MATH 21, FALL, 2009, MAKE-UP EXAM # 2

Name:_____.

ID #_____.

Section #_____.

Instructor_____.

TA_____.

Instructions: All cellphones, calculators, computers, translating devices, and music players must be turned off.

Do all work on the test paper. ***Show all work***. You may receive no credit, even for a correct answer, if no work is shown. You may use the back if you need extra space. ***Do not*** simplify your answers, unless you are explicitly instructed to do so. This does not apply to evaluation of elementary functions at standard values or functional expressions, so that you **would** be expected to simplify $\sin(\pi/6)$ to $\frac{1}{2}$, for example. Do not write answers as decimal approximations; if $\sqrt{2}$ is the answer, leave it that way. *Except where explicitly stated otherwise, you can use the derivative rules and limit rules learned in class.*

You may **not** use a calculator, computer, the assistance of any other students, any notes, crib sheets, or texts during this exam. All cellphones must be switched off and out of sight. No music players, translators, or other electronic devices allowed

You have 60 minutes to complete this exam.

Do not turn to the next page until you are instructed to do so.

Grading:

1._____/15

2._____/15

3._____/10

4._____/10

5._____/10

6._____/10

7._____/10

8._____/20

Total._____/100

(1) Find the indicated derivatives. *(5 points/part)*

(a) $(\ln(2x+3))' =$

(b) $\left(x^2 \sinh(x)\right)' =$

(c) $\left(\tan^{-1}(x^2)\right)' =$ *(remember,* $\tan^{-1}(x)$ *is the inverse tangent of* x*, or* $\arctan(x)$*)*

(2) Find the following limits. Justify each step. *(5 points/part)*

(a) $\lim_{x\to 0} \dfrac{\sin(3x)}{\sinh(5x)} =$

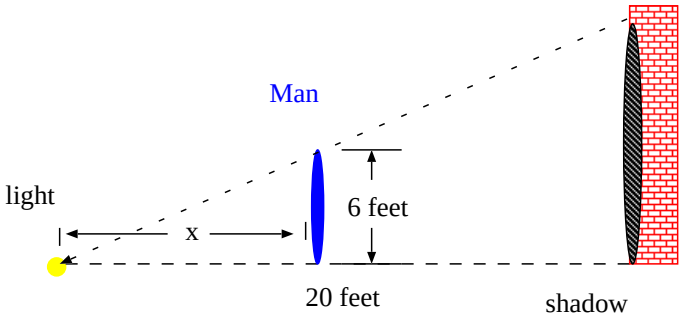
(b) $\lim_{x\to\infty} x\left(e^{1/x}-1\right) =$

(c) $\lim_{x\to 0} \dfrac{2\cos(x)-2+x^2}{x^4} =$

(3) Find an equation of the tangent line to the graph of $x^3 - xy - y^2 + 5 = 0$ at $(1, 2)$. *(10 points)*

(4) Inspector Clousseau has found a murder victim in the Paris Metro. When he first discovered the corpse, he measured the temperature of the body as 32 degrees Celsius. One hour later he measured it again to find that the temperature had dropped to 28 degrees Celsius. If the deceased had a slightly elevated body temperature of 38 degrees Celsius when he met his untimely end (he had *la grippe*, as Clousseau determined by the quantity of used tissues and cough drops found on the body), find how long the poor chap had been dead when Clousseau came across him. The Paris Metro maintains a constant temperature of 20 degrees Celsius. Use Newton's law of cooling, which says that the rate of change of temperature of a dead body is proportional to the difference between the temperature of the body and the ambient temperature. *(10 points)*

(5) The side of a tall building is illuminated by a floodlight mounted on the lawn in front of the building. The floodlight is at ground level, and is 20 feet from the building. A man, 6 feet tall, is walking towards the floodlight at 4 feet/sec. How fast is his shadow on the wall growing when he is 10 feet from the floodlight?



(6)

(a) State the Mean Value Theorem. *(5 points)*

(b) Suppose $f(x)$ is differentiable every every real number x and $|f'(x)| \leq 7$ for all x. Show, using the Mean Value Theorem, that

$$|f(x_2) - f(x_1)| \leq 7|x_2 - x_1|$$

for all x_1, x_2. *(5 points)*

(7) Approximate $\sqrt[3]{7.5}$ using differentials (or, equivalently, linear approximation). *(10 points)*

(8) Let $f(x) = \dfrac{e^x}{x-1}$, then *(20 points)*

(a) Find the domain of $f(x)$.

(b) Find all x- and y- intercepts.

(c) Find any horizontal or vertical asymptotes.

Continuing with problem 8. Recall that $f(x) = \dfrac{e^x}{x-1}$

(d) Find where the curve is increasing and decreasing, and find any critical points.

(e) Find on what regions the curve is concave up, and where it is concave down, and find the x-coordinants of any points of inflection (don't try to find the y-coordinates of the inflection points).

Continuing with problem 8. Recall that $f(x) = \dfrac{e^x}{x-1}$

(f) Then, sketch the curve, showing each of these features.