Math 21 Final Exam December 16, 2005

Name _____

Section _____

Grading

1. _____	9. _____
2. _____	10. _____
3. _____	11. _____
4. _____	12. _____
5. _____	13. _____
6. _____	14. _____
7. _____	15. _____
8. _____	16. _____

Total _____

This is a closed book exam. No books, notes or other aids may be used.

Cell phones must be off. Cell phones, calculators, books, notes etc that are out during the exam can be considered cheating.

Unless otherwise indicated, explain your answers for full credit. You will not get full credit for a problem without appropriate explanation of your work. A correct answer alone is not enough.

Point values for the problems are indicated as [*]

1: [12] Determine the following limits. You may use any method you like, but be clear to indicate what you are doing.

(a) $\lim_{x \to -8} \dfrac{\sqrt{x+12}-2}{x+8}$

(b) $\lim_{x \to 2} \dfrac{x^2+3x-10}{x-2}$

(c) $\lim_{x \to 0} \dfrac{e^x-1-x}{x^2}$

2: [7] If $f''(x) = 13$ with $f(1) = 1$ and $f'(1) = 0$ what is $f(x)$?

3: [16] Determine the following derivatives:

(a) $\dfrac{d}{dx}\left(xe^{x}\right)$

(b) $\dfrac{d}{dx}\left(xe^{x}\ln x\right)$

(c) $\dfrac{d}{dx}\left(\ln\left(\sin(x^{5})\right)\right)$

(d) $\dfrac{d}{d\theta}\left(\dfrac{\mu W}{\mu\sin\theta+\cos\theta}\right)$

4: [15] Evaluate the following definite integrals:

(a) $\int_{-7}^{9} 13 \, dx$

(b) $\int_{0}^{h} \left(-\frac{b}{h} y + b \right) dy$

(c) $\int_{1}^{e} \frac{lnx}{x} \, dx$

5: [12] (a) Find y' if $xy^2 + 3y = 13$.

(b) Find y' if $y = x^{\ln x}$

6: [15] Evaluate the following integrals:

(a) $\int \left(\sqrt[5]{x^3} - \cos x - \frac{1}{x} + \frac{1}{x^2} \right) dx$

(b) $\int \left(x^3 + 5x\right) \left(x^4 + 10x^2\right)^{17} dx.$

(c) $\int \frac{e^x}{1 + e^x} dx$

7: [8] Find an equation for the tangent line to $f(x) = x^3 + 2x$ at the point $(2, 12)$.

8: [8,8,4] (a) Show that $\cos\left(\tan^{-1} x\right) = \dfrac{1}{\sqrt{1+x^2}}$. (Thus, if $y = \tan^{-1} x$ then $\sec y = \sqrt{1+x^2}$.

(b) Show that $\dfrac{d}{dx}\left(\tan^{-1} x\right) = \dfrac{1}{1+x^2}$. (You may use the result from part (a) even if you have not answered it.)

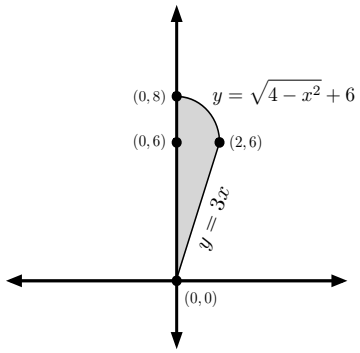
(c) Determine $\displaystyle\int \frac{e^{2x}}{1+e^{2x}}\,dx$. (You may use the result of part (b) even if you have not answered it.)

9: [9] A closed box is to be made with volume Vm^3. If its base has length equal to twice its width, what dimensions will minimize the amount of material used? Be sure to justify that your answer is indeed minimum.

10: [9] A manager of a 100 unit apartment complex knows from experience that all units will be occupied if the rent is \$800 per month. A market survey suggests that, on average, one additional unit will remain vacant for each \$10 increase in rent. What rent should the manager charge to maximize revenue?

11: [18] If $f(x) = x^4 + 4x^3$ then $f'(x) = 4x^3 + 12x^2$ and $f''(x) = 12x^2 + 24x$.
(a) On which interval(s) is $f(x)$ decreasing?
(b) What are the critical numbers?
(c) For each critical number does it correspond to a local maximum or local minimum or neither? In each case justify your answer with the first or second derivative test.
(d) What are the inflection points?
(e) On which interval(s) is is $f(x)$ concave up?
(f) Sketch $f(x)$.

12: [15] Consider the region shown in the figure: it is bounded by the y-axis, the line $y = 3x$ and a portion of $y = 6 + \sqrt{4 - x^2}$ (one quarter of a circle of radius 2 centered at $(0, 6)$). For each of the problems below, you do not need to evaluate the integrals, just write them down.



(a) Write down two integrals, one integrating with respect to x (using vertical rectangles) and one integrating with respect to y (using horizontal rectangles) which give the area of the region.
(b) Write down two integrals, one using the method of washers and one using the method of shells which give the volume of the solid obtained by rotating this region about the y-axis.
(c) Write down two integrals, one using the method of washers and one using the method of shells which give the volume of the solid obtained by rotating this region about the line $x = -13$.

13: [9] Prove, using the δ, ϵ definition of limits that $\lim_{x \to -2}(x^2 + 3x + 5) = 3$. You must write this clearly including appropriate explanations in words.

14: [9] Prove, using the definition of derivative that if $f(x) = 3x^2$ then $f'(x) = 6x$

15: [7,3,5,4] (a) Find $\lim_{n\to\infty} \frac{10}{n}\left(7+\frac{5i}{n}\right)^2$ directly using $\sum_{i=1}^{n} i = \frac{n^2+n}{2}$ and $\sum_{i=1}^{n} i^2 = \frac{2n^3+3n^2+n}{6}$.

(b) Describe an area (under a curve) which is equal to the expression in part (a).

(c) Give two different definite integrals equal to the expression in part (a).

(d) Use the fundamental theorem of calculus applied to one of the integrals in part (c) to check your answer to part (a).

16: [7] The volume obtained by rotating $y = \sqrt{r^2 - x^2}$ about the x-axis is a sphere of radius r. Use calculus to show that this volume is $\frac{4}{3}\pi r^3$. (That is, set up and evaluate an appropriate integral.)