Abstract: A graph is edge-transitive (vertex-transitive) if its automorphism group acts transitively on its edges (vertices). Edge-transitive graphs are either vertex-transitive or biregular bipartite. While much research has been done on vertex transitivity, less is known about edge-transitive graphs. We are the first to completely determine all (connected) edge-transitive graphs on 12 through 20 vertices, results which have been added to the Online Encyclopedia of Integer Sequences.

Furthermore we determined with proof infinite classes of edge-transitive graphs, including all graphs of the form $\overline{K_m \times K_n}$; several classes of 3-circulants; and a construction for an edge-transitive and connected biregular (non-trivial) subgraph of $K_{m,n}$ for every pair (m, n) with $gcd(m, n) > 2$. We further prove necessary and sufficient conditions under which edge-transitive (connected) (r, s) biregular subgraphs of $K_{m,n}$ exist for $s = 2$, as well as conditions for uniqueness. These results include a construction for an edge-transitive (connected) $(r, 2)$ biregular subgraph of $K_{m,n}$ for $mr = 2n$ and m even, and a construction for an edge-transitive and connected (non-trivial) biregular subgraph of $K_{4,n}$ where $gcd(4, n) = 2$ and $6|n$, which we show is unique. We also show that there does not exist a non-trivial edge-transitive (connected) biregular subgraph of $K_{4,n}$ when $gcd(4, n) = 2$ and 6 does not divide n. In addition, we investigate necessary and sufficient conditions for edge-transitivity of circulants in general.

We also investigate uniform edge betweenness centrality, a necessary condition for edge-transitivity. Loosely, the betweenness centrality of an edge is the fraction of shortest paths between all pairs of vertices passing through that edge. A graph is said to have uniform edge betweenness centrality if the edge betweenness centrality is the same value for all edges. Edge-transitivity implies uniform edge betweenness centrality. Graphs that have uniform edge betweenness centrality but are not edge-transitive are rare; of the over 11.9 million (connected) graphs on less than or equal to 10 vertices, only 4 have uniform edge betweenness centrality but not edge-transitivity. We have been able to identify infinite classes of circulant graphs that have uniform edge betweenness centrality but are not edge-transitive, including the 2-circulants $C_{18n-3}(1, 6n)$ and $C_{18n+3}(1, 6n)$, $n \in \mathbf{N}$. In doing so, we develop an original method for demonstrating uniform edge betweenness centrality without making explicit centrality calculations.