MATH 21, FALL, 2008, EXAM # 1

(1) Find the indicated limits: Show the steps involved. *(5 points/part)*
  (a) $\lim_{x\to -4} \dfrac{x^2+5x+4}{x^2+3x-4}$
  (b) $\lim_{x\to\infty} \sqrt{x^2+2x-1} - x$
  (c) $\lim_{y\to\infty} \dfrac{2y^2-5y-3}{5y^2+4y}$

(2) Show that there is a solution of the equation $\cos(x) = x$ in the interval $[0, \pi/2]$. You can presume that the function $\cos(x)$ is continuous. Explicitly note which theorems you are using. *(10 points)*

(3) Find an equation of the tangent line to the curve $y = x^4$ at the point $(2, 16)$.

(4) Derivatives by the definition:
  (a) Write down the definition of the derivative $f'(x)$ of a function $f(x)$ at x. *(5 points)*
  (b) Show, using the *definition* of the derivative as a limit, that $\left(5x^2+3x-2\right)' = 10x+3$. *(10 points)*

(5) Find the following derivatives, using the rules we have discussed in class. *(5 points/part)*
  (a) $(4x^3 - x^2 + 1)''$
  (b) $\left(\dfrac{2x+3}{5x-1}\right)' =$
  (c) $\left(e^x\left(x^3+4x\right)\right)' =$

(6) Let

$$f(x) := \begin{cases} x^2, & \text{if } x \le 2 \\ mx+b, & \text{if } x > 2 \end{cases}.$$

Find the values of m and b for which the function f will be differentiable everywhere. *(10 points)*

(7) Show that

$$\lim_{x\to 3} \left(x^2 + x - 4\right) = 8$$

by using an $\epsilon - \delta$ argument. *(10 points)*

(8) A ball is tossed up in the air so that its height above the ground t seconds after being tossed is $s(t) = -16t^2 + 32t + 5$ feet. *(15 points)*
  (a) How fast was the ball moving at the instant when it was tossed?
  (b) How high was the ball above the ground one second after it was tossed?
  (c) What was its instantaneous velocity at $t = 1$?