

LECTURE 12 : A survey of ab initio based MD

- Finding approximations
- Density functional theory
- Functionals
- Car Parrinello MD

Goals ...

- ❑ We want to study glasses or glass-forming liquids without any prerequisite e.g. models coming from the force field (approximation in classical MD).
- ❑ Charge transfer in covalent systems or in metallic glasses.
q=cst in most classical MD (Coulomb interaction)
- ❑ Description of other (e.g. spectroscopic) properties
- X-ray, Raman, IR, XPS,...

How ?

- ❑ Just solve Schrödinger equation $H\Phi = E\Phi$
- ❑ For large systems, there are basically 2 options
 - **Adiabatic approximation**: e⁻ move faster than nucleus which can be considered as fixed.
 - **Mean field approximation** for the e⁻ -e⁻ correlation (DFT)

A. BASICS

1. Finding approximations

- ❑ We write the Hamiltonian for molecules or solids
- ❑ Contains the interaction and the kinetic energy of N atoms (nuclei of mass M) at positions \mathbf{R}_i , and n electrons of mass m at positions \mathbf{r}_i
- ❑ Can be also writtent under the simpler form:

$$\mathcal{H} = \sum_{i=1}^n \frac{P_i^2}{2m} + \frac{1}{2} \sum_{j,i \neq j}^n \frac{e^2}{|\mathbf{r}_i - \mathbf{r}_j|} - \sum_{i=1}^n \sum_{l=1}^N \frac{Z_l e^2}{|\mathbf{r}_i - \mathbf{R}_l|} + \sum_{l=1}^N \frac{P_l^2}{2M_l} + \frac{1}{2} \sum_{k,l \neq k}^N \frac{Z_k Z_l e^2}{|\mathbf{R}_k - \mathbf{R}_l|}$$

$$\mathcal{H} = T_e(\{p\}) + V_{ee}(\{r\}) + V_{eN}(\{r\}, \{R\}) + T_N(\{P\}) + V_{NN}(\{R\})$$

↑
Electronic kinetic E
↑
Electron-electron potential E
↑
Electron-nucleus potential E
↑
Nuclear kinetic E
↑
Nucleus-nucleus potential E

A. BASICS

1. Finding approximations

□ We then solve Schrödinger equation: $\mathcal{H} \Psi_{(i,l)}(r, R) = E_{(i,l)} \Psi_{(i,l)}(r, R)$

□ **Adiabatic approximation.** Since $M=2000-100.000m_e$, there are two different time scales: electrons and nuclei (ions).

➤ Vibrational excitations: $\hbar\omega \sim 10 \text{ meV}$

➤ Electronic excitations: $\hbar\omega_e \sim 1 \text{ eV}$

➤ $\tau_{\text{ion}} \gg \tau_e$...on the time scale of electrons, ions can be considered as frozen.

□ **Consequence.** We can split the wavefunction into an e-part and a ion-part:

$$\Psi_{(i,l)}(r, R) \equiv \chi_{i,l}(R) \cdot \Phi_e(r, R)$$

$$\left[T_e + V_{ee} + V_{eN}(R) + V_{NN}(R) \right] \Phi_e(r, R) = E_e(R) \Phi_e(r, R)$$

$$\left[T_n + E_e(R) \right] \chi_{i,l}(R) = E_{i,e} \chi_{i,l}(R)$$

A. BASICS

1. Finding approximations

- ❑ Adiabatic approximation, continued. **1)** We solve for a parameter R

$$\left[T_e + V_{ee} + V_{eN}(R) + V_{NN}(R) \right] \Phi_e(r, R) = E_e(R) \Phi_e(r, R)$$

to obtain $\Phi_e(r, R)$ and $E_e(R)$ (**Born-Oppenheimer surface**)

2) Knowing $E_e(R)$, we solve $\left[T_n + E_e(R) \right] \chi_{i,l}(R) = E_{i,e} \chi_{i,l}(R)$

to obtain $\chi_{i,l}(R)$ and $E_{i,e}$

- ❑ **Common approximations:**

- At low temperature, harmonic approximation. $\chi_{i,l}(R)$ only non-zero around

$$E_e(R) \simeq E_e(R_0) + \frac{1}{2!} (R - R_0)^2 \frac{\partial^2 E_l(R)}{\partial R^2} + \dots$$

- At high temperature, nuclei behaves as a classical particle $M \frac{d^2 M}{dt^2} = - \frac{\partial E_e(R)}{\partial R}$

A. BASICS

2. Solutions for the electronic Hamiltonian

- ❑ Reduction of complexity. We want to compute $\Phi(r_1, r_2, \dots, r_{N_{el}})$
- ❑ Discretize space of each variable on a grid of N_G grid points. To define Φ , we need to specify $(N_G)^{N_{el}}$ values. Ugh ! **Many body problem !**
- ❑ Many properties can be reproduced by an independent electron approach. Description of the system with N_{el} single particle wavefunctions Φ_1 having $N_G N_{el}$ values. For total energy of the ground state, the best available method is **density functional theory**
- ❑ Many static or quasi-static can be obtained from derivatives of the total energy

- Forces $-\frac{\partial E_e(\mathbf{R})}{\partial \mathbf{R}}$
- Dynamic matrix (vibrations) $-\frac{\partial^2 E_e(\mathbf{R})}{\partial \mathbf{R}_i \partial \mathbf{R}_j}$
- Polarisation $-\frac{\partial E_e(\mathbf{R})}{\partial \mathbf{E}_{elect}}$

B. DENSITY FUNCTIONAL THEORY

- ❑ **Idea:** Systems with **interacting** electrons in a potential $V(\mathbf{r}) = - \sum_{i=1}^N \frac{Z_i e^2}{|\mathbf{r} - \mathbf{R}_i|}$

can be mapped onto a system with **non-interacting** electrons in a fictitious potential $V'(\mathbf{r}) \neq V(\mathbf{r})$ that has the same total energy and the same electronic density.

- ❑ Definition : *Density is the basic variable.*

- ❑ N interacting electrons in an external potential $V_{\text{ext}}(\mathbf{r})$ in a non degenerate groundstate. (GS) The ground state density is unique (non degenerate):

$$n(\mathbf{r}) = N \int \int \Psi_{GS}^*(\mathbf{r}, \mathbf{r}_2, \mathbf{r}_3, \dots, \mathbf{r}_N) \Psi_{GS}(\mathbf{r}, \mathbf{r}_2, \mathbf{r}_3, \dots, \mathbf{r}_N) d^3 \mathbf{r}_2 \cdot d^3 \mathbf{r}_3 \dots d^3 \mathbf{r}_N$$

- ❑ It can be shown (Hohenberg- Kohn th.) that for the non-interacting electronic system, one has :

$$V'_{\text{ext}}(\mathbf{r}) = V_{\text{ext}}(\mathbf{r}) + \text{const}$$

B. DENSITY FUNCTIONAL THEORY

□ The DFT functional is given by: $F[n] \equiv \langle \Psi_{GS}^{[n]} | T_e + V_{ee} | \Psi_{GS}^{[n]} \rangle$

with: $\int n(\mathbf{r}) d^3 \mathbf{r} = N$

Universal functional (does not depend on a specific $V_{ext}(\mathbf{r})$).

□ Energy of the system is : $E[n] = F[n] + \int n(\mathbf{r}) V_{ext}(\mathbf{r}) d^3 \mathbf{r}$

and minimized by the ground state density $E[n_{GS}] = E_{GS}$

□ **Kohn-Sham approach:**

➤ Given an interacting GS charge density $n_{GS}(\mathbf{r})$ of a N electron system

➤ There exists just one external potential $V_{KS}(\mathbf{r})$ for a non-interacting N-elec. System that has $n_{GS}(\mathbf{r})$ as the ground state charge density.

➤ Defines the Kohn-Sham Hamiltonian $\mathcal{H}_{KS} \equiv \frac{p^2}{2m} + V_{KS}(\mathbf{r})$

B. DENSITY FUNCTIONAL THEORY

□ Given the KS Hamiltonian $\mathcal{H}_{KS} \equiv \frac{p^2}{2m} + V_{KS}(\mathbf{r})$

we find the eigenvector $|\Psi_{KS}^i\rangle$ with eigenvalue ϵ_{KS}^i satisfying

$$n(\mathbf{r}) = 2 \sum_{i=1}^{N/2} \langle \Psi_i^{KS} | r \rangle \langle r | \Psi_i^{KS} \rangle$$

□ The KS exact functional then writes:

$$E[n] = F[n] + \int n(\mathbf{r}) V_{ext}(\mathbf{r}) d^3 \mathbf{r} \quad \text{Ordinary DFT functional}$$

But we define:

$$T_0[n] = \sum_{i=1}^{N/2} 2 \langle \Psi_{KS}^{i[n]} | \frac{p^2}{2m} | \Psi_{KS}^{i[n]} \rangle \quad \leftarrow \begin{array}{l} \text{Kinetic energy of the KS system} \\ \text{Universal functional of } n \end{array}$$

$$E_H[n] = \frac{1}{2} \int \int d^3 \mathbf{r} d^3 \mathbf{r}' \frac{e^2 n(\mathbf{r}) n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \quad \leftarrow \begin{array}{l} \text{Hartree Coulomb} \\ \text{interaction} \\ \text{Classical interaction of} \\ \text{the electrons} \end{array}$$

B. DENSITY FUNCTIONAL THEORY

- By definition, the exchange-correlation (XC) functional is:

$$E_{XC}[n] = F[n] - T_0[n] - E_H[n]$$

- The total energy functional then writes:

$$\begin{aligned} E[n] &= T_0[n] + E_H[n] + E_{XC}[n] + \int n(\mathbf{r})V_{ext}(\mathbf{r})d^3\mathbf{r} \\ &= T_0[n] + E_{KS}[n] \end{aligned}$$

- Note that T_0 is not the kinetic part of the real interacting system but the fictitious non-interacting one.

- By construction, one has for the electronic density:

$$n(\mathbf{r}) = 2 \sum_{i=1}^{N/2} \langle \Psi_i^{KS} | r \rangle \langle r | \Psi_i^{KS} \rangle$$

- **Conclusion:** All the difficult part is hidden in E_{xc}

□ C. EXCHANGE-CORRELATION

□ We want to calculate :
$$E_{XC}[N] = \frac{1}{2} \int n(\mathbf{r}) d^3 \mathbf{r} \int d^3 \mathbf{r}' \frac{n_{XC}(\mathbf{r}, \mathbf{r}' - \mathbf{r})}{|\mathbf{r}' - \mathbf{r}|}$$

with
$$n_{XC}(\mathbf{r}, \mathbf{r}' - \mathbf{r}) \equiv n(\mathbf{r}') \int_0^1 d\lambda [g(\mathbf{r}, \mathbf{r}', \lambda) - 1]$$

Involving the pair correlation function $g(\mathbf{r}, \mathbf{r}', \lambda)$ of a system with density $n(\mathbf{r})$ and electron-electron interaction λ .

- n_{XC} is the **exchange-correlation hole**, a quantum-mechanical zone surrounding every electron in an interacting system that reduces the probability $P(\mathbf{r}, \mathbf{r}')$ of finding other electrons within the immediate vicinity.
- Classically, one would have $P(\mathbf{r}, \mathbf{r}') \sim n(\mathbf{r})n(\mathbf{r}')$ (Hartree-Coulomb energy). But this neglects the possibility of (spin) symmetry (Pauli **exchange interaction**)
- Exact results when $g(\mathbf{r}, \mathbf{r}') \rightarrow 1$ (i.e. at infinity) or for other selected cases

C. EXCHANGE-CORRELATION

❑ Otherwise...approximations

➤ Local density approximation (LDA) $E_{XC}[n] \simeq E_{XC}^{LDA}[n]$

➤ Generalized gradient approximation (GGA) $E_{XC}[n] \simeq E_{XC}^{GGA}[n]$

❑ **Local density approximation:** $E_{XC}^{LDA}[n] \equiv \int d^3\mathbf{r} n(\mathbf{r}) \varepsilon(n(\mathbf{r}))$

assuming that $\varepsilon(n(\mathbf{r}))$ is just a function of the local electronic density.

❑ Advantage: $\varepsilon(n(\mathbf{r}))$ is the exchange and correlation energy per electron of a homogeneous interacting electron gas. Exact for this system.

❑ Allows reproducing shell structure of atoms, bond lengths, vibrational frequencies.

C. EXCHANGE,-CORRELATION

- ❑ **Generalized gradient approximation:** the functional now also depends on the local gradient of $n(\mathbf{r})$.

$$E_{XC}^{GGA}[n] \equiv \int d^3\mathbf{r} f\left(n(\mathbf{r}), \nabla n(\mathbf{r})\right)$$

- ❑ Gradient expansion of the exchange-correlation hole

Perdew et al. PRL 1999

- ❑ Provides better geometries for weak bonds which are too short in LDA. Better total energies (e.g. atomisation).

Solid	a^{LDA}	a^{GGA}	a^{expt}
Na	4.05	4.20	4.23
NaCl	5.47	5.70	5.64
Al	3.98	4.05	4.05
Si	5.40	5.47	5.43
Ge	5.63	5.78	5.66
GaAs	5.61	5.76	5.65
Cu	3.52	3.63	3.60
W	3.14	3.18	3.16
Mean abs. error	0.078	0.051	...

Lattice constants a (in Å) for some solids

C. EXCHANGE,-CORRELATION

□ Generalized gradient approximation:

E_{xc} often break up into exchange (e.g. Becke, B) and correlation functionals (e.g. Lee, Yang and Parr, LYP). Or Perdew-Wang (PW91).

$$\varepsilon_{xc}(\rho(\vec{r})) = \varepsilon_x(\rho(\vec{r})) + \varepsilon_c(\rho(\vec{r}))$$

LDA $\varepsilon_x(\rho(\vec{r})) = -\frac{3}{4} \left(\frac{3\rho(\vec{r})}{\pi} \right)^{1/3}$

Coulomb correlation
Interpolated by **Ceperley et Alder 1980**

$$E_x^{CGA} = E_x^{LDA} - \sum_{\sigma} \int F(s_{\sigma}) \rho_{\sigma}^{4/3}(\vec{r}) d\vec{r} \quad s_{\sigma} = \frac{|\nabla \rho|}{\rho^{4/3}}$$

**Becke & Perdew/Wang:
BPW91**

Becke & Lee/Parr/Yang: BLYP



$$F^B = \frac{\beta s_{\sigma}}{1 + 6\beta s_{\sigma} \text{Sinh}^{-1}(s_{\sigma})}$$

Becke, 1988

C. EXCHANGE-CORRELATION

Perspectives and conclusions:

- ❑ Modelling of such XC functions beyond the scope of the lecture.
- ❑ Generation of E_{XC} approximations is still a large and still rapidly expanding field of research.

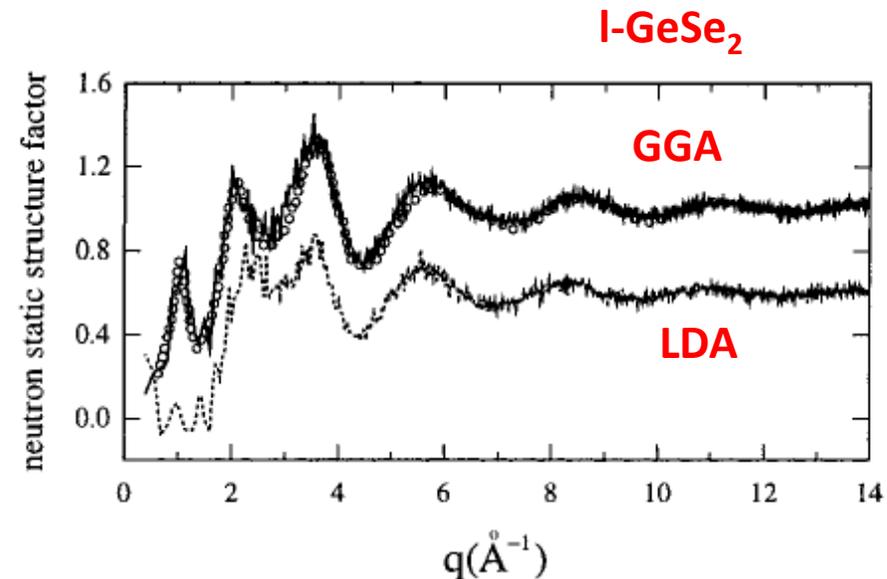


Figure 1. Neutron structure factor $S(q)$ as a function of magnitude of momentum transfer q for liquid GeSe_2 , obtained within GGA (solid line) and LDA (dots), compared to experiment (circles), ref 8. For clarity, the LDA curve is displaced downward by 0.4. We used scattering lengths of $b^{\text{Ge}} = 8.189 \text{ fm}$ and $b^{\text{Se}} = 7.97 \text{ fm}$.

Massobrio et al. JACS 1999

- ❑ Many different flavours of functional available which are more or less appropriate for any particular study.
- ❑ Ultimately the choice E_{XC} of must be made in terms of results (direct comparison with experimental data).

D. IMPLEMENTATION

Plane waves. We are now trying to obtain the ground state (GS) energy and the electronic density.

$$E_{GGA}^{GS} = \min_{\Psi_i^{KS}} \left[T_0[\{\Psi_i^{KS}\}] + E_{KS}^{GGA}[n(\mathbf{r})] \right]$$

$$n(\mathbf{r}) = 2 \sum_{i=1}^{N/2} \langle \Psi_i^{KS} | r \rangle \langle r | \Psi_i^{KS} \rangle$$

- ❑ **In a practical calculation**, one uses a variational approach and expands Ψ_i^{KS} on a finite basis (Hilbert space), e.g. with **atomic orbitals** (most common approach in quantum chemistry):

I ions, n Q numbers

$$|\Psi_i^{KS}\rangle = \sum_I \sum_n C_{In}^i |\Phi_{I,n}\rangle$$

- ❑ **Advantages:** smart basis and small size. Tight-binding approach. Properties are calculated in terms of atomic orbitals.
- ❑ **Disadvantages:** Basis depends on atomic position. No systematic way to increase the basis, delocalized states in an empty region are hard to describe (e- in a crystal vacancy).

D. IMPLEMENTATION

- ❑ **Plane-wave basis set.** Alternative for periodic systems (Bloch's theorem): Ψ_i^{KS} can be written as the product of the lattice periodic part (e.g. reciprocal lattice vectors \mathbf{G}) and a wavelike part.

$$|\Psi_{i,\mathbf{q}}^{KS}\rangle = e^{i\mathbf{q}\cdot\mathbf{r}} \sum_{\mathbf{G}} C_{\mathbf{G}}^{i,\mathbf{q}} |\mathbf{G}\rangle$$

Exact if the basis set is infinite, otherwise fixed by an energy cutoff E_{cutoff} .

- ❑ Advantage: Basis covers all space uniformly, does not depend on position (good for weak bonds), simple and fast evaluation, fixed by a single parameter E_{cutoff} .
- ❑ Disadvantages: To have a converged result with reasonable small E_{cutoff} , one describes only valence electrons. **Need of pseudopotentials.**

Pseudopotentials. Concept related to replacing the effects of the core electrons with an effective potential.

D. IMPLEMENTATION

Pseudopotentials. Concept related to replacing the effects of the core electrons with an effective potential. Just the outer shell electrons participate in the chemical bonding.

- The pseudopotential generation procedure starts with the solution of the atomic problem (all electrons) using the Kohn-Sham approach.
- Once the KS orbitals are obtained, one makes an arbitrary distinction between **valence and core states**.
- The core states are assumed to change very little due to changes in the environment so their effect is replaced by a model potential derived in the atomic configuration and it is assumed to be transferable.
- Outside the core region, the pseudo-wavefunction coincides with the all electron calculation.

Refs. Blöchl, PRB 50, 17953 (1994).

D. IMPLEMENTATION

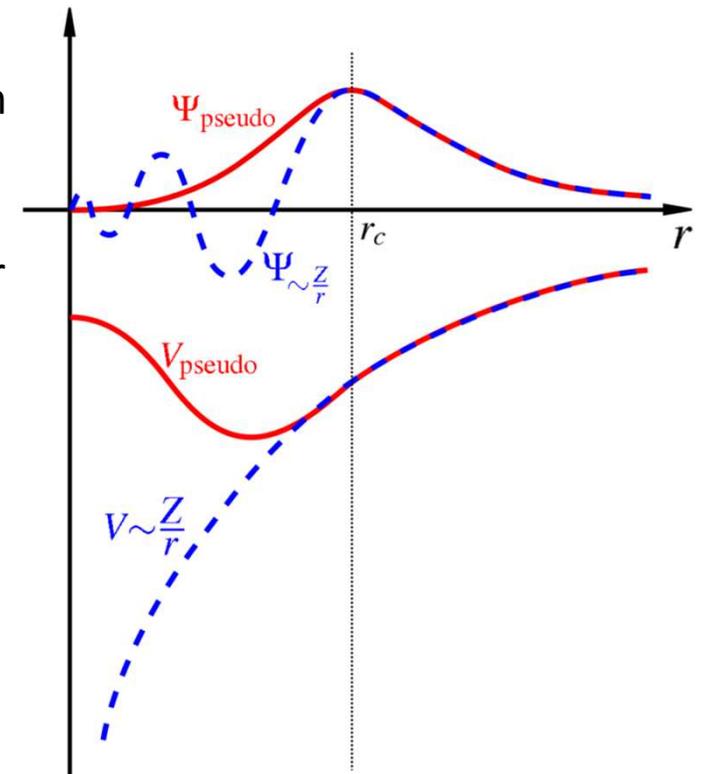
□ General conditions for pseudo-potential generation (Hamann, et al. PRL 43, 1494 (1979)).

1. All-electron and pseudo eigenvalues agree for the reference configuration.
2. AE and PS wavefunctions agree beyond a certain cutoff, r_c .

□ **Two opposing considerations :**

1. Good transferability \Rightarrow small r_c .
2. Large $r_c \Rightarrow$ smoother pseudopotentials.

□ A good pseudopotential is one that strikes a balance between these two constraints.



Unified Approach for Molecular Dynamics and Density-Functional Theory

R. Car

International School for Advanced Studies, Trieste, Italy

and

M. Parrinello

*Dipartimento di Fisica Teorica, Università di Trieste, Trieste, Italy, and
International School for Advanced Studies, Trieste, Italy*

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We present a unified scheme that, by combining molecular dynamics and density-functional theory, profoundly extends the range of both concepts. Our approach extends molecular dynamics beyond the usual pair-potential approximation, thereby making possible the simulation of both covalently bonded and metallic systems. In addition it permits the application of density-functional theory to much larger systems than previously feasible. The new technique is demonstrated by the calculation of some static and dynamic properties of crystalline silicon within a self-consistent pseudopotential framework.

PACS numbers: 71.10.+x, 65.50.+m, 71.45.Gm

Electronic structure calculations based on density-functional (DF) theory¹ and finite-temperature computer simulations based on molecular dynamics² (MD) have greatly contributed to our understanding of condensed-matter systems. MD calculations are able to predict equilibrium and nonequilibrium properties of condensed systems. However, in all practical appli-

very large and/or disordered systems and to the computation of interatomic forces for MD simulations.

We wish to present here a new method that is able to overcome the above difficulties and to achieve the following results: (i) compute ground-state electronic properties of large and/or disordered systems at the level of state-of-the-art electronic structure calcula-

E. MOLECULAR DYNAMICS

- “We present a unified scheme that, by combining molecular dynamics and density-functional theory, profoundly extends the range of both concepts.”

R. Car and M. Parrinello, Phys. Rev. Lett. 55, 2471-2474 (1985).

- **Car-Parrinello Molecular Dynamics (CPMD)**. Idea: Split the dynamics (as expected from Lagrangian) into **ion (classical) and electronic (fictitious) dynamics**. Newton’s equation of motion **for the ions** is solved:

$$M_I \frac{d^2 \mathbf{R}_I}{dt^2} = \mathbf{F}_I$$

the forces being calculated from the electronic density, via:

$$\mathbf{F}_I = - \frac{\partial}{\partial \mathbf{R}_I} V \left(\{ \mathbf{R}_I \} \right)$$
$$V \left(\{ \mathbf{R}_I \} \right) = \min_{\Psi_{KS}} E^{GGA} \left[\{ \Psi_{KS} \}, \{ \mathbf{R} \} \right] = E^{GGA} \left[\{ \Psi_{KS}^{GS} \}, \{ \mathbf{R} \} \right]$$

plus some usual (classical) MD ingredients (Verlet algorithm, time step,...)

E. MOLECULAR DYNAMICS

□ Minimization technique in CPMD

1. For a given set of $\{\mathbf{R}\}$, minimize $E[\Psi_{KS}]$ to obtain Ψ_{KS}^{GS}
2. Compute the forces on the ions from Ψ_{KS}^{GS}
3. Move the ions with the Newton equation

□ Car Parrinello fictitious dynamics:

μ Fictitious electronic mass

$$\mu \frac{d^2 |\Psi_{KS}^i\rangle}{dt^2} = - \frac{\partial}{\partial \langle \Psi^i |} E^{GGA}[\Psi_{KS}, \mathbf{r}]$$

plus

$$M_I \frac{d^2 \mathbf{R}_I}{dt^2} = - \frac{\partial}{\partial \mathbf{R}_I} E^{GGA}[\{\Psi_{KS}\}, \{\mathbf{R}\}] = -2 \sum_{i=1}^{N/2} 2 \langle \Psi_{KS}^i | \frac{\partial V_{ext}}{\partial \mathbf{R}} | \Psi_{KS}^i \rangle$$

□ Conserved energy is:

$$\frac{1}{2} M_I \left| \frac{d\mathbf{R}_I}{dt} \right|^2 + \mu \sum_i \left\langle \frac{d\Psi_{KS}^i}{dt} \middle| \frac{d\Psi_{KS}^i}{dt} \right\rangle + E^{GGA}[\Psi_{KS}, \mathbf{R}]$$

E. MOLECULAR DYNAMICS

Meaning of the fictitious electronic mass μ :

- ❑ "Non-physical" parameter controlling the time scale of the "classical" Car Parrinello electronic dynamics.
- ❑ Close to the minimum Ψ_{KS}^i , Ψ_{KS} oscillates at frequencies

$$\omega_{electron} = \sqrt{\frac{2(E_e^{KS} - E_o^{KS})}{\mu}}$$

with E_e^{KS} and E_o^{KS} the eigenvalues of empty/occupied states, so that a minimum frequency is

$$\omega_{min}^{electron} \simeq \sqrt{\frac{2E_{gap}}{\mu}}$$

- ❑ To be sure of adiabatic separation between artificial electronic and real ionic motion, one must have :

$$\omega^{ions} \ll \omega_{min}^{electron} \simeq \sqrt{\frac{2E_{gap}}{\mu}}$$

In this case, the electronic system remains near the ground state and K_e is small.

E. MOLECULAR DYNAMICS

CPMD simulation of 200 atomic As_2Se_3

$t=10$ a.u. (0.24 fs)

$L=17.98$ Å

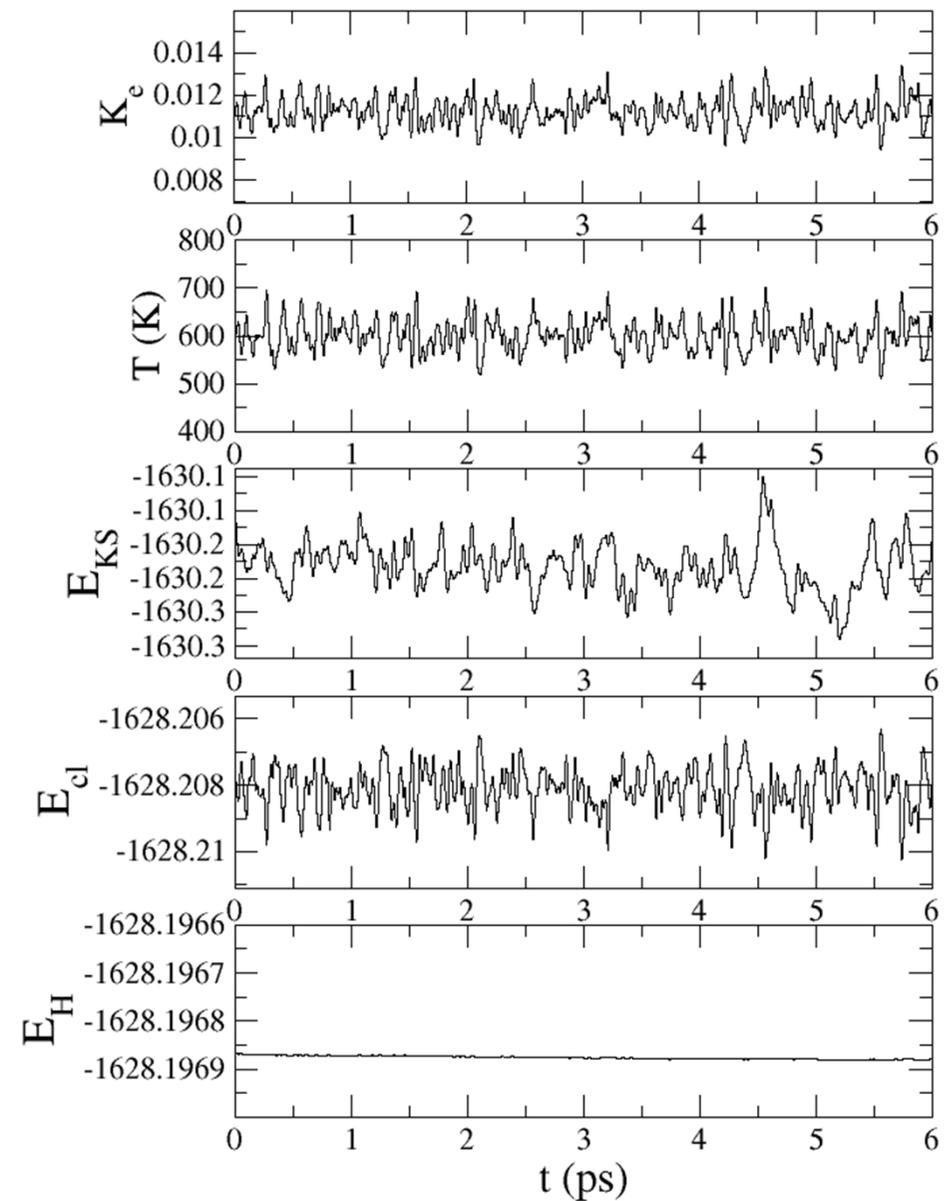
Trouillier-Martins pseudos

$E_{\text{cut}}=20$ Ry

BLYP functional

$m=2000$ a.u.

- Kinetic energy of the electrons K_e remains small.
- Total energy E_H conserved



F. GETTING STARTED

❑ What you need to specify to run a DFT computation

- Basis set
- Coordinates
- Exchange functional: B, B3, etc.
- Correlation functional: LYP, PW91, etc.

❑ Packages

- ❖ CPMD www.cpmc.org
- ❖ ABINIT www.abinit.org
- ❖ VASP <https://www.vasp.at>
- ❖ Fireball <http://fireball-dft.org>

❑ Lectures. Need of a detailed course on quantum chemistry, DFT, and ab initio methods.

- CECAM tutorials www.cecam.org/
- Salanne-Vuilleumier lectures on CPMD- CP2K
<http://www.pecsa.upmc.fr/spip.php?rubrique155>

Conclusion

- ❑ Ab initio methods allow to investigate materials properties using atomic number as only input.
- ❑ Theoretical background provided by density functional theory (DFT)
- ❑ Additional approximations are needed to make DFT useful and applicable
- ❑ The combination of DFT with MD is provided through the Car-Parrinello approach involving the fictitious dynamics of the electron.

Next (last) lecture: Application of ab initio methods to glasses

Home reading: The impact of DFT on Materials research

Useful references:

- D. Marx and J. Hutter, *Ab Initio Molecular Dynamics : Basic Theory and Advanced Methods*. Cambridge University Press, 2009.
- R. Vuilleumier, *Density functional theory based ab initio molecular dynamics using the Car-parrinello approach*, Lect. Notes Phys., 703(1) :223–285, 2006.