MATH 21, FALL, 2008, FINAL EXAM

Name:____________________________________.

ID #____________________________________.

Section #____________________________________.

Instructor____________________________________.

TA____________________________________.

Instructions: All cellphones, calculators, computers, translating devices, and music players must be turned off.

Do all work on the test paper. ***Show all work***. You may receive no credit, even for a correct answer, if no work is shown. You may use the back if you need extra space. ***Do not*** simplify your answers, unless you are explicitly instructed to do so. This does not apply to evaluation of elementary functions at standard values or functional expressions, so that you **would** be expected to simplify $\sin(\pi/6)$ to $\frac{1}{2}$, for example. Do not write answers as decimal approximations; if $\sqrt{2}$ is the answer, leave it that way. *Except where explicitly stated otherwise, you can use the derivative rules, limit rules, and integral rules learned in class.*

You may **not** use a calculator, computer, the assistance of any other students, any notes, crib sheets, or texts during this exam.

You have 3 hours to complete this exam.

Do not turn to the next page until you are instructed to do so.

Grading:

1.__________/15	6.__________/10	11.__________/10
2.__________/25	7.__________/10	12.__________/10
3.__________/25	8.__________/10	13.__________/25
4.__________/10	9.__________/10	14.__________/15
5.__________/10	10.__________/15	

Total.__________/200

(1) Find the indicated limits: Show the steps involved. *(5 points/part)*

(a) $\lim_{t\to 1} \dfrac{t^2 - 3t + 2}{t^2 - 1}$

(b) $\lim_{x\to 0^+} \sqrt{x}\ln x$

(c) $\lim_{x\to \infty} x - \dfrac{x^2 + 1}{x + 1}$

(2) Find the indicated derivatives: Show the steps involved. Do not simplify. *(5 points/part)*

(a) $\big((x^2+2x-3)(x^3-5)\big)'$

(b) $\left(\dfrac{x^2+3x-2}{\sqrt{x+2}}\right)'$

(c) $\big(\sin^3 x + \sin(x^3)\big)'$

(d) Assume that $y = f(x)$ satisfies the equation

$$y^5 + 3x^2y^2 + 5x^4 = 12.$$

Find dy/dx in terms of x and y.

(e) $\dfrac{d}{dx}\left(\displaystyle\int_1^{x^2} \frac{1}{t^3+1}\,dt\right)$

(3) Evaluate the following integrals. *(5 points/part)*

(a) $\displaystyle\int (x^3 + 3x^2 - 4x + 2)\,dx$

(b) $\displaystyle\int_0^{\pi/2} \sin(x)\cos(x)\,dx$

(c) $\displaystyle\int_{-1}^{1} \frac{x}{1+x^2}\,dx$

(d) $\int x\sqrt{x-1}\,dx$

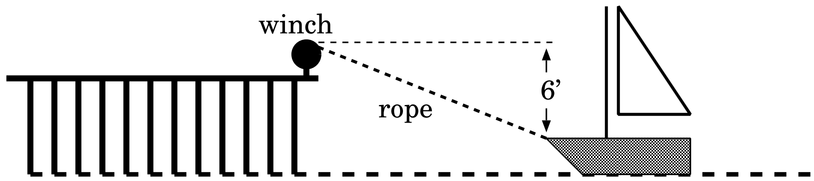
(e) $\int_3^{11} \frac{dx}{\sqrt{2x+3}}$

(4) (a) State the definition of the derivative of a function $f(x)$ (as a limit): $f'(x) =$ *(5 points)*

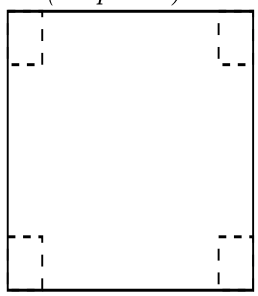
(b) Using only the definition of the derivative, find the derivative function $f'(x)$ for the function $f(x) = \sqrt{x}$. *(5 points)*

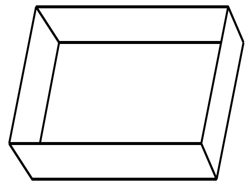
(5) Find $\lim_{n\to\infty} \frac{\pi}{n} \sum_{i=1}^{n} \cos\left(\frac{i\pi}{n}\right)$. [Hint: This limit is an integral. Figure out what function is being integrated, and over what interval.] *(10 points)*

(6) A boater has his boat tied to the pier by a long rope. He feeds the pier-end of the rope through a winch and pulls his boat in towards the pier. The tide is out, so the bow of the boat is 6 feet below the level of the pier. If the winch is pulling in the line at a constant rate of 2 feet per second, how fast is the boat moving towards the pier when there is 10 feet of rope between the boat and the winch? *(10 points)*



(7) Take a 12 inch by 12 inch rectangle of tin, cut equal squares out of each corner and bend up the sides, then solder the seams on the edges to make a rectangular box with no top. How large a square should you cut out of each corner to maximize the volume of the resulting box? *(10 points)*





(8) Find the linear approximation of $f(x) = \sqrt{x+1}$ for x near 0. Your answer should be in the form $g(x) = Ax + B$. *(10 points).*

(9) 100 grams of pure Doublemintium (D_m), a new radioactive substance, was found under the seats of Neville II. After 1 day there was 90 grams left, the rest having decomposed into micronite and tar. Presuming that D_m, like any radioactive substance, decomposes at a rate proportional to the amount present *(10 points)*:

(a) Find a formula for the amount present at time t.

(b) When will half of the original amount be present?

(10) *(5 points/part)*

(a) Approximate the integral $\int_0^4 \left(x^2 - 2x - 3\right)\, dx$ by using a Riemann sum with 4 subintervals, using right-hand endpoints.

(b) Approximate the integral $\int_0^4 \left(x^2 - 2x - 3\right)\, dx$ by using a Riemann sum with an arbitrary number n of subintervals, using right-hand endpoints. Leave your answer as a sum.

(c) State the second part of the Fundamental Theorem of Calculus, and use that to find the exact value of $\int_0^4 \left(x^2 - 2x - 3\right)\, dx$.

(11) Find the integral

$$\int_{-3}^{3} \left(1+\sqrt{9-x^2}\right)\,dx$$

by interpreting it in terms of areas. *(10 points)*

(12) If a particle is moving along a line so that its acceleration $a(t) = t^2 + 3\cos(t)$, with velocity $v(0) = 3$ at time $t = 0$ and position $s(0) = 2$ at time $t = 0$, find $s(t)$. *(10 points)*

(13) (a) Let $g(x) = \int_0^x e^{t^2}\, dt$. State the conclusion of the first part of the Fundamental Theorem of Calculus for this function $g(x)$, and then find $g''(x)$ (note, two derivatives) and all intervals where the graph of $y = g(x)$ is concave up, and all intervals where the graph is concave down. *(15 points)*

(b) Where is the function $G(x) = \int_0^x \frac{(t+1)(t+2)}{t^4+1} dt$ increasing, and where is it decreasing? What is $G(0)$? Is $G(-1)$ positive or negative? *(10 points)*

(14) Find the area of the region bounded by the parabola $y = x^2$ and the line $y = x + 2$. *(15 points)*