MATH 21, FALL, 2008, REVIEW FOR EXAM # 2

The material that will be covered on this exam is from section 3.4 (the chain rule) to section 4.5 (curve sketching), inclusive.

You should use this review as an indication of what sorts of problems you should expect on the exam. However, there is no guarantee that if a certain type of problem does not occur on this review, that it will not be on the exam. All topics covered in the text or lecture from the indicated sections are within the scope of the exam.

(1) Find the following derivatives
   (a) $\dfrac{d}{dx}\left(x^{\sin x}\right)$.
   (b) $(\ln(\tan(x)+\sec(x)))'$.
   (c) $(\cosh(2x+3))'$.
   (d) $\dfrac{d}{dx}\left(\sin^{-1}(x^2)\right)$.
   (e) $\left(x^2\ln(x)\right)' =$
   (f) $\left(\tan^{-1}(3x+2)\right)'$.
   (g) $\left((\ln(x)+1)(4x-1)^3\right)'$
   (h) $f(x) = \frac{3x^2-2}{x+3}$. Find $f'(x)$.
   (i) Find $\frac{d}{dx}\left((x^2+2)e^{(x^2+3)}\right)$.
   (j) If $f(x) = \sqrt{\ln(x)}$, find $f'(x)$.
   (k) $\left(e^{2x}\sin(3x)\right)' =$
   (l) $\left(\frac{2x-3}{4x+1}\right)' =$
   (m) $\left((x^2+2x+1)^3(x^2+5x)\right)' =$
   (n) If $f(x) := \dfrac{\left(x^2-2\right)^2}{x^3+2x+3}$, $f'(x) =$
   (o) $\frac{d}{dx}\ln(e^x+1)|_{x=0}$
   (p) $\dfrac{d}{dx}e^{\arctan x}|_{x=1}$
   (q) $\dfrac{d}{dx}\dfrac{\ln|x|}{x}|_{x=-1}$
   (r) $\dfrac{d}{dx}(6^x)|_{x=2}$
   (s) $\dfrac{d}{dx}(\log_3 x)|_{x=7}$

(2) Suppose that $f(x)$ and $g(x)$ are differentiable functions so that $f(1) = 2$, $f'(1) = 3$, $f(4) = 2$, $f'(4) = 3$, $f(5) = -3$, $f'(5) = 1$, $g(1) = 4$, $g'(1) = 5$, $g(2) = 1$, and $g'(2) = 3$. Find $\frac{d}{dx}\left[f(g(x))\right]|_{x=1}$.

(3) Suppose that $f(x)$ and $g(x)$ are differentiable functions so that $f(1) = 1$, $f'(1) = 4$, $f(5) = 3$, $f'(5) = 2$, $f(4) = -3$, $f'(4) = 1$, $g(1) = 5$, $g'(1) = 4$, $g(2) = 1$, and $g'(2) = 3$. Find $\frac{d}{dx}\left[f(g(x))\right]\big|_{x=1}$

(4) Assume that $y = f(x)$ satisfies the equation

$$y^5 + 3x^2y^2 + 5x^4 = 12.$$

Find dy/dx in terms of x and y.

(5) If $x^2 - 3xy + y^2 = 5$, find $\frac{dy}{dx}$ at the point (1,-1). *(10 points).*

(6) Find an equation of the tangent line to the graph of $x^3 - xy - y^2 + 5 = 0$ at $(1, 2)$.

(7) Show, using the definition of the inverse tangent and implicit differentiation, that

$$\left(\tan^{-1}(x)\right)' = \frac{1}{1+x^2}.$$

(8) *(5 points/part)*

(a) Simplify $\tan(\arcsin(x))$, that is, write that expression without using any trigonometric or inverse trigonometric functions.

(b) Use the definition of $\sinh x$ to write $\sinh(\ln(x))$ in terms of algebraic expressions in x.

(c) Use the definition of $\cosh x$ to write $\cosh(\ln(x))$ in terms of algebraic expressions in x.

(d) Find

$$\sin(\tan^{-1}\frac{1}{2})$$

(9) The population of certain bacteria grows at a rate proportional to its size. It increases by 60% after 3 days. How long does it take for the population to double? (12 points)

(10) A new radioactive substance, Doublemintium (D_m), has been found sticking to the undersides of the seats in Packard Auditorium. 70 grams of pure D_m was collected initially. 5 days later only 60 grams of the stuff was still D_m; the rest had decayed into lead and tar. Assuming that the rate of decay of D_m is proportional to the amount present, how much will there be after 20 days?

(11) I bought a cup of coffee at McBurgers. It was far too hot to drink, 90^o Celsius. After 10 minutes, the coffee is at 80^o. The air in McBurger's is kept at an air-conditioned constant of 25^o. How long will I have to wait until the coffee is 70^o and thus cool enough to drink?

(12) A spherical snowball melts in such a way that its volume decreases at the rate of 2 cubic centimeters per minute. At what rate is the radius decreasing when the volume is 400 cubic centimeters?

(13) Egbert is flying his kite. It is 75 feet off the ground, moving horizontally away from Egbert at a rate of 5 feet per second. How fast is Egbert letting out the string when the kite is 100 feet (horizontally) downwind of him?

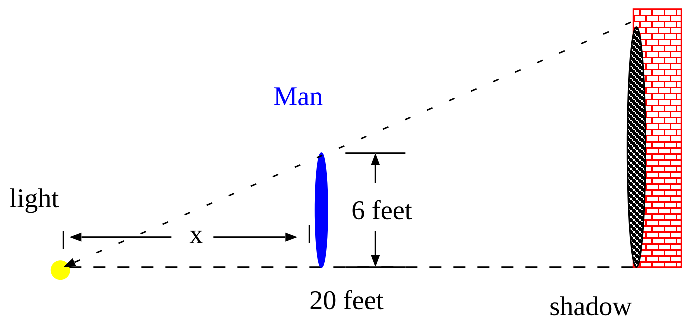
(14) A 20 foot ladder is leaning against a wall. A painter stands on the top of the ladder, minding his own business. Some fool comes by and ties his dog to the base of the ladder, a cat comes along, and the dog chases after the cat, dragging the base of the ladder with him at a rate of 2 feet per second directly away from the wall. How fast is the painter falling when he is 12 feet from the ground?

(15) Egbert is drinking a daquiri (non-alcoholic) out of a glass that is a cone with a radius at the top of 3 inches and a height of 5 inches. He drinks his daquiri at a constant rate of 2

cubic inches per second (through a straw). How fast is the top of the daquiri falling when it is 4 inches above the bottom of the glass?

(16) A man, walking at night, is walking directly towards a streetlight. The light is 10 feet off the ground, and the man is 6 feet tall and walking at 4 feet per second. When the man is 8 feet from the streetlight, how fast is the length of his shadow changing?

(17) The side of a tall building is illuminated by a floodlight mounted on the lawn in front of the building. The floodlight is at ground level, and is 20 feet from the building. A man, 6 feet tall, is walking towards the floodlight at 4 feet/sec. How fast is his shadow on the wall shrinking when he is 10 feet from the floodlight?



(18) Use differentials to approximate

$$\sqrt{24}.$$

(19) Use differentials to approximate $\sqrt{97}$. *(10 points).*

(20) If the radius of a circle is measured to be 12 inches, with an error of $\pm 1/8$ inch, then how much error might there be in the calculation of the area of the circle? (Use differentials). *(10 points)*

(21) Approximate $\sqrt[3]{7.5}$ using differentials.

(22) Find the critical numbers (critical points) of the function $f(x) = 2x^3 + 3x^2 - 12x + 1$.

(23) For the function

$$f(x) = x^2 - 3x + 1,$$

defined on the interval $[0, 2]$, find the maximum value and where it occurs. *(10 points)*

(24) Find the critical numbers of the function $F(x) = x^{4/5}(x - 4)^2$.

(25) Without using a calculator, find the absolute maximum and absolute minimum values of $f(x)$ on the given interval.
   (a) $f(x) = x^2 - 6x + 10$, $[2, 5]$
   (b) $f(x) = (x + 1)/\sqrt{x^2 + 1}$, $[0, 3]$
   (c) $f(x) = \sin x - \cos x$, $[0, \pi]$
   (d) $f(x) = (x - 2)\ln(x - 2)$, $[\frac{7}{3}, 3]$
   (e) $f(x) = x\sqrt{e} - e^x + 2$, $[0, 1]$

(26) Find the critical points of the function $f(x) = 4x^3 + 3x^2 - 6x + 2$.

(27) Find the maximum value of the function

$$f(x) = \begin{cases} x, & \text{if } 0 \le x \le 1 \\ x^2 - 3x + 3, & \text{if } 1 < x \le 2 \end{cases}$$

on the interval $[0, 2]$.

(28) State the Mean Value Theorem. *(5 points)*

(29) Use the Mean Value Theorem to show that the function
$$f(x) := x^3 + 4x - 5$$
only has a root at $x = 1$, and nowhere else. *(5 points)*

(30) Verify that the function $f(x) = x + \ln x$ satisfies the hypotheses of the Mean Value Theorem on the interval $1 \le x \le e$ and find all numbers c which satisfy the conclusion of the Mean Value Theorem.

(31) Suppose $f(x)$ is differentiable every every real number x and $|f'(x)| \le 7$ for all x. Show, using the Mean Value Theorem, that
$$|f(x_2) - f(x_1)| \le 7|x_2 - x_1|$$
for all x_1, x_2. *(12 points)*

(32) If all you know about a (differentiable) function $f(x)$ is that its derivative $f'(x)$ is zero for only two values of x, show that the equation $f(x) = 3$ can have no more than three solutions. *(10 points)*

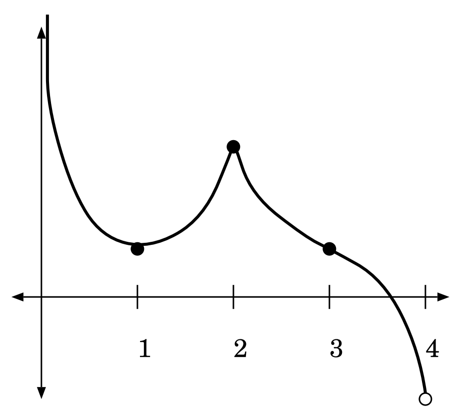
(33) Sketch the graph of the function $f(x) = \frac{1}{4}x^4 - x^3$. Include (with labels) all local extreme points, all absolute extreme points, where the curve is increasing or decreasing, all inflection points and concavity, and all intercepts.

(34) Sketch the graph of the function $f(x) = 3(x^2 - 1)/(x + 2)$, including (with labels) all intercepts, asymptotes (including slant-asymptotes), local extreme points, and where the curve is increasing and decreasing.

(35) Sketch the graph of $f(x) = 2x^3 - 3x^2 + 1$, showing all intercepts, asymptotes, where the curve is increasing or decreasing, any critical points, concavity, and any points of inflection. As a hint, $f(1) = 0$. *(15 points)*

(36) Sketch the graph of the function $f(x) = \dfrac{e^x}{x+1}$ (including everything).

(37) Below is the graph of a function f defined on the open interval $0 < x < 4$. Find
  (a) The critical numbers
  (b) The local extrema
  (c) The absolute extrema
  (d) Where the graph is concave up and where concave down.
  (e) The inflection points.



(38) Let $f(x) = \frac{x^2-4}{x^2-1}$, then
  (a) Find the domain of $f(x)$.

(b) Find all x- and y- intercepts.
(c) Find any horizontal or vertical asymptotes.
(d) Find where the curve is increasing and decreasing, and find any critical points.
(e) Find on what regions the curve is concave up, and where it is concave down, and find any points of inflection.
(f) Then, sketch the curve, showing each of these features. *(20 points)*

(39) Find the following limits: *(5 points/part)*
(a) $\lim_{x\to 0+}(1+2x)^{1/x}$
(b) $\lim_{x\to\infty}\dfrac{x+\sin(3x)}{x-\sin(2x)}$
(c) $\lim_{x\to 0^+} x\ln x.$
(d) $\lim_{x\to 0}\dfrac{1-\cos(3x)}{x^2} =$
(e) $\lim_{x\to 0}\dfrac{\tan(3x)}{\sinh(2x)} =$
(f) $\lim_{x\to\infty}\left(1-\dfrac{1}{x}\right)^x =$
(g) $\lim_{x\to\infty}\dfrac{\sinh x}{3e^x+x^2-1}.$
(h) $\lim_{x\to 0^+} x(\ln x)^2.$
(i) $\lim_{x\to\infty}(1-\dfrac{3}{x})^x$.

(40) Find an equation of the tangent line to the curve $y = e^{x^2}$ at the point where $x = 1$.

(41) Sketch the graphs including the intervals on which the function is increasing/decreasing, the intervals on which the graph is concave upward/downward, the y-intercepts, the x-intercepts, all relative (i.e., local) extreme points, all inflection points, all absolute extreme points, and all asymptotes.
(a) $f(x) = x^4 - 4x^3 + 6x^2$
(b) $f(x) = x(x+1)^4$
(c) $f(x) = (\ln|x|)/x$
(d) $f(x) = e^{2x} + 3x$
(e) $f(x) = \sin x + \cos x.$