Math 163 Introductory Seminar - Lehigh University - Spring 2008 - Assignment 4 Solutions Due Monday February 11

11. We can consider perfect matchings in graphs in general, not just bipartite graphs. For this we use Tutte's condition: $odd(G - S) \leq |S|$ for all $S \subseteq V$. Do not worry what odd(G - S) means or what the condition is about for now. It is known that Tutte's condition is necessary and sufficient for a graph G to have a perfect matching.

(a) State a condition equivalent to: Tutte's condition is a necessary condition for G to have a perfect matching.

(b) State the contrapositive of the statement in (a).

(c) State a condition equivalent to: Tutte's condition is a sufficient condition for G to have a perfect matching.

(d) State the contrapositive of the statement in (a).

For (a) and (c) one of your statements should be of the form 'If G has a perfect matching then ...' and the other should be of the form 'If ... then G has a perfect matching.' The '...' should be statements using terms like 'for some $S \subseteq V$ ', 'for all $S \subseteq V$ ', |S|, odd(G-S), \leq and >. For (b) and (c) you should give 'if ... then' statements using terms like those in the previous sentence (and you should not use the word 'not') and 'G does not have a perfect matching'.

(a) If G has a perfect matching then $odd(G - S) \leq |S|$ for all $S \subseteq V$.

(b) If odd(G-S) > |S| for some $S \subseteq V$ then G does not have a perfect matching.

(c) If $odd(G-S) \leq |S|$ for all $S \subseteq V$ then G has a perfect matching.

(d) If G does not have a perfect matching then odd(G-S) > |S| for some $S \subseteq V$.