it is more effective for the firm to choose high investment risk allocation in the states where the payment delta is relative high, ceteris paribus. The key difference is that, when we are considering the dynamic problem for a single firm, there is a internal balancing across states and times. The firm decides to allocate more to the risky asset in some states and times to make up the low allocation in some other states and times. In contrast, there is no such balancing effect across firms, that is, the two firms are not dealing with each other to reach a mutually beneficial agreement to jointly solve the the pension optimization problem. Furthermore, we focus only on the comparison of the average allocation across firms.

As we reduce the firm's share of the surplus, we see the allocation varies from the charts of Panel A to the charts in C. The main result is that the allocation to the risky asset is reduced as the firm's share of the surplus reduces. This pattern is consistent with what we observe in Panel B of Figure 2 for the one-period model. As the firm's share of the surplus increases, the employee's share naturally decreases. Consequently, the pension payment's delta to the pension's investment value is lowered. To provide the desirable level of the risk-return trade-off, in other words, to provide the desirable exposure to the investment risk for the employee, the asset allocation has to increase accordingly to compensate for the decline in delta.

VI. Conclusions

This paper provides a perspective to understand the risky asset allocation policies pursued by corporate pensions. In our model, pension risk taking is driven by employees' preference for systematic risk exposure, while the firm balances employees' preference with its concern for reducing financing cost. The pension investment risk is shared between shareholders and employees. The firm's decisions on pension benefits and pension funding are endogenous to such risk sharing. For a reasonable set of parameter values, the optimal pension investment risk and its relations with a firms' bankruptcy probability and pension funding ratio predicted by the model are consistent with empirical observations.

The stakeholder approach we take combines two polarized views on the objective function of pension decisions – the shareholder value maximization view and the employee utility maximization view. The former has been a prevalent approach in recent academic studies to evaluate corporate pension decisions. The challenge for this view is the extreme implication that pension investment should be riskfree, which contrasts dramatically with the observed pension investment behavior. The employee utility maximization view is recognized by Bodie (1990) as an alternative interpretation of the risky pension investment behavior. Perhaps implicitly, under this view a DB plan should invest in a way similar to what employees were to manage their portfolios outside a DB plan – a mean-variance efficient portfolio, for example. However, this does not take into account the convoluted cash flow rights (in particular, the guarantee by the firm on pension benefits) a DB plan offers to employees, which may substantially alter the investment decisions. The stakeholder approach appears to be successful in meshing the two views to deliver a reasonable interpretation of the observed investment policies by corporate pensions.

Our analysis highlights the inefficient risk sharing in typical DB plans and its important consequences. The defined benefit plans, out of a motive to protect employees from firm-specific risk, make employees' pension payoffs relatively insulated from systematic risk, despite the preference by employees for systematic risk. Therefore, defined benefit pension contracts are suboptimal in allocating the systematic component of pension investment risk between shareholders and employees. A more efficient contract would let employees to shoulder all the pension investment risk while keeping them insulated from firm-specific risks. Interestingly, this arrangement resembles what a defined contribution plan offers (although DC plans lack the longevity risk sharing feature of DB plans). Our analysis shows that such an arrangement may substantially reduce firms' pension funding costs. Perhaps this is one of the reasons that many firms have switched from DB plans to DC plans.

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Appendix

Proof of Proposition 1

Proof. First we define pension plan funding ratio δ as the ratio of initial pension asset W_0 to the present value of the firm's pension liability F with discount rate R_f, thus we have $W_0 = FR_f^{-1}\delta$.

In the first case with no bankruptcy risk, the pension payment is $S = F$. Define function $\phi(x)$ as $\phi(x) = C_1(x)$ if $x > 0$, and $\phi(x) = 0$ otherwise. The firm's objective function is specified as

$$G(w, \delta) = -C_0((W_0 - H_0)^+) - FR_f^{-1} - R_f^{-1}E^Q\left(\phi(z)\right)$$

where $z = F - W_T$.

For the case of $\delta \geq 1$, we have $G(0, \delta) = -C_0((W_0 - H_0)^+) - FR_f^{-1}$. Since $\phi(z) \geq 0$, we have $G(w, \delta) \leq -C_0((W_0 - H_0)^+) - FR_f^{-1}$. Thus $(0, \delta) = \operatorname{argmax} G(w, \delta)$ for $\delta \geq 1$.

For the case of $\delta < 1$, $G(0, \delta) = -C_0((W_0 - H_0)^+) - FR_f^{-1} - R_f^{-1}\phi(z_0)$ where $z_0 = F(1 - \delta)$. Under the assumption that $\phi(z)$ is convex, we have $\phi(z) \geq \phi(z_0) + c(z - z_0)$ for some constant c.[16] Therefore,

$$\begin{aligned}
G(w, \delta) \leq & - C_0((W_0 - H_0)^+) - FR_f^{-1} - R_f^{-1}E^Q\left[(\phi(z_0) + c(z - z_0))\right] \\
= & G(0, \delta) - cR_f^{-1}E^Q[(z - z_0)] = G(0, \delta) - cR_f^{-1}E^Q\left[(F - W_T - F + F\delta)\right] \\
= & G(0, \delta) - cW_0R_f^{-1}E^Q\left[(R_f - R(w))\right] = G(0, \delta)
\end{aligned}$$

Given that $\phi(z)$ is strictly convex at least at one point (otherwise it would be constant 0), the above inequalities will be strict when $w \neq 0$. Hence $G(0, \delta)$ is the unique maximum for any δ.

Now turn to the case with bankruptcy risk. We need the following two lemmas for the proof.

Lemma 1. *Given any promised retirement benefit fixed, more risky asset investment will raise the present value of external financing cost regardless of the initial funding status.*

Need to show that

$$\frac{\partial E^Q\left[C_T\left((F - W_T)^+\right)\right]}{\partial w} > 0, \text{ for any given } F$$

Taking the derivative with respect to w, we have

$$\frac{\partial E^Q\left[C_T\left((F - W_T)^+\right)\right]}{\partial w} = -\int_{-\infty}^{R_m^c} C_T'(F - W_T)W_0R_ef^Q(R_m)dR_m$$

where R_m^c is the realized risky return such that $F = W_T$.

If the pension plan is initially overfunded, $\delta > 1$, $R_m^c < R_f$, the risky asset's excess return R_e is always negative in the integration. Thus the derivative is positive.

If the pension plan is initially underfunded, $\delta > 1$, $R_m^c > R_f$, the derivative can be written as

[16] Where c is the slope of a support of the convex function. If $\phi(x)$ is differentiable at z_0, then the support is unique and $c = \phi'(z_0)$.

$$
\begin{aligned}
\frac{\partial E^Q\left[C_T\left((F-W_T)^+\right)\right]}{\partial w} &= -\int_{-\infty}^{R_f} C'_T(F-W_T)W_0R_e f^Q(R_m)dR_m - \int_{R_f}^{R_m^c} C'_T(F-W_T)W_0R_e f^Q(R_m)dR_m \\
&> -\int_{-\infty}^{R_f} C'_T(F-F\delta)W_0R_e f^Q(R_m)dR_m - \int_{R_f}^{R_m^c} C'_T(F-F\delta)W_0R_e f^Q(R_m)dR_m \\
&= -\int_{-\infty}^{R_m^c} C'_T(F-F\delta)W_0R_e f^Q(R_m)dR_m > -C'_T(F-F\delta)W_0\int_{-\infty}^{+\infty} R_e f^Q(R_m)dR_m = 0
\end{aligned}
$$

The second inequality holds because $\frac{\partial C'_T(F-W_T)}{\partial R_m} = C''_T(F-W_T)(-W_0w) < 0$ as long as $w > 0$. Therefore, $C'_T(F-W_T) > C'_T(F-F\delta)$ for $R_m < R_f$, and $C'_T(F-W_T) < C'_T(F-F\delta)$ for $R_m > R_f$.

Next, consider the employees' participation constraint

$$
(1-p)U(F) + pEU(W_T(w,\delta)) = \mathbb{U}
$$

which implies $F = F(\mathbb{U}, w, \delta)$. We can prove the following lemma.

Lemma 2. *$\frac{\partial F(\mathbb{U},0,\delta)}{\partial w} < 0$, $\frac{\partial F(\mathbb{U},w_e,\delta)}{\partial w} = 0$, and $\frac{\partial F(\mathbb{U},\hat{w}_e,\delta)}{\partial w} > 0$, for all $\mathbb{U} > 0$ and $\delta > 0$. w_e is employee's optimal investment allocation of self-management portfolio, and $\hat{w}_e > w_e$.*

With the other parameters exogenously given, we can obtain the general form of $\frac{dF}{dw}$ using implicit function theorem.

$$
\frac{dF}{dw} = \frac{\partial F}{\partial w} = -\frac{pE[U'(W_T)W_0R_e]}{(1-p)U'(F) + pE[U'(W_T)R_f^{-1}\delta R(w)]}
$$

Now let's figure out the sign of $\frac{dF}{dw}$ evaluated at three critical w's: 0, w_e, and $\hat{w}_e$.

$$
\left.\frac{dF}{dw}\right|_{w=0} = -\frac{pU'(F\delta)W_0E(R_e)}{(1-p)U'(F) + pU'(F\delta)\delta} < 0
$$

The CRRA employee chooses w_e such that $\frac{dEU(W_T)}{dw} = E[U'(F, w_e)W_0R_e] = 0$ for any F. Therefore,

$$
\left.\frac{dF}{dw}\right|_{w=w_e} = -\frac{pE[U'(F,w_e)W_0R_e]}{(1-p)U'(F) + pE[U'(F,w_e)R_f^{-1}\delta R(w_e)]} = 0
$$

For any given F,

$$
\frac{dE[U'(F,w)R_e]}{dw} = E[U''(F,w)R_e^2]W_0 < 0
$$

which implies $E[U'(F,\hat{w}_e)R_e] < E[U'(F,w_e)R_e] = 0$. Further we are able to show

$$
\begin{aligned}
\frac{dE[U'(F,w)R(w)]}{dw} =& E[U''(F,w)W_0R_eR(w) + U'(F,w)R_e] \\
=& E[U''(W_T)W_TR_e + U'(W_T)R_e] = E[(1-\gamma)U'(F,w)R_e] \geq 0, \forall w \geq w_e
\end{aligned}
$$

Hence $E[U'(F,\hat{w}_e)R(\hat{w}_e)] > E[U'(F,w_e)R(w_e)] = EU'(F,w_e)R_f > 0$. So we have

$$
\left.\frac{dF}{dw}\right|_{w=\hat{w}_e} = -\frac{pE[U'(F,\hat{w}_e)W_0R_e]}{(1-p)U'(F) + pE[U'(F,\hat{w}_e)R_f^{-1}\delta R(\hat{w}_e)]} > 0
$$

Now, to finish the proof of Proposition 1, we need to sign $G'(w)$ for $w = 0$, w_e, and $\hat{w}_e$. Let's start from $w = w_e$ and $w = \hat{w}_e$.

$$\begin{aligned}
G'(w_e) = &- \left.\frac{dF}{dw}\right|_{w=w_e} R_f^{-1}(p\delta + (1-p)) - \left.\frac{dF}{dw}\right|_{w=w_e} C_0'(W_0 - H_0)R_f^{-1}\delta \\
&- (1-p)R_f^{-1}\int_{-\infty}^{R_m^c(w_e)} C_T'(F - W_T(w_e))(1 - R_f^{-1}\delta R(w_e)) \left.\frac{dF}{dw}\right|_{w=w_e} f^Q(R_m)dR_m \\
&+ (1-p)R_f^{-1}\int_{-\infty}^{R_m^c(w_e)} C_T'(F - W_T(w_e))W_0R_e f^Q(R_m)dR_m \\
=&(1-p)R_f^{-1}\int_{-\infty}^{R_m^c(w_e)} C_T'(F - W_T(w_e))W_0R_e f^Q(R_m)dR_m < 0
\end{aligned}$$

The last inequality comes from Lemma 1.

$$\begin{aligned}
G'(\hat{w}_e) = &- \left.\frac{dF}{dw}\right|_{w=\hat{w}_e} R_f^{-1}(p\delta + (1-p)) - \left.\frac{dF}{dw}\right|_{w=\hat{w}_e} C_0'(W_0 - H_0)R_f^{-1}\delta \\
&- (1-p)R_f^{-1}\int_{-\infty}^{R_m^c(\hat{w}_e)} C_T'(F - W_T(\hat{w}_e))(1 - R_f^{-1}\delta R(\hat{w}_e)) \left.\frac{dF}{dw}\right|_{w=\hat{w}_e} f^Q(R_m)dR_m \\
&+ (1-p)R_f^{-1}\int_{-\infty}^{R_m^c(\hat{w}_e)} C_T'(F - W_T(\hat{w}_e))FR_f^{-1}\delta R_e f^Q(R_m)dR_m < 0
\end{aligned}$$

For $w = 0$, we discuss the results separately with different δ:

1. If the pension plan is initially underfunded, $\delta < 1$, then $R_m^c|_{w=0} = +\infty$,

$$\begin{aligned}
G'(0) = &- \left.\frac{dF}{dw}\right|_{w=0} R_f^{-1}(p\delta + (1-p)) - \left.\frac{dF}{dw}\right|_{w=0} C_0'(W_0 - H_0)R_f^{-1}\delta \\
&- (1-p)R_f^{-1}\int_{-\infty}^{+\infty} C_T'(F - F\delta)(1-\delta) \left.\frac{dF}{dw}\right|_{w=0} f^Q(R_m)dR_m \\
&+ (1-p)R_f^{-1}\int_{-\infty}^{+\infty} C_T'(F - F\delta)W_0R_e f^Q(R_m)dR_m \\
= &- \left.\frac{dF}{dw}\right|_{w=0} R_f^{-1}\left[p\delta + C_0'(W_0 - H_0)\delta + (1-p)(1 + C_T'(F - F\delta)(1-\delta))\right] > 0
\end{aligned}$$

2. If the pension plan is initially overfunded, $\delta > 1$, then $R_m^c|_{w=0} = -\infty$,

$$\begin{aligned}
G'(0) = &- \left.\frac{dF}{dw}\right|_{w=0} R_f^{-1}(p\delta + (1-p)) - \left.\frac{dF}{dw}\right|_{w=0} C_0'(W_0 - H_0)R_f^{-1}\delta \\
&- (1-p)R_f^{-1}\int_{-\infty}^{-\infty} C_T'(F - F\delta)(1-\delta) \left.\frac{dF}{dw}\right|_{w=0} f^Q(R_m)dR_m \\
&+ (1-p)R_f^{-1}\int_{-\infty}^{-\infty} C_T'(F - F\delta)V_0R_e f^Q(R_m)dR_m \\
= &- \left.\frac{dF}{dw}\right|_{w=0} R_f^{-1}\left[(p\delta + (1-p)) + C_0'(W_0 - H_0)\delta\right] > 0
\end{aligned}$$

3. If the pension plan is initially fully funded, $\delta = 1$, then $R_m^c|_{w=0} = R_f$, $\frac{dF}{dw}\Big|_{w=0} = -pFR_f^{-1}E[R_e]$,

$$\begin{aligned}G'(0) = &- \left.\frac{dF}{dw}\right|_{w=0} R_f^{-1}(p + (1-p)) - \left.\frac{dF}{dw}\right|_{w=0} C_0'(W_0 - H_0)R_f^{-1} \\ &- (1-p)R_f^{-1}\int_{-\infty}^{R_f} C_T'(F-F)(1-1)\left.\frac{dF}{dw}\right|_{w=0} f^Q(R_m)dR_m \\ &+ (1-p)R_f^{-1}\int_{-\infty}^{R_f} C_T'(F-F)FR_f^{-1}R_e f^Q(R_m)dR_m \\ =&FR_f^{-1}\left\{pR_f^{-1}E[R_e]\left[1 + C_0'(FR_f^{-1} - H_0)\right] - (1-p)R_f^{-1}C_T'(0)E^Q[R_e^+]\right\}\end{aligned}$$

If the cost function is in quadratic form, $C_{0,T}'(0) = 0$, and $G'(0) > 0$; otherwise $V'(0) > 0$ if and only if $C_T'(0) < \frac{pR_f^{-1}E[R_e]\left[1+C_0'(FR_f^{-1}-H_0)\right]}{(1-p)R_f^{-1}E^Q[R_e^+]} \equiv C$.

Given the sign of $G'(0)$, $G'(w_e)$, and $G'(\hat{w}_e)$, we can conclude that there exists global maximization solution $w^* \in (0, w_e)$ such that $G'(w^*) = 0$. □

Proof of Corollary I

Proof. The first pension payment schedule implies a positive bankruptcy risk and that the firm takes all the surplus if any.

The firm's objective function $G(w, \delta)$ becomes,

$$\begin{aligned}G(w,\delta) = &- W_0(\delta) - C_0\left((W_0(\delta) - H_0)^+\right) + pR_f^{-1}E^Q\left[(W_T - F)^+\right] \\ &+ (1-p)R_f^{-1}E^Q\left[W_T(w,\delta) - F - C_T\left((F - W_T(w,\delta))^+\right)\right]\end{aligned}$$

Employee's participation constraint becomes,

$$\mathbb{U} = (1-p)U(F) + pEU(F + (W_T(w,\delta) - F)^-) \equiv EU(w,\delta)$$

The firm needs to maximize the following Lagrange function $\mathbb{L} = G(w,\delta) + \lambda(EU(w,\delta) - \mathbb{U})$. The first-order derivative w.r.t. w is

$$\frac{\partial \mathbb{L}}{\partial w} = \frac{\partial G(w,\delta)}{\partial w} + \lambda\frac{\partial EU(w,\delta)}{\partial w}$$

Look at the first term, the marginal utility of firm,

$$\frac{\partial G(w,\delta)}{\partial w} = R_f^{-1}\int_{-\infty}^{R_m^c} W_0R_e\left[C_T'(F - W_0R(w))(1-p) - p\right]f^Q(R_m)dR_m$$

where $R_m^c = \left(\frac{F}{W_0} - R_f\right)\frac{1}{w} + R_f$.

At $w = 0$, $R_m^c \to +\infty$ if underfunded; $R_m^c \to -\infty$ if overfunded, either way we have

$$\frac{\partial G(0,\delta)}{\partial w} = 0$$

Now look at the second term, the marginal utility of employee,

$$\frac{\partial EU}{\partial w} = p\int_{-\infty}^{R_m^c} U'(W_0R(w))W_0R_e f(R_m)dR_m$$

If pension is underfunded, $W_0 R_f < F$, $R_m^c > R_f$,

$$\frac{\partial EU(0,\delta)}{\partial w} = p\int_{-\infty}^{\infty} U'(W_0R_f)W_0R_e f(R_m)dR_m = pU'(W_0R_f)E(R_m - R_f) > 0$$

Therefore, when pension is underfunded,

$$\left.\frac{\partial \mathbb{L}}{\partial w}\right|_{w=0} = \frac{\partial G(0,\delta)}{\partial w} + \lambda \frac{\partial EU(0,\delta)}{\partial w} > 0$$

The second pension payment schedule implies no bankruptcy risk and that the firm and the employee shares the surplus if any. The firm's objective function $G(w,\delta)$ becomes

$$G(w,\delta) = -W_0(\delta) - C_0\left((W_0(\delta) - H_0)^+\right) + R_f^{-1}E^Q\left[(W_T - F)^+\alpha + (W_T - F)^- - C_T((F - W_T)^+)\right]$$

Employee's participation constraint becomes

$$\mathbb{U} = EU\left(F + (W_T - F)^+(1-\alpha)\right)$$

Following the same procedure as above,

$$\frac{\partial G}{\partial w} = R_f^{-1}\int_{-\infty}^{R_m^c} W_0R_e\left[(1-\alpha) + C_T'(F - W_T)\right] f^Q(R_m)dR_m$$

where $R_m^c = \left(\frac{F}{W_0} - R_f\right)\frac{1}{w} + R_f$.

At $w = 0$, $R_m^c \to +\infty$ if underfunded; $R_m^c \to -\infty$ if overfunded, either way we have

$$\frac{\partial G(0,\delta)}{\partial w} = 0$$

Similarly,

$$\frac{\partial EU}{\partial w} = \int_{R_m^c}^{+\infty} U'(F\alpha + W_T(1-\alpha))(1-\alpha)W_0R_e f(R_m)dR_m$$

If fully funded, $R_m^c = R_f$,

$$\frac{\partial EU(0,1)}{\partial w} = \int_{R_f}^{+\infty} U'(F\alpha + W_0R_f(1-\alpha))(1-\alpha)W_0R_e f(R_m)dR_m > 0$$

If overfunded, $R_m^c \to -\infty$,

$$\frac{\partial EU(0,\delta)}{\partial w} = \int_{-\infty}^{+\infty} U'(F\alpha + W_0R_f(1-\alpha))(1-\alpha)W_0R_e f(R_m)dR_m > 0$$

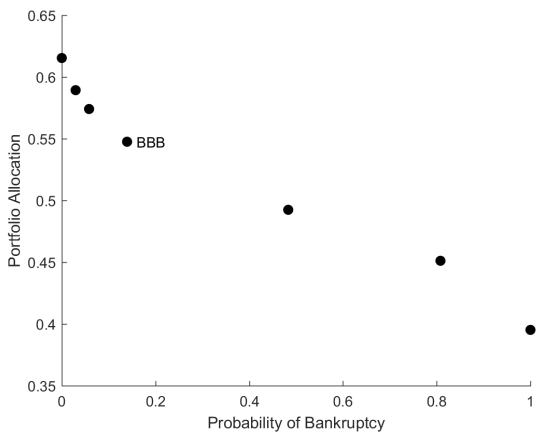
Therefore, when pension is not underfunded,

$$\left.\frac{\partial \mathbb{L}}{\partial w}\right|_{w=0} = \frac{\partial G(0,\delta)}{\partial w} + \lambda \frac{\partial EU(0,\delta)}{\partial w} > 0$$

□

Figure 1 Impact of Bankruptcy Probability on Pension Investment and Pension Funding Decisions

This figure illustrates the impact of bankruptcy probability on pension investment and funding decisions. Panel (1) shows the pension allocation to the risky asset. Panel (2) shows the optimal pension PBO and optimal pension contribution. The bankruptcy probabilities are calibrated to those for firms with credit ratings of AAA, AA, A, BBB, BB, and B, respectively.



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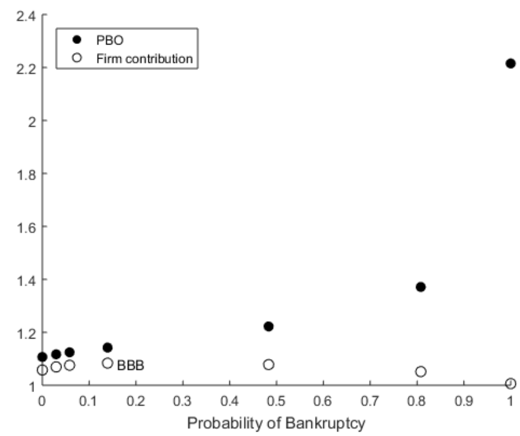

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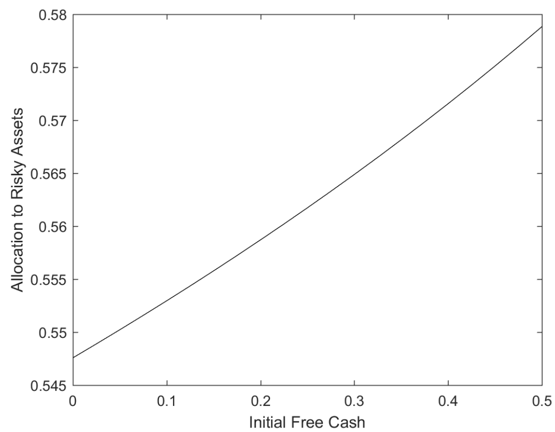
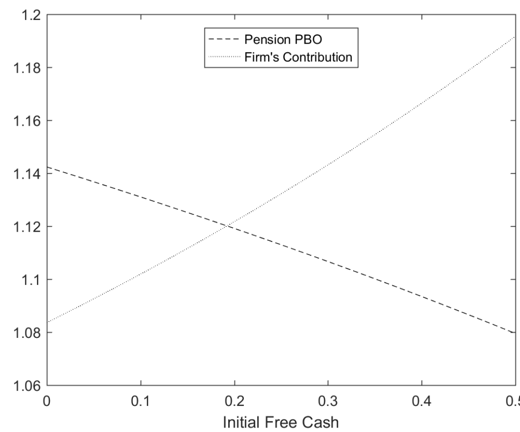
Figure 2 Impact of Additional Factors on Pension Investment and Pension Funding Decisions

This figure illustrates the impact of an additional set of factors on pension investment and funding decision. Panel A shows the impact of firm's initial free cash on pension allocation to risky assets (A1) and optimal PBO and the firm's pension contribution (A2). Panel B shows the impact of pension surplus sharing between the firm and the employee. Panel C shows the impact of the linear component of financing cost. Panel D shows the impact of the quadratic component of financing cost. Panel E shows the impact of risk free rate. Panel F shows the impact of the risk premium of the risky asset. Panel G shows the impact of the volatility of the risky asset.

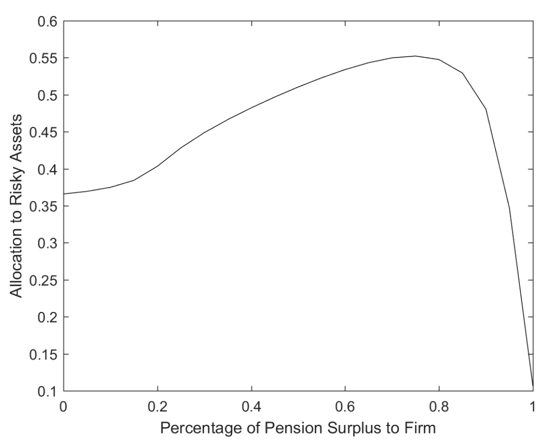
A.1 Impact of Initial Free Cash on Allocation



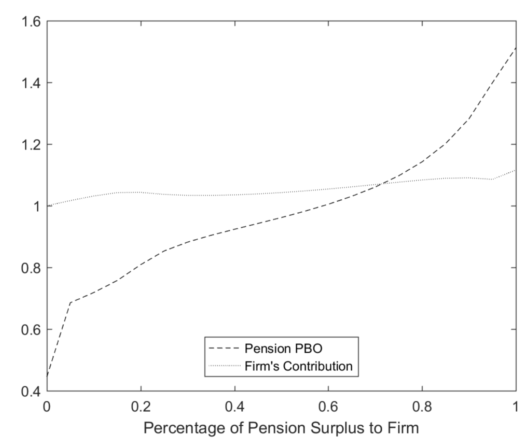
A.2 Impact of Initial Free Cash on PBO and Contribution



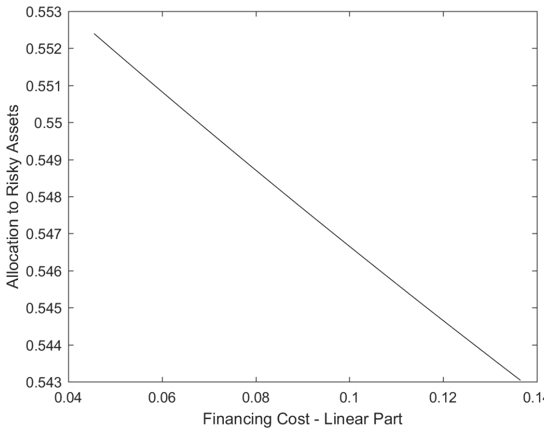
B.1 Impact of Surplus Sharing on Allocation



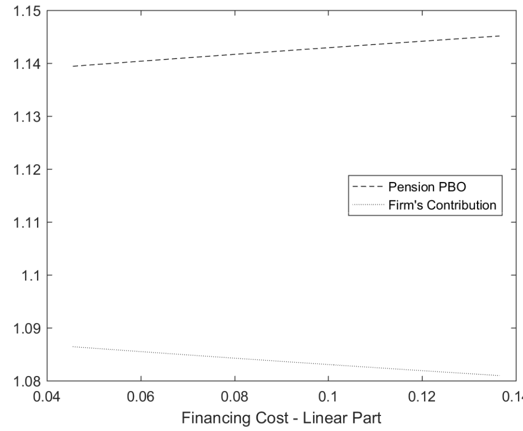
B.2 Impact of Surplus Sharing on PBO and Contribution



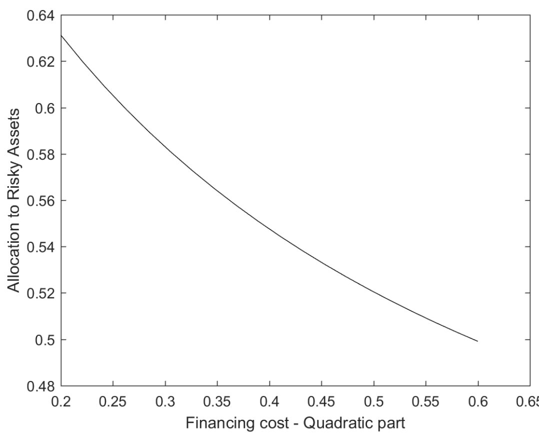
C.1 Impact of Linear Part of Financing Cost on Allocation



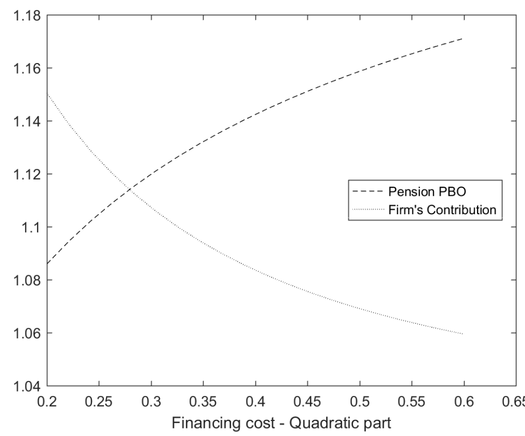
C.2 Impact of Linear Part of Financing Cost on PBO and Contribution



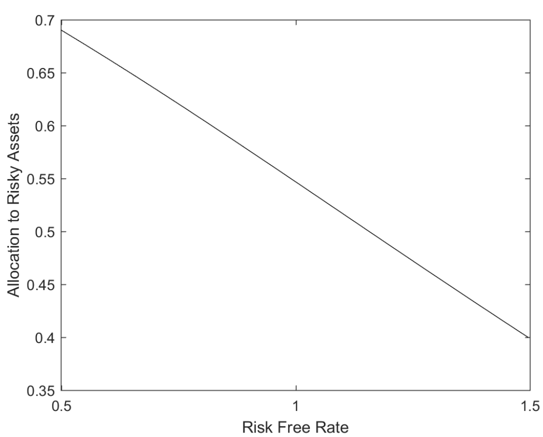
D.1 Impact of Quadratic Part of Financing Cost on Allocation



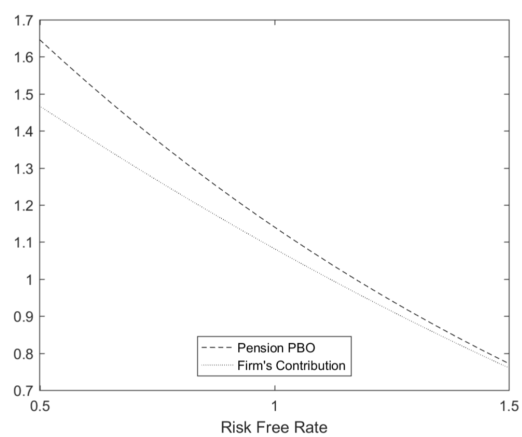
D.2 Impact of Quadratic Part of Financing Cost on PBO and Contribution



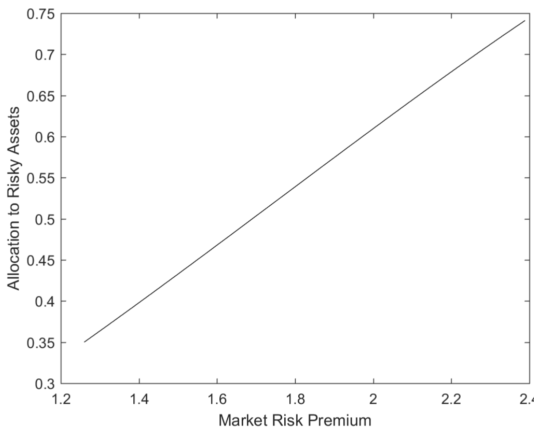
E.1 Impact of Riskfree Rate on Allocation



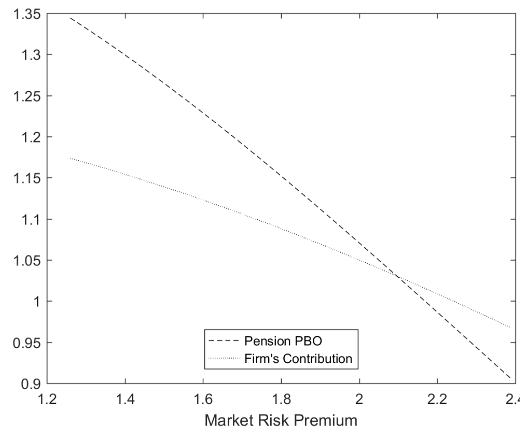
E.2 Impact of Riskfree Rate on PBO and Contribution



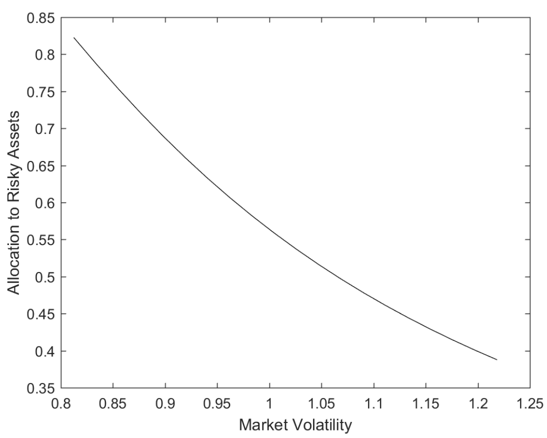
F.1 Impact of Risk Premium on Allocation



F.2 Impact of Risk Premium on PBO and Contribution



G.1 Impact of Volatility on Allocation



G.2 Impact of Volatility on PBO and Contribution

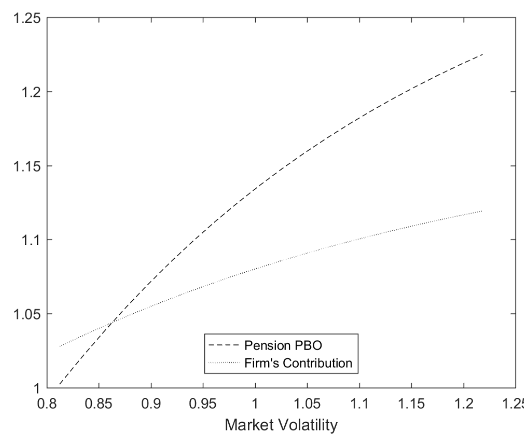


Figure 3 Relation Between Optimal Investment and Optimal Funding Ratio

This figure illustrates the relation between optimal pension investment and optimal funding ratio under varying parameter values. Plot (1) shows the relation under various parameter values of bankruptcy probability, initial free cash, and surplus sharing. Plot (2) shows the relationship under varying values of time 0 linear financing cost and time T linear financing cost.

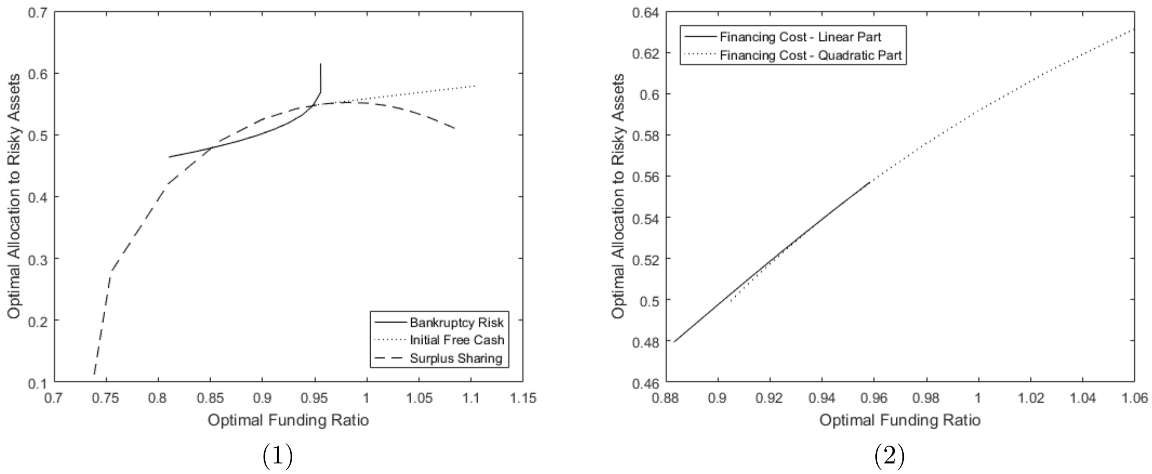
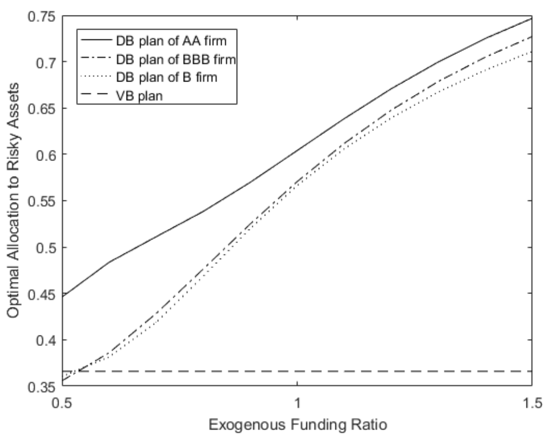


Figure 4 Pension Investment and Pension Funding Cost Under Exogenous Funding Ratios

This figure illustrates pension investment and funding cost under exogenous funding ratios. The pension funding cost is the firm's cash outflow at time 0 plus the present value of expected cash outflow at time T. Panel A shows pension investment and funding cost with varying exogenous funding ratios where the bankruptcy probabilities are calibrated to those of AA-rated firms, BBB-rated firms, and B-rated firms. Panel B shows the impact of varying surplus sharing schemes, where employees receive 30% ($\alpha = 0.7$), 20%($\alpha = 0.8$), and 10%($\alpha = 0.9$) of pension surplus. For comparison, both panels include pension investment and pension funding cost of variable benefit plans, in which the employee receives 100% of pension assets and no firm guarantee on pension benefits (essentially a defined contribution plan).

Panel A Bankruptcy Risk



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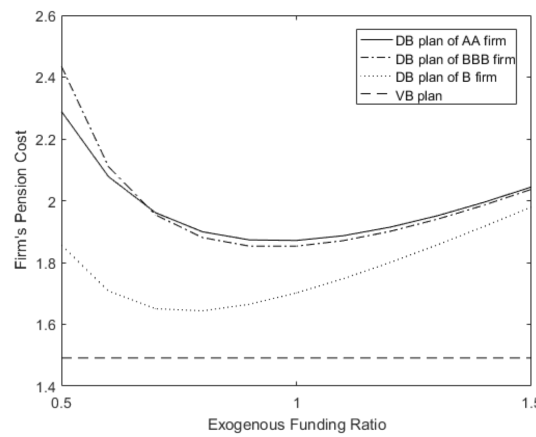

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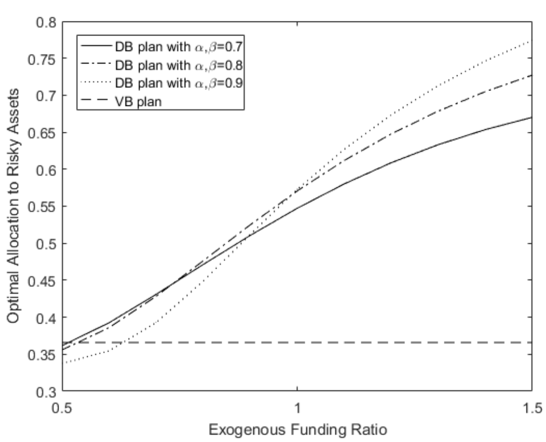
Panel B Surplus Sharing



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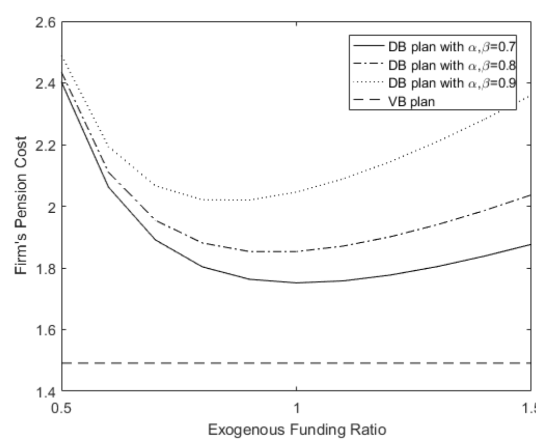
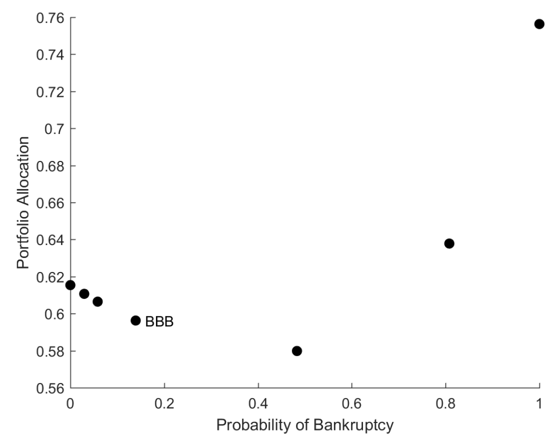


(2)

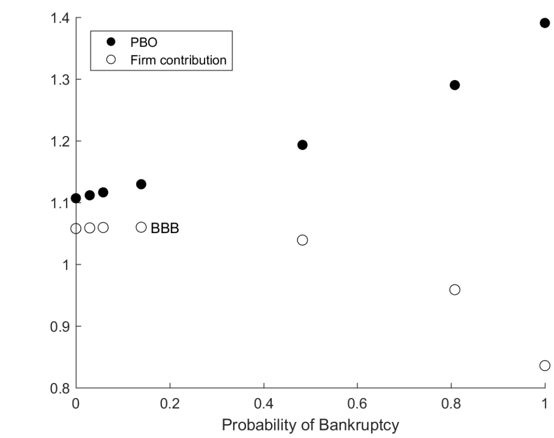
Figure 5 Pension Investment and Pension Funding Decisions with PBGC Pension Insurance

This figure illustrates the impact of various factors on pension investment and funding decisions under PBGC pension insurance coverage. Panel A shows the impact of bankruptcy probability on allocation to risky assets (Panel A1), and the optimal PBO and pension contribution (Panel A2). Panel B shows the impact of firm's initial free cash. Panel C shows the impact of pension surplus sharing between the firm and the employee. Panel D shows the impact of PBGC's linear component of financing cost. Panel E shows the impact of underpriced and overpriced PBGC pension insurance. Panel F shows the impact of PBGC pension insurance coverage.

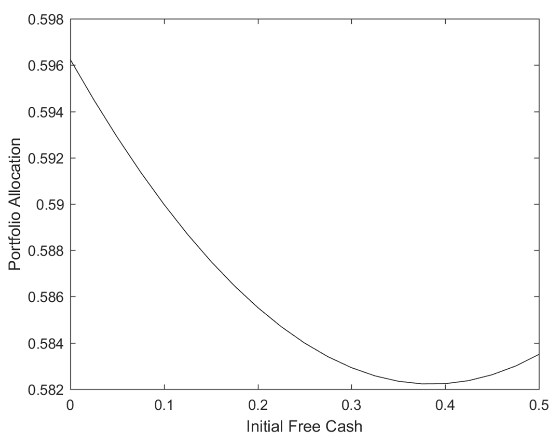
A.1 Impact of Bankruptcy Risk on Allocation



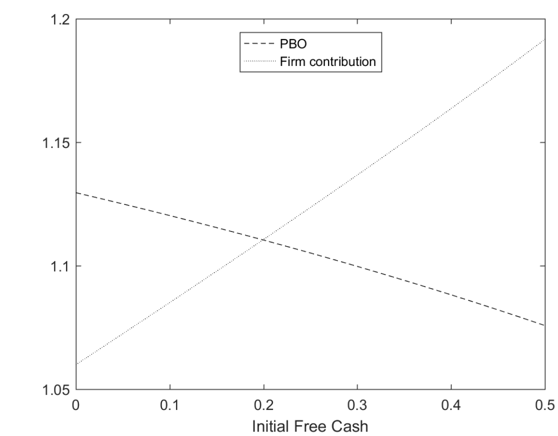
A.2 Impact of Bankruptcy Risk on PBO and Contribution



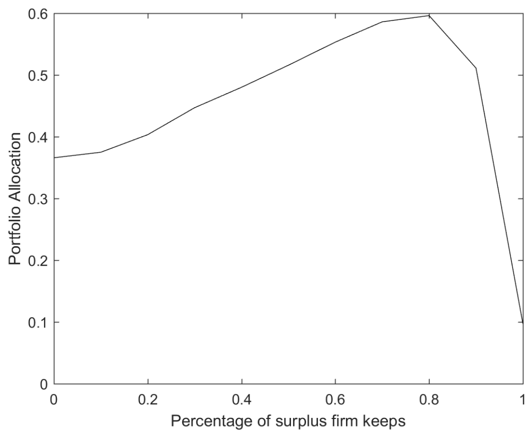
B.1 Impact of Initial Free Cash on Allocation



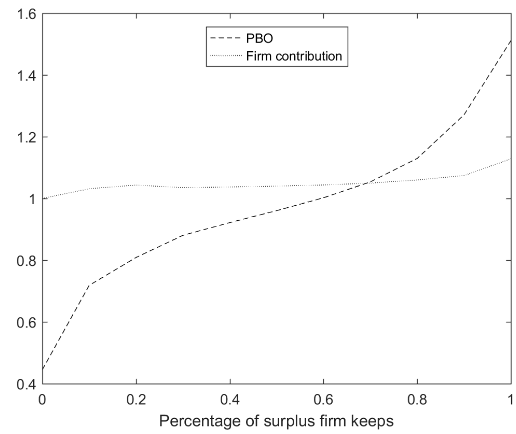
B.2 Impact of Initial Free Cash on PBO and Contribution



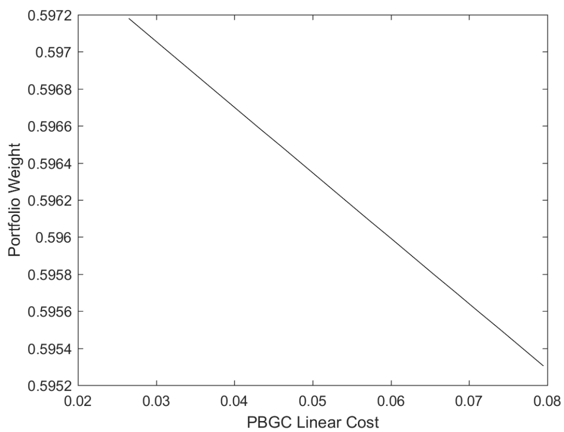
C.1 Impact of Surplus Sharing on Allocation



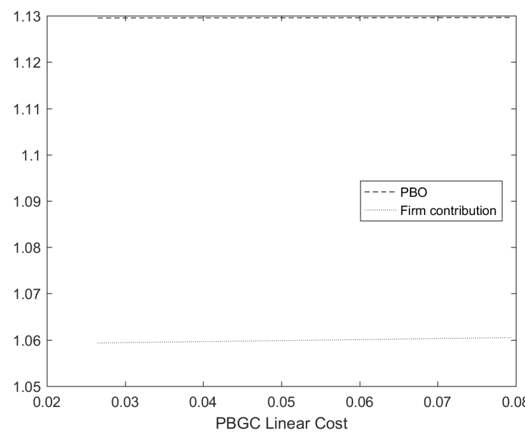
C.2 Impact of Surplus Sharing on PBO and Contribution



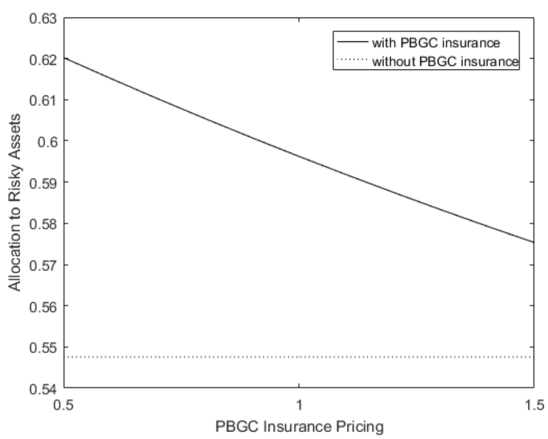
D.1 Impact of PBGC Linear Financing Cost on Allocation



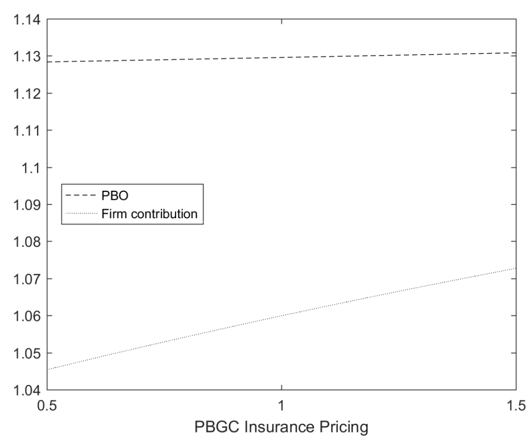
D.2 Impact of PBGC Linear Financing Cost on PBO and Contribution



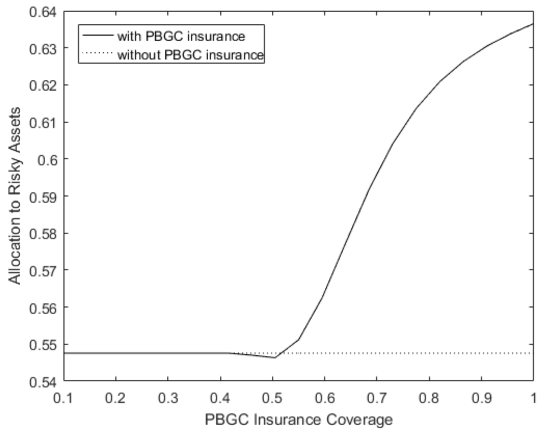
E.1 Impact of PBGC Insurance Pricing on Allocation



E.2 Impact of PBGC Insurance Pricing on PBO and Contribution



F.1 Impact of PBGC Insurance Coverage on Allocation



F.2 Impact of PBGC Insurance Coverage on PBO and Contribution

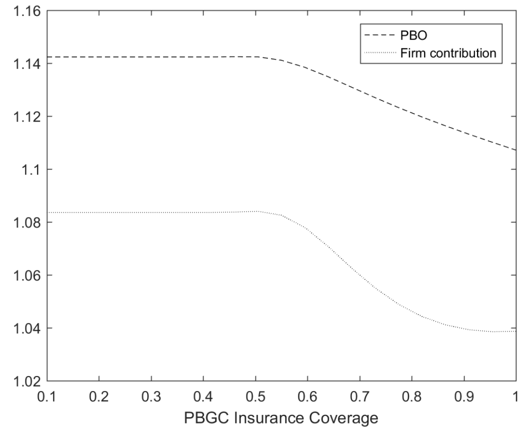
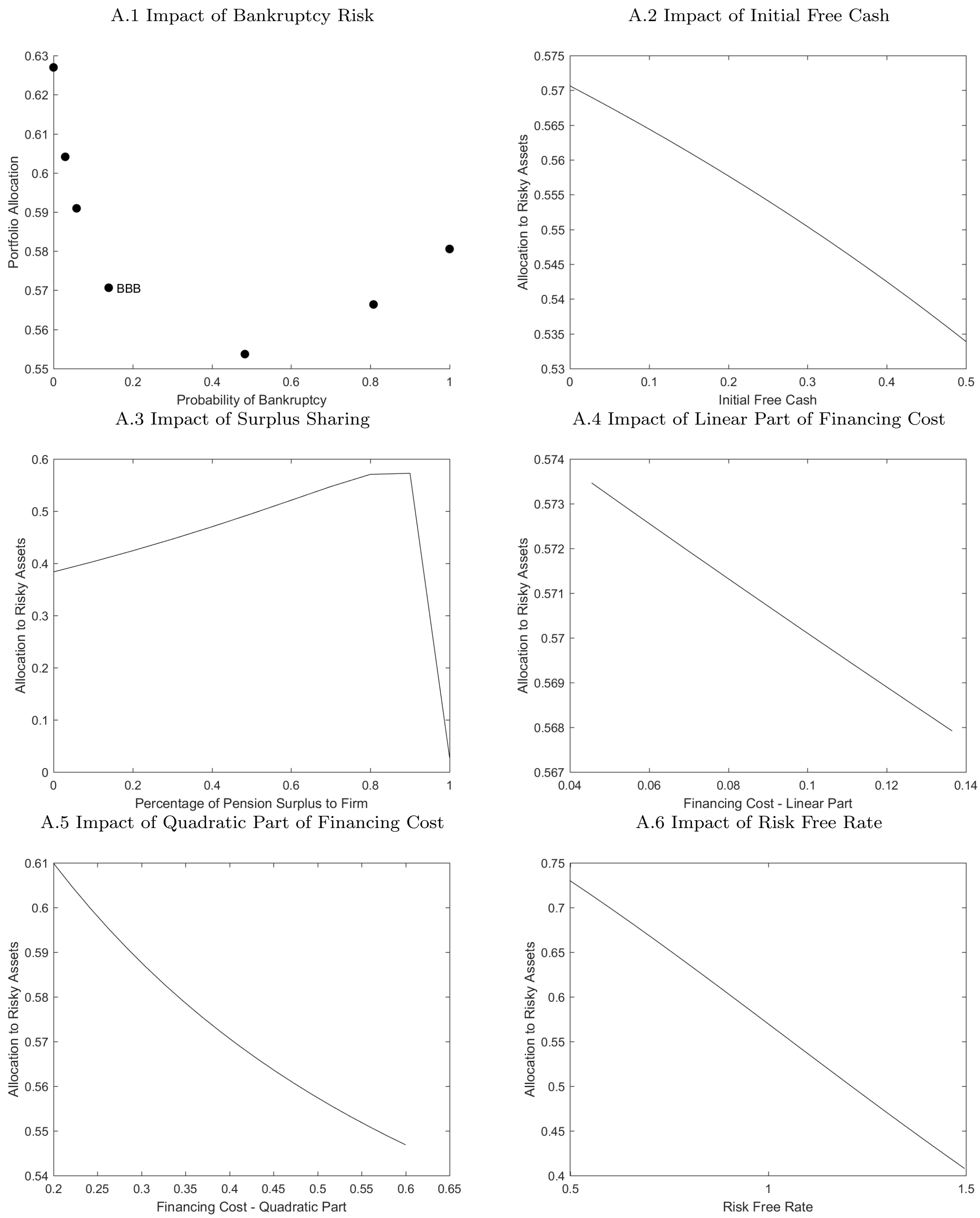
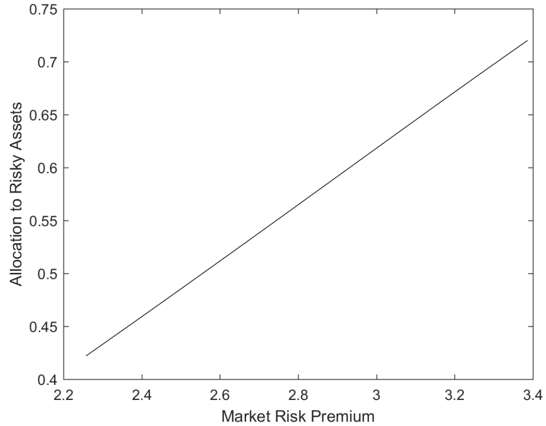


Figure 6 Investment Decisions by Initially Fully-funded Pensions

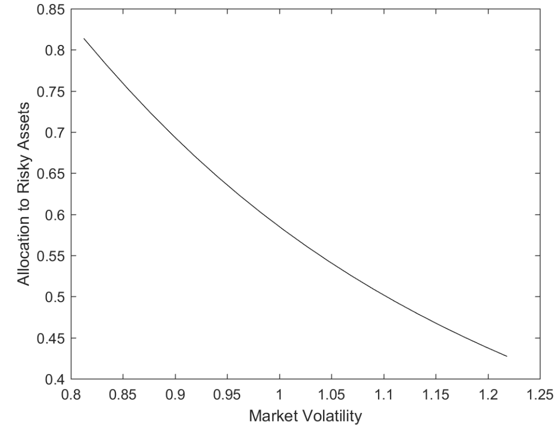
This figure illustrates the impact of various factors on pension asset application when a pension is initially fully-funded.



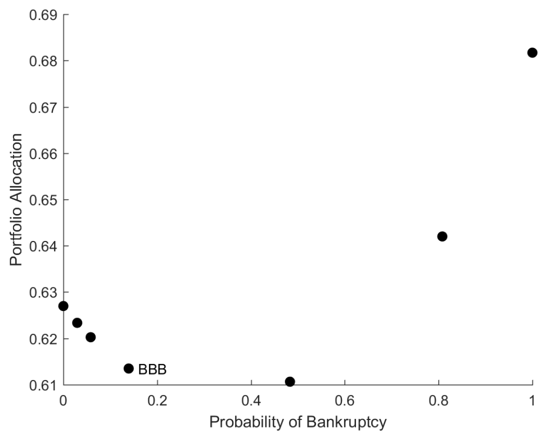
A.7 Impact of Market Risk Premium



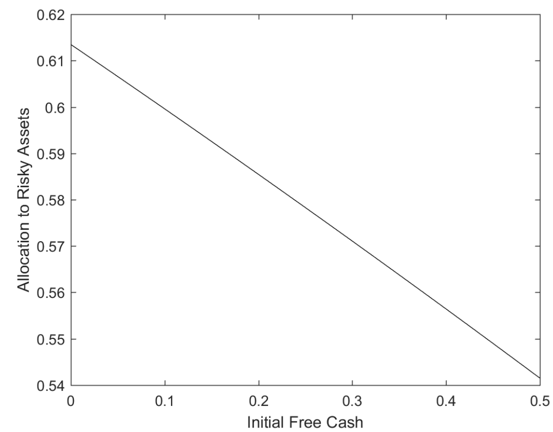
A.8 Impact of Market Volatility



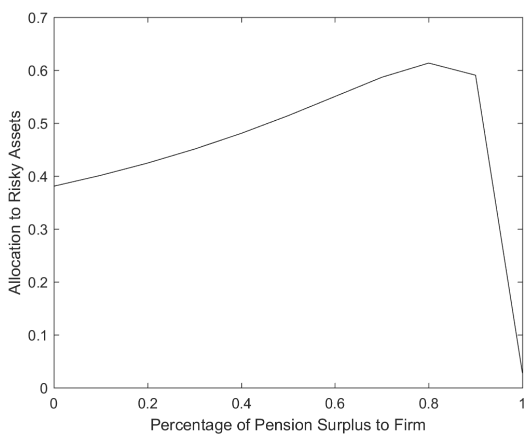
B.1 Impact of Bankruptcy Risk



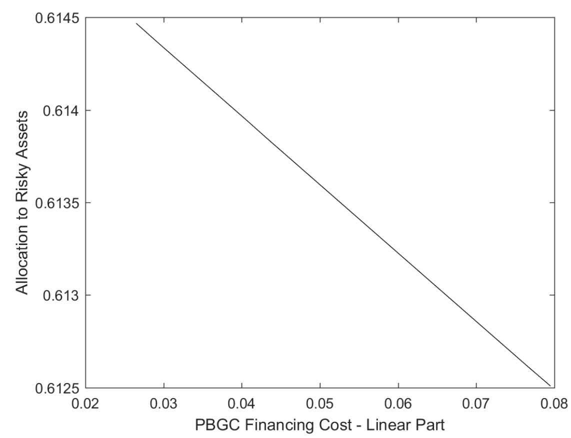
B.2 Impact of Initial Free Cash



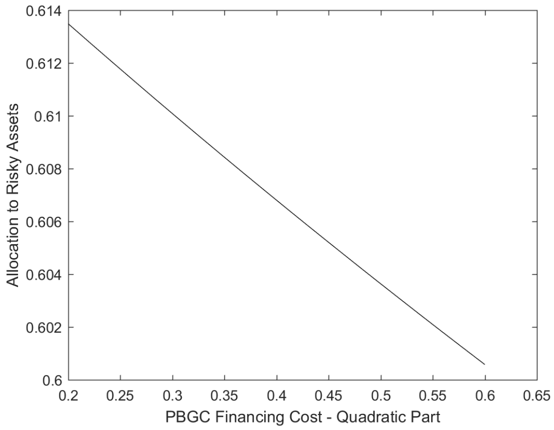
B.3 Impact of Surplus Sharing



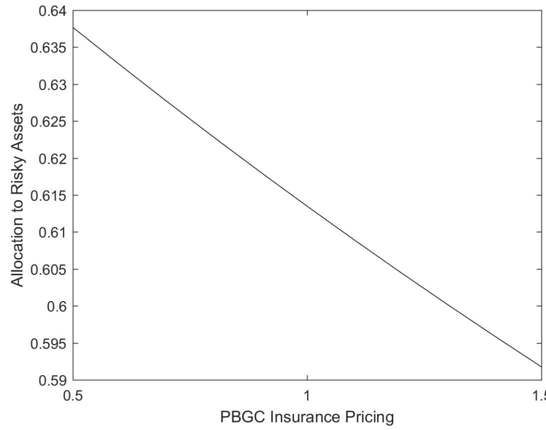
B.4 Impact of Linear Part of Financing Cost



B.5 Impact of Quadratic Part of Financing Cost



B.6 Impact of PBGC Insurance Pricing



B.7 Impact of PBGC Insurance Coverage

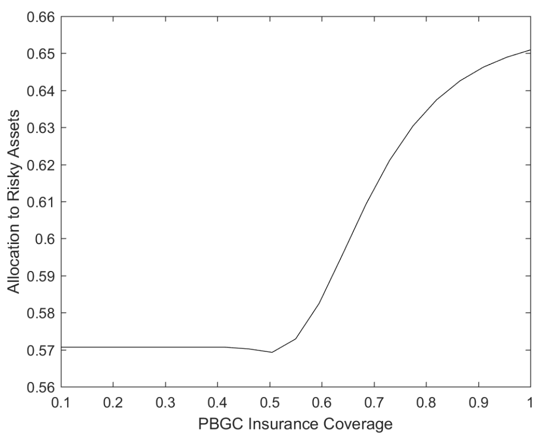


Figure 7 Pension Investment and Funding Decisions Under Surplus Reversion Tax

This figure illustrates pension investment and plan funding ratio under different bankruptcy probabilities (Panel A), different pension surplus sharing schemes (Panel B), and different tax rates on surplus reversion (Panel C).

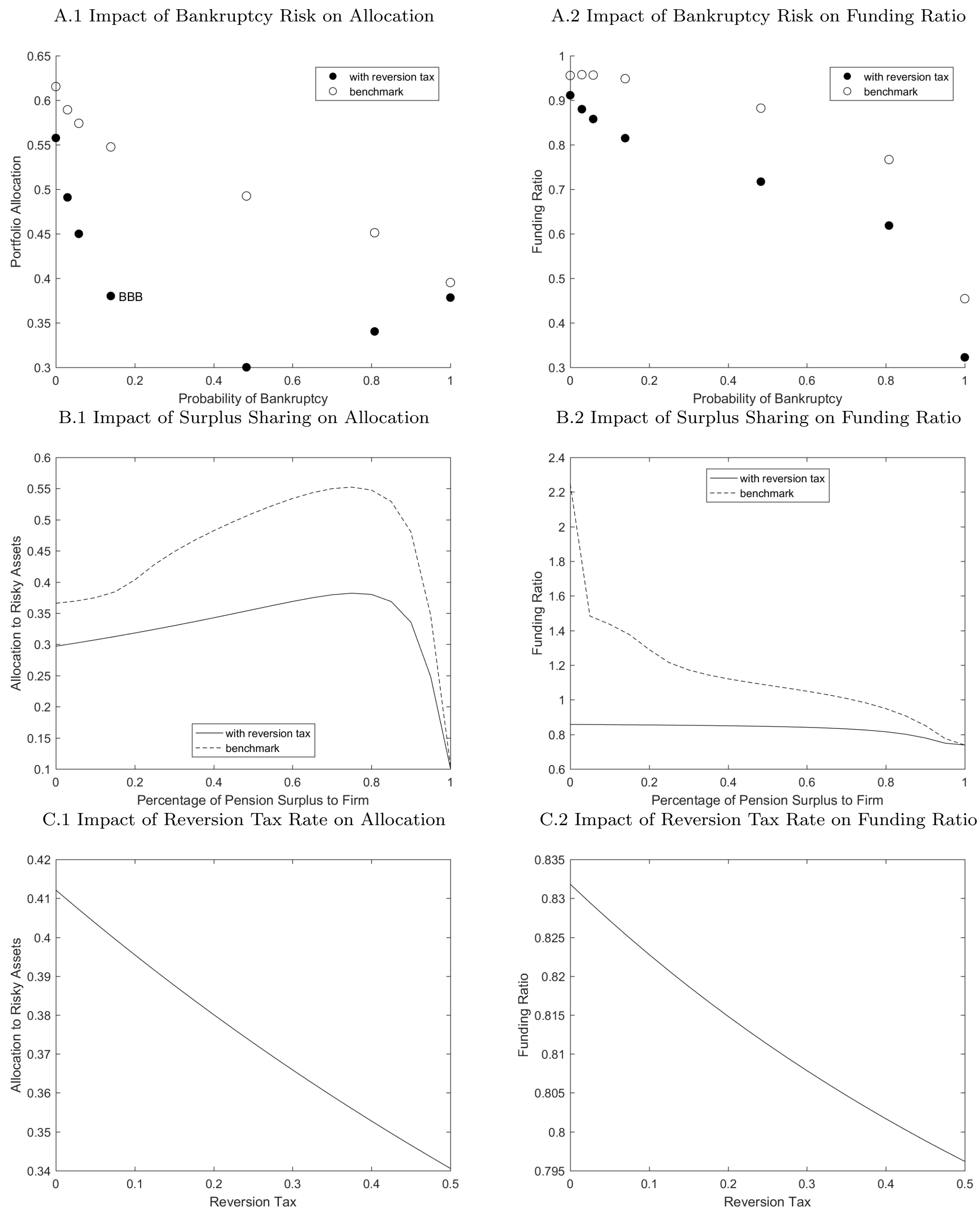
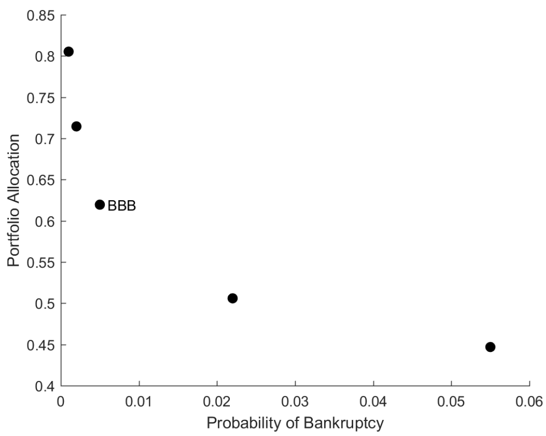


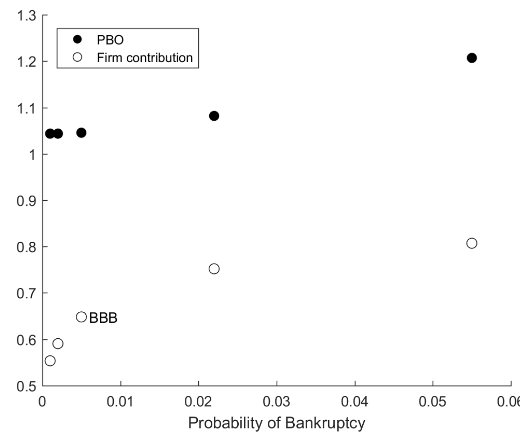
Figure 8 Initial Investment and Initial Cash Contribution Decisions in the Dynamic Model

This figure illustrates the firm's asset allocation and cash contribution at time 0 in the dynamic model. Panel A shows the optimal allocations and cash contributions at time 0 under different bankruptcy probabilities. Panel B shows the optimal allocation and cash contribution at time 0 under different amounts of initial free cash. Panel C shows the optimal allocation and cash contribution at time 0 under different surplus sharing ratios.

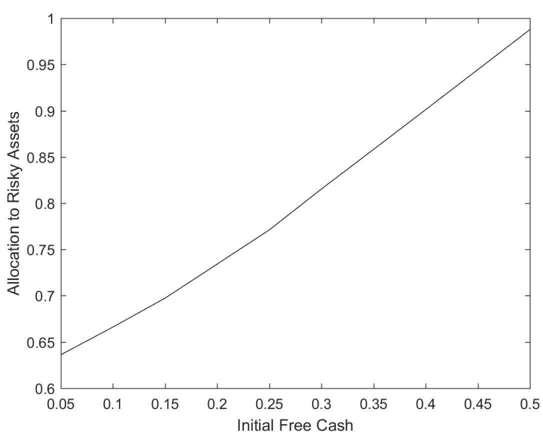
A.1 Impact of Bankruptcy Risk on Allocation



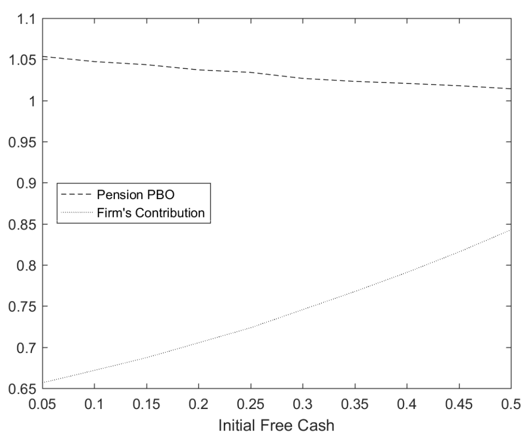
A.2 Impact of Bankruptcy Risk on PBO and Contribution



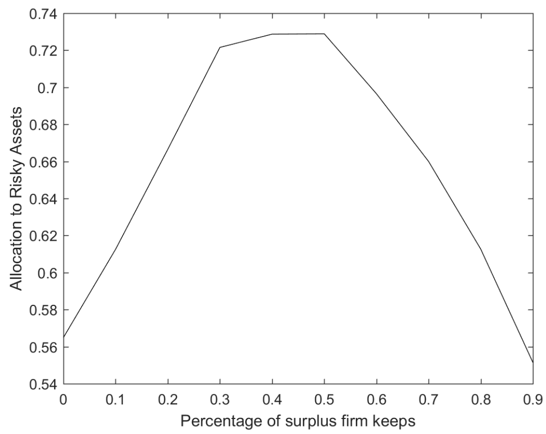
B.1 Impact of Initial Free Cash on Allocation



B.2 Impact of Initial Free Cash on PBO and Contribution



C.1 Impact of Surplus Sharing on Allocation



C.2 Impact of Surplus Sharing on PBO and Contribution

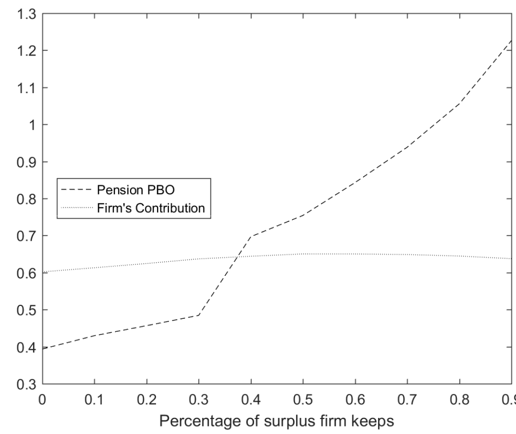
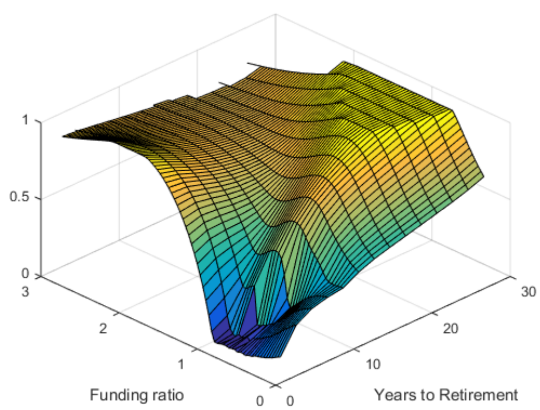


Figure 9 Optimal Investment and Pension Funding Decisions Over Time

This figure illustrates pension investment and pension contribution decisions across time and states (funding ratio before cash contribution). Panel A shows the optimal allocations and cash contributions under the benchmark parameters, where the bankruptcy probability is calibrated to that of a BBB-rated firm and the firm keeps 80% of pension surplus. Panel B shows the optimal allocations and cash contributions under a lower probability of bankruptcy (for an AAA-rated firm). Panel C shows the optimal allocations and cash contributions when the firm keeps a lower (60%) fraction of the pension surplus.

Panel A Benchmark



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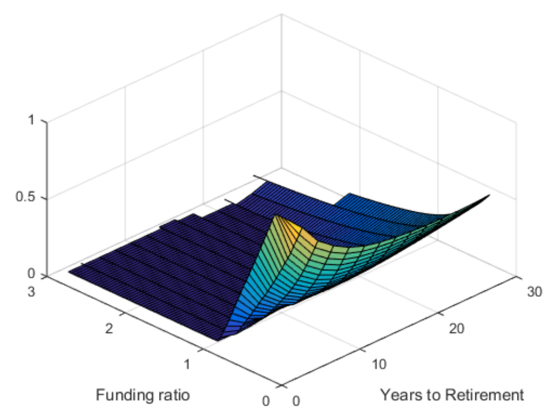

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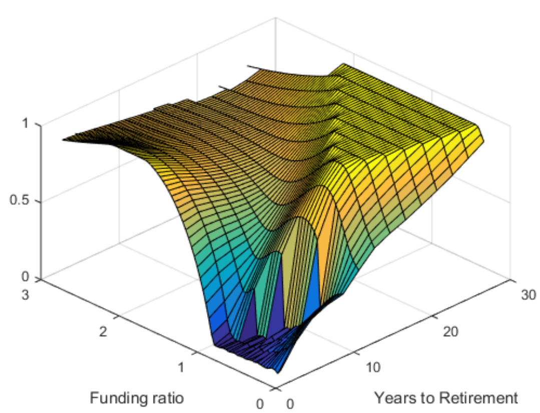
Panel B Lower Bankruptcy Probability (AAA-rated firm)



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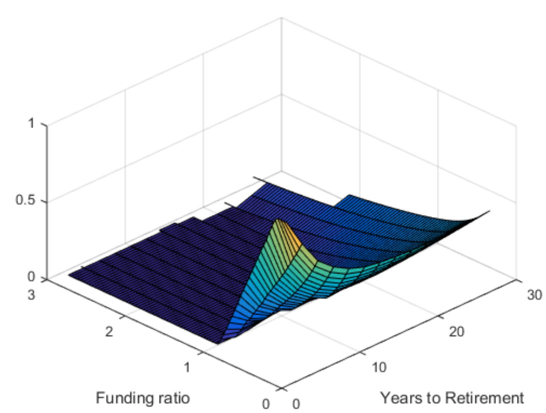
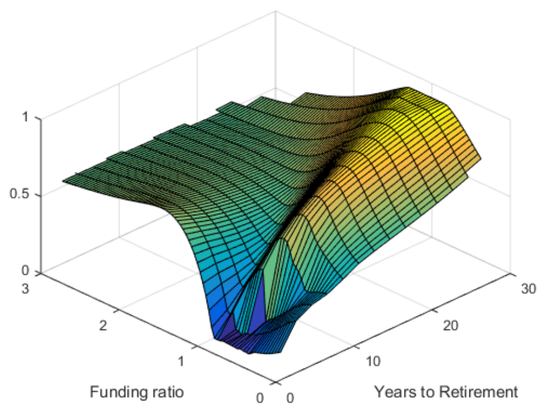


(2)

Panel C More Pension Surplus to Employee (firm keeping 60%)



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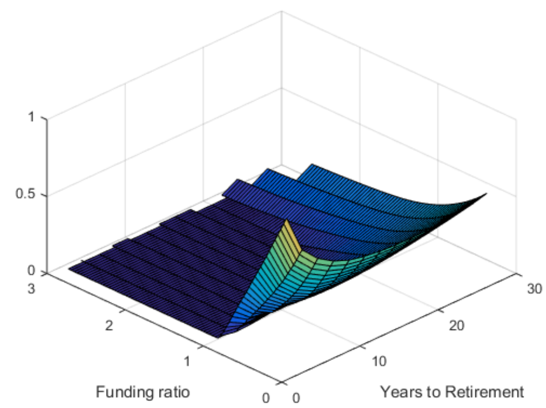

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