Regular Functionals on Biparabolic Lie Subalgebras of $\mathfrak{gl}(n)$

Aria Dougherty

2/25/2019

In Lie theory, there is an important algebraic invariant referred to as the index of a Lie algebra $\mathfrak{g}$ with bracket $[\cdot, \cdot]$. This invariant is defined by

$$\min_{F \in \mathfrak{g}^*} \dim \ker B_F = \min_{F \in \mathfrak{g}^*} \dim \{x \mid B_F(x, y) = 0 \ \forall \ y \in \mathfrak{g}\}$$

where B_F is the Kirillov Form, defined by $B_F(x, y) = F[x, y]$. This algebraic invariant is traditionally difficult to compute, but in certain classes of Lie algebras there are combinatoric methods of computing it. However, finding explicit regular functionals (functionals F such that $\dim \ker B_F = \text{ind } \mathfrak{g}$) is still a widely open problem. We solve this problem for all biparabolic subalgebras of $\mathfrak{gl}(n)$ under the commutator bracket.