MATH 21, FALL, 2008, MAKE-UP EXAM # 2

Name:_____.

ID #_____.

Section #_____.

Instructor_____.

TA_____.

Instructions: All cellphones, calculators, computers, translating devices, and music players must be turned off.

Do all work on the test paper. ***Show all work***. You may receive no credit, even for a correct answer, if no work is shown. You may use the back if you need extra space. ***Do not*** simplify your answers, unless you are explicitly instructed to do so. This does not apply to evaluation of elementary functions at standard values or functional expressions, so that you **would** be expected to simplify $\sin(\pi/6)$ to $\frac{1}{2}$, for example. Do not write answers as decimal approximations; if $\sqrt{2}$ is the answer, leave it that way. *Except where explicitly stated otherwise, you can use the derivative rules and limit rules learned in class.*

You may **not** use a calculator, computer, the assistance of any other students, any notes, crib sheets, or texts during this exam.

You have 60 minutes to complete this exam.

Do not turn to the next page until you are instructed to do so.

Grading:

1._____/10

2._____/10

3._____/5

4._____/10

5._____/5

6._____/10

7._____/10

8._____/10

9._____/10

10._____/20

Total._____/100

(1) Find the indicated derivatives: Show the steps involved. *(5 points/part)*

(a) $(\ln(\cos(x)))'$

(b) $\frac{d}{dx}(\sinh(\ln(2x)))$

(2) Find the indicated limits: Show the steps involved. *(5 points/part)*

(a) $\lim_{x\to 0} \dfrac{3x - \sin(3x)}{x - x\cos(x)}$

(b) $\lim_{x\to 0^+} (1 + \sinh(2x))^{\frac{1}{x}}$

(3) Suppose that $f(x)$ and $g(x)$ are differentiable functions so that $f(1) = 1$, $f'(1) = 4$, $f(5) = 3$, $f'(5) = 2$, $f(4) = -3$, $f'(4) = 1$, $g(1) = 5$, $g'(1) = 4$, $g(2) = 1$, and $g'(2) = 3$. Find $\frac{d}{dx}\left[f(g(x))\right]|_{x=1}$. *(5 points)*

(4) Show, using the definition of the inverse sine and implicit differentiation, that

$$(\arcsin(x))' = \frac{1}{\sqrt{1-x^2}}.$$

(10 points)

(5) Use linear approx or differentials to find an approximate value of $e^{0.2}$. *(5 points)*

(6) MVT

(a) State the Mean Value Theorem. *(5 points)*

(b) Use the Mean Value Theorem to show that the function

$$f(x) := x^3 - 4x - 3$$

has only one root in $[-1, 1]$. *(5 points)*

(7) The population of Decatur, Texas grows at a rate proportional to its size. It increased by 40% after 3 years. How long does it take for the population to triple? *(10 points)*

(8) A man, walking at night, is walking directly towards a streetlight. The light is 10 feet off the ground, and the man is 6 feet tall and walking at 4 feet per second. When the man is 8 feet from the streetlight, how fast is the length of his shadow changing?*(10 points)*

(9) Find the absolute maximum and absolute minimum of $f(x) = f(x) = 4x^3 + 3x^2 - 6x + 2$ on $[-2, 4]$. *(10 points)*

(10) Let $f(x) = \dfrac{x^2 - 1}{x - 3}$. *(20 points)*

(a) Find the domain of $f(x)$.

(b) Find all x- and y- intercepts.

Continuing with problem (10). Recall that $f(x) = \dfrac{x^2 - 1}{x - 3}$.

(c) Find any horizontal, vertical, or slant asymptotes.

(d) Find on what intervals the curve is increasing and decreasing, and find any critical points and local extrema.

(e) Find on what intervals the curve is concave up, and where it is concave down, and find any points of inflection.

Continuing with problem (10). Recall that $f(x) = \dfrac{x^2 - 1}{x - 3}$.

(f) Then, sketch the curve, showing each of these features.