MATH 21, FALL, 2008, EXAM # 2

(1) Find the indicated derivatives: Show the steps involved. *(5 points/part)*
  (a) $(\ln(\tan(x) + \sec(x)))'$
  (b) $\frac{d}{dx}\left(\arcsin(x^2)\right)$

(2) Find the indicated limits: Show the steps involved. *(5 points/part)*
  (a) $\lim\limits_{x \to 0} \dfrac{3x - \sin(3x)}{x - x\cos(x)}$
  (b) $\lim\limits_{x \to 0^+} (1 + \sin(2x))^{\frac{1}{x}}$

(3) Suppose that $f(x)$ and $g(x)$ are differentiable functions so that $f(1) = 1$, $f'(1) = 4$, $f(5) = 3$, $f'(5) = 2$, $f(4) = -3$, $f'(4) = 1$, $g(1) = 5$, $g'(1) = 4$, $g(2) = 1$, and $g'(2) = 3$. Find $\frac{d}{dx}\left[f(g(x))\right]\big|_{x=1}$. *(5 points)*

(4) Show, using the definition of the inverse tangent and implicit differentiation, that

$$(\arctan(x))' = \frac{1}{x^2 + 1}.$$

(10 points)

(5) Use linear approx or differentials to find an approximate value of $(7.9)^{\frac{1}{3}}$. *(5 points)*

(6) MVT
  (a) State the Mean Value Theorem. *(5 points)*
  (b) Use the Mean Value Theorem to show that the function

$$f(x) := x^3 + 4x - 5$$

  only has a root at $x = 1$, and nowhere else. *(5 points)*

(7) The population of Decatur, Texas grows at a rate proportional to its size. It increased by 60% after 3 years. How long does it take for the population to triple? *(10 points)*

(8) A 20 foot ladder is leaning against a wall. A painter stands on the top of the ladder, minding his own business. Some fool comes by and ties his dog to the base of the ladder, a cat comes along, and the dog chases after the cat, dragging the base of the ladder with him at a rate of 2 feet per second directly away from the wall. How fast is the painter falling when he is 12 feet from the ground? *(10 points)*

(9) Find the absolute maximum and absolute minimum of $f(x) = \sin(x) + \cos(x)$ on $\left[0, \frac{2\pi}{3}\right]$. *(10 points)*

(10) Let $f(x) = \dfrac{x^2 - 1}{x^2 - 9}$. *(20 points)*
  (a) Find the domain of $f(x)$.
  (b) Find all x- and y- intercepts.
  (c) Find any horizontal or vertical asymptotes.
  (d) Find on what intervals the curve is increasing and decreasing, and find any critical points and local extrema.
  (e) Find on what intervals the curve is concave up, and where it is concave down, and find any points of inflection.
  (f) Then, sketch the curve, showing each of these features.